

DISCUSSION PAPER SERIES

IZA DP No. 12387

**Sufficient Statistics for Frictional Wage
Dispersion and Growth**

Rune Vejlin
Gregory F. Veramendi

MAY 2019

DISCUSSION PAPER SERIES

IZA DP No. 12387

Sufficient Statistics for Frictional Wage Dispersion and Growth

Rune Vejlin

Aarhus University and IZA

Gregory F. Veramendi

Arizona State University

MAY 2019

Any opinions expressed in this paper are those of the author(s) and not those of IZA. Research published in this series may include views on policy, but IZA takes no institutional policy positions. The IZA research network is committed to the IZA Guiding Principles of Research Integrity.

The IZA Institute of Labor Economics is an independent economic research institute that conducts research in labor economics and offers evidence-based policy advice on labor market issues. Supported by the Deutsche Post Foundation, IZA runs the world's largest network of economists, whose research aims to provide answers to the global labor market challenges of our time. Our key objective is to build bridges between academic research, policymakers and society.

IZA Discussion Papers often represent preliminary work and are circulated to encourage discussion. Citation of such a paper should account for its provisional character. A revised version may be available directly from the author.

ISSN: 2365-9793

IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Sufficient Statistics for Frictional Wage Dispersion and Growth*

This paper develops a sufficient statistics approach for estimating the role of search frictions in wage dispersion and lifecycle wage growth. We show how the wage dynamics of displaced workers are directly informative of both for a large class of search models. Specifically, the correlation between pre- and post-displacement wages is informative of frictional wage dispersion. Furthermore, the fraction of displaced workers who suffer a wage loss is informative of frictional wage growth, independent of the job-offer distribution. Applying our methodology to US data, we find that search frictions account for less than 20 percent of wage dispersion. In addition, we estimate an employed job-offer to job-destruction ratio less than one, implying little frictional wage growth. We finish by estimating two versions of a random search model to show how at least two different mechanisms — involuntary job transitions or compensating differentials — can reconcile our results with the job-to-job mobility seen in the data. Regardless of the mechanism, the estimated models show that frictional wage growth accounts for about 15% of lifecycle wage growth.

JEL Classification: E24, J31, J64

Keywords: search models, wage dispersion, wage growth, sufficient statistics, displacement

Corresponding author:

Rune Vejlin
Department of Economics and Business
Aarhus University
Fuglesangs Allé 4
8210 Aarhus V
Denmark
E-mail: rvejlin@econ.au.dk

* Rune Vejlin, Department of Economics and Business, Aarhus University, Denmark, rvejlin@econ.au.dk; Gregory Veramendi, Department of Economics, Arizona State University, gregory.veramendi@asu.edu. We would like to thank Katarína Borovičková, James Heckman, Espen Moen, Dale Mortensen, Robert Shimer, Gianluca Violante, and numerous seminar and conference participants for helpful discussions and comments. We also thank Henry Farber for kindly sharing the earlier CPS-DWS data and Santiago Garcia Couto, Christian Frandsen, and Ernesto Rivera Mora for excellent research assistance. We acknowledge financial support from the Danish Social Sciences Research Council (grant no. DFF 6109-00037). The paper was previously circulated under the title "A Quantitative Assessment of Wage Dispersion and Wage Growth in Search Models".

1 Introduction

Search models are considered foundational models in the macro-economist’s toolbox (Blanchard 2018). These models have been used to help economists understand, among other things, wage inequality (*e.g.* Mortensen 2005), wage growth (*e.g.* Burdett and Mortensen 1998), the allocation of workers to firms (*e.g.* Shimer and Smith 2000), and the Philips Curve (*e.g.* Moscarini and Postel-Vinay 2019). Two core elements of search models are important for understanding the role of search frictions in labor markets. First, how disperse are the job opportunities that workers face? Second, how easily do workers find better job opportunities during an employment spell? In other words, how important are the frictional components of wage dispersion and wage growth. These elements are complementary for many questions that search models seek to answer, but are not easy to identify in typical labor market datasets.

In the spirit of the sufficient statistics literature, we propose two simple statistics that are informative of frictional wage dispersion and frictional wage growth in search models.¹ The statistics have the advantage that they only require information on the wage dynamics of displaced workers and can be estimated from cross-sectional or short panel datasets. The main identifying assumption is that the frictional component of the post-displacement wage is statistically independent of the frictional component of the pre-displacement wage. Inference using these statistics is independent of wage offer distributions and other labor market parameters.

The proposed *wage-dispersion statistic* is the correlation between the pre- and post-displacement wage. We show that the wage-dispersion statistic describes the fraction of the variance of wages accounted for by the between-worker variation in the cross-sectional wage distribution. The remaining variation can be considered an upper bound for the contribution of frictional wage dispersion as it could include variation due to firm-specific human capital, measurement error, compensating wage differentials, *etc.* The wage-dispersion statistic is informative of frictional wage dispersion across most search models in the literature.

The proposed *wage-growth statistic* is the fraction of displaced workers earning lower wages after displacement. In order to make inference, we need to additionally assume, first, that job offers are *iid* draws from a wage offer distribution and, second, workers choose the job with the higher wage. In other words, the pre-displacement wage is the maximum of *iid* draws from a wage offer distribution. Without making assumptions about the offer arrival technology (*e.g.* a Poisson process), we show that the wage-growth statistic can be used to place bounds on the number of job offers workers receive during an employment spell. Assuming a Poisson offer arrival

¹See Chetty (2009) for a survey of the sufficient statistics literature.

technology, the wage-growth statistic can be directly related to the ratio of the rate of job offers while employed to the rate of job separations, often represented in the literature by $\kappa = \lambda^e/\delta$. The ratio κ is a key parameter in search models as it describes labor market competition and wage growth.² As before, wage distributions and other labor market parameters—such as κ —are allowed to be worker-specific and we do not need to make steady-state assumptions.

We estimate the statistics using two U.S. datasets: the Displaced Worker Supplement to the Current Population Survey (CPS-DWS) and the Survey of Income and Program Participation (SIPP). The CPS-DWS is a cross-sectional dataset that identifies workers who were displaced by a plant closure and measures weekly wages. The SIPP measures monthly wages and identifies workers who were displaced by permanent layoffs, slack work conditions, or firm bankruptcy. We view the datasets as complementary, since they survey displaced workers differently (retrospective vs. panel) and record post-displacement wages at different points in time. In both datasets, we select a sample of full-time private sector workers who were displaced and found full-time private sector jobs after displacement. We correct both statistics for measurement error. We find a correlation between pre-displacement and post-displacement wages (the wage-dispersion statistic) of 0.68 and 0.72 in the SIPP and CPS-DWS, respectively. After correcting for measurement error, we infer that frictional wage dispersion accounts for less than 20% of the variance of wages. In addition, we find that 58% of displaced workers earn lower wages after displacement in both the SIPP and CPS-DWS (the wage-growth statistic).³ The estimated wage-growth statistics imply that at least 40% of workers receive zero job offers during an employment spell. Assuming a Poisson offer arrival technology, the wage-growth statistic implies values of κ of 0.76 and 0.82, for the SIPP and CPS-DWS respectively. In other words, workers are more likely to receive a job separation shock than a job offer. We also estimate the statistics by education groups. While the wage-dispersion statistic does not differ much by education, the wage-growth statistic is substantially lower for college graduates. The estimates for college graduates imply that at least 75% of college graduates receive zero job offers during an employment spell and are three times more likely to receive a job separation shock than a job offer.

Our estimates of low relative offer rates may seem to be at odds with the large job-to-job flows observed in the US labor market. We consider two mechanisms that can reconcile our results on frictional wage growth and observed job-to-job transition rates: involuntary job offers

²See *e.g.* [Burdett and Mortensen \(1998\)](#).

³This is not the first paper to find that a significant fraction of workers earn higher wages after displacement. See *e.g.* [Fallick, Haltiwanger, and McEntarfer \(2012\)](#) and [Hyatt and McEntarfer \(2012\)](#).

and compensating differentials.⁴ We show how an involuntary job offer "resets" the frictional wage growth process and we relate our wage growth statistic to these classes of models with minor modifications. A compensating differential model is one where jobs offer a non-pecuniary benefit and workers may change jobs in order to get a higher non-pecuniary benefit even if it means a lower wage. Both extensions can accommodate large job-to-job flows with low frictional wage growth. Both mechanisms highlight that one should be careful inferring frictional wage growth from job-to-job transition rates alone.

We proceed by setting up and estimating two versions of a random search model with two-sided heterogeneity and general human capital. The first version includes involuntary job offers and the second version includes compensating differentials. The goals of structurally estimating the two models are three-fold. First, we show that either of the two extensions can reconcile our results with observed transition rates. Second, we show that inferring job offer rates from observed transition rates is highly model-dependent. In other words, a large set of job offer rates can be consistent with the same observed job transition rate and frictional wage growth rate depending on modelling assumptions. Finally, we use the models to quantify the fraction of total wage growth over the life-cycle explained by frictional wage growth.

Both estimated models match the frictional wage statistics and the observed transition rates in the data. The inferred job offer rates when employed are quite different though. The compensating-differentials model estimates a job offer rate that is three times higher than the model with involuntary job offers. This is expected as workers accept a higher fraction of job offers if their frictional wage growth path is occasionally reset by involuntary job offers. Since workers accept a higher fraction of job offers, a lower job offer rate is needed to match the observed job-to-job transition rate. Regardless of the mechanism, the estimated models show that frictional wage growth accounts for about 15% of lifecycle wage growth and frictional wage dispersion accounts for about 26% of wage dispersion over the first 25 years of the lifecycle.⁵ Finally, we use the estimated model to consider more general models where wages are not an order statistics of the value of a job. We provide evidence that the wage growth statistic is also

⁴An involuntary job offer is one where the outside option for the worker is not staying with the current employer, but to go to unemployment. One can think of this as capturing *e.g.* advance notice shocks.

⁵The estimated models give slightly higher contributions for frictional wage dispersion because they focus on the first 25 years in the labor market, while the statistics reported earlier are calculated for workers across the full life-cycle. The contribution from frictional wage dispersion is expected to decrease over the lifecycle as human capital becomes more important and the variance in the frictional component declines due to on-the-job search (see *e.g.* [Rubinstein and Weiss 2006](#)).

informative of wage growth in sequential-auction models and compensating-differential model.⁶

This paper relates to a large empirical literature studying frictional wage dispersion and frictional wage growth. The papers most related to ours study these topics without estimating a full structural model (*e.g.* without assuming functional forms for the wage offer distribution). One of the most well-known examples is [Hornstein, Krusell, and Violante \(2011\)](#), which use unemployment durations (*i.e.* job offer rates) to infer the importance of frictional wage dispersion. On one hand, they find that statistics of labor-market turnover rates imply very little frictional wage dispersion in a basic search model with no on-the-job search. On the other hand, they find that models with on-the-job search can be consistent with both the turnover rates and important contributions from frictional dispersion. [Alvarez, Borovickova, and Shimer \(2014\)](#) use an estimator related to our wage-dispersion statistic to study heterogeneity in unemployment duration and wages using data on workers who experience two different unemployment spells in administrative data from Austria. Lastly, [Barlevy \(2008\)](#) uses detailed job history data in a novel way to identify κ , which allows him to non-parametrically estimate the offer distribution in the Burdett-Mortensen model.⁷ Finally, our paper is also connected to the empirical literature pioneered by [Abowd, Kramarz, and Margolis \(1999\)](#) (AKM) that estimate fixed effect models with firm and worker fixed effects. The estimated models using the AKM framework are typically used to answer questions about the drivers of wage dispersion through different decompositions of wage variation. Our approach has several advantages compared to the AKM approach, since we do not need to worry about incidental parameters bias for the fixed effects, endogenous mobility, or assuming fixed types over long periods. Thus, the ability of our method to deal with short panels is an important improvement over AKM. Our methodology complements the literature more generally in using wage statistics instead of labor-market turnover and it can be estimated using publicly-available labor-market datasets compared to *e.g.* AKM, which require detailed matched employer-employee datasets.

This paper relates to a parallel literature estimating structural search models to understand the role of frictional wage dispersion. Some of the earliest contributions to this literature are [Van den Berg and Ridder \(1998\)](#) and [Bontemps, Robin, and van den Berg \(1999\)](#). Both papers build on the wage posting model of [Burdett and Mortensen \(1998\)](#) and add firm heterogeneity in productivity. [Van den Berg and Ridder \(1998\)](#) assumes that worker heterogeneity comes from

⁶See *e.g.* [Cahuc, Postel-Vinay, and Robin 2006](#) and [Taber and Vejlin \(2016\)](#). To the best of our knowledge, directed-search models with on-the-job search (*e.g.* [Garibaldi, Moen, and Sommervoll 2016](#)) predict that *all* workers receive weakly lower wages after displacement. Hence, they do not do a good job of reproducing the empirical distribution of wage dynamics of displaced workers that is the focus of this paper.

⁷Another recent example is [Gottfries and Teulings \(2017\)](#), who use variation in job-finding rates and time since last lay-off to separate frictional wage growth from human capital wage growth.

observable differences, while [Bontemps, Robin, and van den Berg \(1999\)](#) assumes that worker heterogeneity arise from differences in worker’s opportunity costs of employment. Both studies find that search frictions play an important role. Specifically, [Van den Berg and Ridder \(1998\)](#) finds that search frictions explain around 20 percent of the variance of wage offers. [Postel-Vinay and Robin \(2002\)](#) were the first to estimate a sequential auction model with unobserved worker and firm heterogeneity. They estimate that search frictions explain around 40-60 percent of cross-sectional wage variation. More recently, [Taber and Vejlin \(2016\)](#) extended the model of [Postel-Vinay and Robin \(2002\)](#) to include human capital, compensating differentials, and multidimensional pre-market skills. They find that search frictions play a minor role in explaining cross-sectional wage variation. [Tjaden and Wellschmied \(2014\)](#) includes an involuntary reallocation shocks in a search model and estimates that around 15 percent of cross-sectional wage dispersion is due to search frictions.

There has also emerged a literature trying to structurally estimate the role of frictions on wage growth. This literature goes back to [Topel and Ward \(1992\)](#). More recently, [Bagger, Fontaine, Postel-Vinay, and Robin \(2014\)](#) extended the model of [Cahuc, Postel-Vinay, and Robin \(2006\)](#) to encompass human capital and decomposed wage growth over the life-cycle into search induced growth and human capital wage growth. They find that the wage-experience profile is explained by both search frictions and human capital, but with search frictions typically playing the main role.⁸

The paper is divided into three parts. In Section 2, we show how our two wage statistics are informative of fundamental parameters in a large class of search models. In Section 3, we estimate these statistics using US data. In Section 4, we estimate two models using our statistics. In Section 5, we conclude the paper.

2 Statistics for Frictional Wage Dispersion and Frictional Wage Growth

In this section, we show how our statistics relate to frictional wage dispersion and frictional wage growth in search models. Before discussing each statistic, we describe the basic environment and the two main identifying assumptions.

Consider an economy populated with heterogeneous workers, where worker heterogeneity is described by discrete types, $i \in \mathcal{I}$. There may be as many types as there are workers. Unemployed

⁸Other papers in this literature include [Yamaguchi \(2010\)](#), [Burdett, Carrillo-Tudela, and Coles \(2011\)](#), [Bowlus and Liu \(2013\)](#), and [Menzio, Telyukova, and Visschers \(2016\)](#).

workers of type i draw job offers from a well-behaved job offer distribution function $F_i(w)$. Let a different well-behaved distribution function, $G_i(w)$, describe the wages of employed workers of type i in the cross-section.

Frictional wage dispersion and frictional wage growth are identified using the wages of displaced workers who experience an employment-unemployment-employment (EUE) transition. Two main assumptions are needed for identification.⁹ The first assumption is that, conditional on worker type, the post-displacement wage is independent of the pre-displacement wage for each worker ($w_i^{pre} \perp\!\!\!\perp w_i^{post}$). The independence assumption is common to most search models in the literature. The second assumption is that displacement by plant closure (or mass layoff) is exogenous ($w_i^{pre} \sim G_i(w)$). While considering plant closures as exogenous layoffs is common in the empirical displacement literature, assuming that these displacements are unrelated to wages is less common. In Section 3.2, we perform a number of empirical tests showing that, conditional on education, the pre-displacement wage distribution of EUE workers "exogenously" displaced (*e.g.* by a plant closure) and the wage distribution of employed workers in the cross-section are quite similar.

2.1 A Statistic for Frictional Wage Dispersion

The wage-dispersion statistic is the correlation between the pre- and post-displacement wage. The correlation measures the persistence of wages across EUE transitions. Intuitively, if frictional wage dispersion is an important component of wage dispersion, then wages will not be as persistent due to the independence assumption.

We first show that the population covariance of pre-displacement and post-displacement wages depends only on the type-specific means of these distributions and is independent of other moments of the wage offer ($F_i(w)$) and pre-displacement wage distributions ($G_i(w)$).

Let $\mu_i = E_{F_i}[w_i^{post}]$ be the conditional mean of the job offer distribution for a worker of type i and $\Delta\mu_i = E_{G_i}[w_i^{pre}] - \mu_i$ be the difference in the conditional means of the $G_i(w)$ and the $F_i(w)$ distributions for a worker of type i . If the post-displacement wage w_i^{post} is statistically independent of the pre-displacement wage w_i^{pre} , then the population covariance is

$$Cov(w^{post}, w^{pre}) = Var(\mu) + Cov(\mu, \Delta\mu). \quad (1)$$

The covariance depends only on the variation in the means of the type-specific distributions

⁹The wage growth statistic requires a third assumption as explained in section 2.2.

and is independent of the shape of the distributions. The derivation of Equation 1 is shown in Appendix Section A.1.

The correlation is then

$$Corr(w^{post}, w^{pre}) = \frac{Var(\mu)}{Var(w)} + \frac{Cov(\mu, \Delta\mu)}{Var(w)}$$

where $Var(w) \equiv \sqrt{Var(w^{pre})Var(w^{post})}$.¹⁰

The first term describes the variance in the means of the offer distributions across worker types (*i.e.* the between-worker variance). It thus captures persistent differences across workers due to human capital and differences in the acceptance sets of jobs (*e.g.* some workers may only work in higher paying jobs). The second term describes the covariance between the mean of the offer distribution and the difference in the means of the pre-displacement and offer distributions. This term will be non-zero if there is a correlation between μ_i and average frictional wage growth. For example, the covariance could be non-zero if workers search with different intensities depending on their type as in Bagger and Lentz (2018). In Section 3.4, we find frictional wage growth to be empirically small and hence the second term will be small relative to the first term.

2.2 A Statistic for Frictional Wage Growth

Frictional wage growth in search models occurs through workers searching on the job. Once workers accept their first job out of unemployment, they continue to search for better jobs while they are employed. The process of finding better jobs is often called climbing the job ladder. We say that a worker exhibits frictional wage growth if their wage grows due to on-the-job search. In this section, we show how the wage-growth statistic, the fraction of displaced workers earning lower wages after unemployment, is informative of three different measures of frictional wage growth, independent of the job offer distribution. First, without making any assumptions about the offer arrival technology, we can use the wage-growth statistic to place bounds (*i.e.* derive an interval estimator) on the number of voluntary job offers a worker received during their last employment spell. This allows us to place a lower bound on the fraction of workers that experienced no frictional wage growth during their last employment spell. Second, assuming that workers receive job offers and experience separations via Poisson arrival rates, we derive an analytical expression for the wage-growth statistic as a univariate function of κ , the ratio of the voluntary job offer rate to the separation rate (commonly denoted by $\frac{\lambda^e}{\delta}$). Finally, we derive

¹⁰We define $Var(w)$ in this way since we find empirically that $Var(w^{CS}) \approx Var(w^F) \approx Var(w^G)$, where $Var(w^{CS})$ is the variance of wages in the cross-section and (w^F, w^G) are measured in a sample of workers who are displaced due to a plant closure. (see section 3.2).

a new measure of wage growth that captures the extent to which workers climb the job ladder and show how it relates to the wage-growth statistic.

Assume that both employed and unemployed workers of type i draw *iid* job offers from the job offer distribution, $F_i(w)$. For the main derivations, we assume that wages are an order statistic of the value of the job. In other words, workers accept any job that offers a higher wage than their current wage.¹¹

Consider workers of type i who drew n independent job offers during their last employment spell. The distribution of pre-displacement wages is just the maximum of those n draws. The pre-displacement wage distribution given n job offers is

$$G_i(w|n) = F_i(w)^n.$$

The fraction of workers suffering a wage loss after displacement is then

$$\begin{aligned} Pr_i(w_i^{post} < w_i^{pre} | n) &= \int F_i(w) dG_i(w|n) \\ &= \int F_i(w) [nF_i(w)^{n-1} dF_i(w)] \\ &= n \int F_i(w)^n dF_i(w) \\ &= n \int_0^1 x^n dx \\ &= \frac{n}{n+1}. \end{aligned} \tag{2}$$

Note that n includes the first job offer that began the previous employment spell. If a worker does not receive any additional job offers during the previous employment spell, then the probability that they will draw a lower wage after displacement is $1/2$. Likewise, if a worker receives one offer during their last employment spell, then the probability that they draw a lower wage is $2/3$. Importantly, notice that the fraction of workers earning lower wages after displacement depends only on the number of job offers they receive (n) and is independent of the wage offer distribution, $F_i(w)$. This is because the probability $Pr_i(w_i^{post} < w_i^{pre})$ depends only on the order statistic of w_i^{pre} and not the actual value of the wage.

Without making additional assumptions, we can use the observed fraction of displaced workers earning lower wages to place bounds on the fraction of workers who received n offers during

¹¹In section 4.4 we consider classes of models that relax this assumption.

their last employment spell. The fraction of displaced workers earning lower wages can be written as a weighted sum

$$Pr_i(w_i^{post} < w_i^{pre}) = \sum_{n=1}^{\infty} \frac{n}{n+1} Pr_i(n), \quad (3)$$

where $Pr_i(n)$ represents the probability that a worker received n offers during an employment spell and must satisfy both $Pr_i(n) \geq 0$ and $\sum_{n=1}^{\infty} Pr_i(n) = 1$. $Pr_i(n)$ will depend on the details of how workers receive job offers and is determined by the offer arrival technology, which we will not put any restrictions on yet. While we cannot derive a point estimator of $Pr_i(n)$ without further assumptions, we can use equation 3 to place bounds on $Pr_i(n)$. For example, if the estimated wage-growth statistic is less than $2/3$,¹² the proportion of workers who received n offers during their last employment spell is

$$Pr_i(n) \in \begin{cases} [4 - 6\hat{Pr}_i, 2 - 2\hat{Pr}_i] & \text{for } n = 1, \hat{Pr}_i \leq 2/3 \\ [0, \frac{n+1}{n-1}(2\hat{Pr}_i - 1)] & \text{for } n > 1, \hat{Pr}_i \leq 2/3, \end{cases} \quad (4)$$

where $\hat{Pr}_i \equiv \hat{Pr}_i(w_i^{post} < w_i^{pre})$ is the estimated wage-growth statistic.¹³ These bounds can be used to place empirical limits for different types of models. For example, $4 - 6\hat{Pr}_i$ describes a lower bound for the fraction of workers who experience no frictional wage growth in the previous employment spell. These bounds can easily be extended to a labor market with involuntary job offers, where n is then the number of voluntary job offers since the most recent unemployment spell *or* involuntary job offer.

In the next step, we additionally assume a Poisson offer arrival technology. Specifically, we assume that workers receive job offers while employed at a Poisson rate of λ_i^e and lose their jobs at a Poisson rate δ_i . These two parameters determine the probability distribution of the number of job offers a worker receives during an employment spell. Specifically, the probability

¹²Different bounds can be calculated if $\hat{Pr}_i(w_i^{post} < w_i^{pre}) > 2/3$, but we do not find evidence for that empirically.

¹³The bounds are calculated by setting the remaining probability to load entirely on extreme values of n . For example, the lower bound for $Pr_i(1)$ occurs when $Pr_i(2) = 1 - Pr_i(1)$ and $Pr_i(m) = 0 \quad \forall m > 2$. The lower bound can then be found by solving for $Pr_i(1)$ using equation 3: $\hat{Pr}_i = \frac{1}{2}Pr_i(1) + \frac{2}{3}[1 - Pr_i(1)]$. Likewise, the upper bound can be found by solving $\hat{Pr}_i = \frac{1}{2}Pr_i(1) + 1[1 - Pr_i(1)]$.

of receiving $n - 1$ additional job offers before a separation shock is¹⁴

$$Pr_i(n) = \left(\frac{\lambda_i^e}{\lambda_i^e + \delta_i} \right)^{n-1} \frac{\delta_i}{\lambda_i^e + \delta_i}.$$

The relationship between the wage growth statistic and the Poisson parameters is then

$$\begin{aligned} Pr_i(w_i^{post} < w_i^{pre}) &= \frac{\delta_i}{\lambda_i^e + \delta_i} \sum_{n=1}^{\infty} \frac{n}{n+1} \left(\frac{\lambda_i^e}{\lambda_i^e + \delta_i} \right)^{n-1} \\ &= 1 - \frac{(\kappa_i + 1) \ln(\kappa_i + 1) - \kappa_i}{\kappa_i^2}, \end{aligned} \quad (5)$$

where $\kappa_i = \lambda_i^e / \delta_i$. The derivation of Equation 5 is found in Appendix Section A.2.¹⁵

The one-to-one relationship between the wage-growth statistic and κ_i is depicted in Figure 1a. As $\kappa_i \rightarrow \infty$ (e.g. $\lambda_i^e \rightarrow \infty$), $Pr_i(w_i^{post} < w_i^{pre}) \rightarrow 1$. As the rate of on-the-job offers increases relative to the job destruction rate, workers, on average, climb further up the job ladder before suffering displacement. The probability that they then earn a lower wage in their first draw from $F_i(w)$ becomes very high. Likewise as $\kappa_i \rightarrow 0$ (e.g. $\lambda_i^e \rightarrow 0$), $Pr_i(w_i^{post} < w_i^{pre}) \rightarrow 1/2$. In other words, as workers become more likely to get a job destruction shock relative to a job offer in their first job, the probability goes to 1/2.

Notice, that any population measure of the probability, $Pr(w^{post} < w^{pre})$, is going to be a weighted sum over the type probabilities, $\sum_i \pi_i Pr_i(w_i^{post} < w_i^{pre})$, where π_i is the fraction of workers of type i .

Our result can easily be extended to the case where workers receive involuntary job offers. An involuntary job offer is a job offer that employed workers must accept or go into unemployment. These have in recent years become common in empirical search models, see e.g. [Bagger and Lentz 2018](#) or [Taber and Vejlin 2016](#). Let λ_i^d represent the Poisson rate of involuntary job offers. The derivation is the same as before, except that we need to calculate the probability of receiving $n - 1$ additional job offers before receiving a job separation or an involuntary job offer,

¹⁴From the mathematics literature on Poisson processes (e.g. [Gallager 2013](#)), the probability that the k^{th} arrival of process 1 occurs before the j^{th} arrival of process 2 is

$$Pr(S_k^1 < S_j^2) = \sum_{i=k}^{k+j-1} \binom{k+j-1}{i} \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} \right)^i \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{k+j-1-i}.$$

¹⁵We show that the result can also be derived using [Burdett and Mortensen \(1998\)](#) steady-state accounting arguments in Section A.3.

$$Pr_i(n) = \left(\frac{\lambda_i^e}{\lambda_i^e + \delta_i + \lambda_i^d} \right)^{n-1} \frac{\delta_i + \lambda_i^d}{\lambda_i^e + \delta_i + \lambda_i^d}.$$

The relationship between the wage growth statistic and the Poisson parameters in a model with involuntary job offers is

$$Pr_i(w_i^{post} < w_i^{pre}) = 1 - \frac{(1 + \tilde{\kappa}_i) \ln(1 + \tilde{\kappa}_i) - \tilde{\kappa}_i}{\tilde{\kappa}_i^2},$$

where $\tilde{\kappa}_i = \frac{\lambda_i^e}{\delta_i + \lambda_i^d}$. In terms of the job ladder, an involuntary job offer functions in a similar way to a job destruction shock, in that workers lose their search capital.

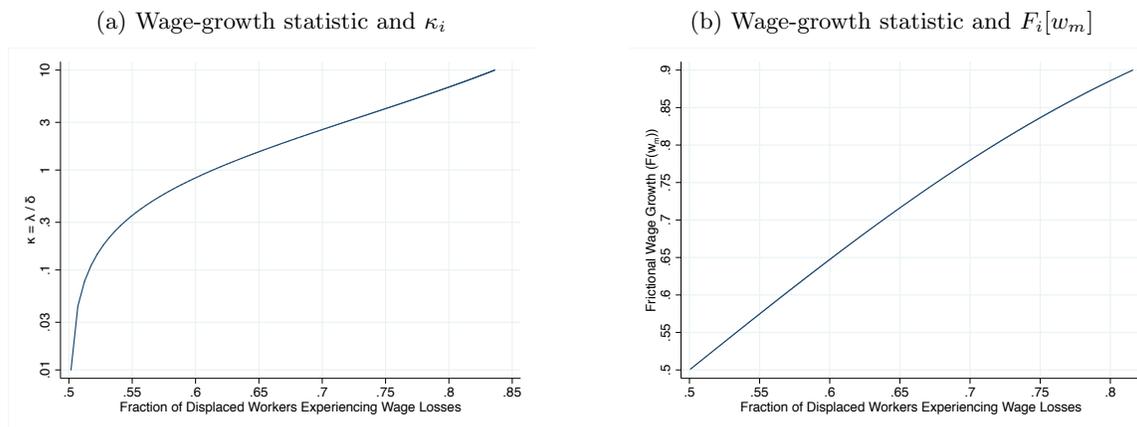
We end this section by showing how our wage-growth statistic can be related to a direct measure of frictional wage growth. One way to measure the extent to which workers climb the job ladder is to calculate the percentile of the wage offer distribution of the median wage of employed workers (*i.e.* $F_i[G_i^{-1}(0.5)]$). If there is no wage growth, then $F_i[G_i^{-1}(0.5)] = 0.5$. This is a measure of wage growth that describes the distance between the wage offer ($F_i[w]$) and wage earned distributions ($G_i[w]$), but does not depend on the shape of the wage distributions. This measure of wage growth is a simple function of κ_i ,¹⁶

$$F_i[G_i^{-1}(0.5)] = \frac{\kappa_i + 1}{\kappa_i + 2}.$$

Figure 1b shows the relationship between our wage growth statistic and $F_i[w_m]$, where w_m is the median wage of employed workers ($w_m = G_i^{-1}[0.5]$). There is a nearly linear relationship between the wage growth statistic and $F(w_m)$. Note, that without further assumptions, $F_i[w_m]$ cannot be measured directly in the data as the function $F_i[\cdot]$ is not known. This wage-growth measure will be useful when evaluating our wage-growth statistic in models where the wage is not an order statistic of the value of the job (see section 4.4).

¹⁶See Appendix section A.4 for the derivation.

Figure 1: Relationship between fraction of workers earning lower wages after displacement and κ



3 Quantitative Implications for Frictional Wage Growth and Wage Dispersion

In this section we present estimates of the two statistics discussed. We start by describing the two datasets in section 3.2. Section 3.3 discusses the effect of measurement error and derives simple corrections for both statistics. Finally, section 3.4 presents and discusses the estimated statistics and their implications.

3.1 Data

We estimate the frictional wage dispersion and frictional wage growth statistics using two US surveys: the Displaced Worker Supplement to the Current Population Survey (CPS-DWS) and the Survey of Income and Program Participation (SIPP). We repeat the analysis in two different datasets for a number of reasons. First, each dataset has its strengths and weaknesses. The SIPP records wages each month, but does not ask about plant closures. The CPS-DWS identifies plant closures (our preferred characterization of exogenously displaced workers), but measures post-displacement wages up to three years after starting the post-displacement job. A large delay in the measurement of wages is clearly not ideal, since we want to measure the wage immediately after finding the first job. Second, using both surveys we are able to show that our estimates are robust to survey design, time period covered, definition of displaced workers, and when and how wages are measured. The goal is to define displacements as involuntary exogenous separations based on the operating decisions of the employer, such as firm/plant closings and permanent layoffs. Other types of separations—*e.g.* due to quits or being fired with cause—are not included,

since these are endogenous to *e.g.* individual wages. The empirical analysis studies prime-aged (25-54 years old), full-time (at least 35 hours/week), private sector workers who are not working in agriculture or construction. All earnings are in 2010 US dollars (deflated by the CPI).

Description of SIPP The Survey of Income and Program Participation (SIPP) is a continuous series of short panels.¹⁷ Our analysis includes the 1996, 2001, 2004, and 2008 panels.¹⁸ The duration of each panel ranges from three to four years. Each individual is surveyed once every four months and is asked about their employment in each month during the previous four months. In particular, if they leave a job they are asked for the reason. In the SIPP, we characterize a displaced worker as a worker who left an employer (and did not return) for one of three reasons: "Layoff," "Employer bankrupt," and "Slack work or business conditions." Unfortunately, workers are not asked about plant closings and separations due to a bankrupt employer are a rare occurrence.

We construct the sample of displaced workers in SIPP via the following steps. The main displaced worker sample consists of prime-aged, full-time, private-sector workers who were displaced at least one year before the last wave of the panel. This is the job-unemployment or "JU" sample. In order to avoid using the earnings for a month in which the worker was not fully employed, we use the last reported monthly wage earnings in the wave previous to displacement as the pre-displacement wage.¹⁹ The job-unemployment-job or "JUJ" sample additionally selects displaced workers who find a full-time, private-sector job within a year of displacement. We choose one year because we want avoid considerations about human capital changing during the unemployment spell. The post-displacement wage is the first reported monthly wage earnings of the post-displacement job. In addition, we construct a cross-section of prime-aged, full-time, private-sector workers that can be compared to the pre-displacement workers in order to investigate the representativeness of displaced workers. This is the cross-section or "CS" sample. For each individual in the "CS" sample, we flag months when they were prime-aged and employed in a full-time private-sector job. Then for each individual, we randomly select a flagged month and record the monthly earnings. We only include observations at least one year before the last

¹⁷We use the Center for Economic and Policy Research SIPP Uniform Extracts: *Center for Economic and Policy Research. 2014. SIPP Uniform Extracts, Version 2.1.7. Washington, DC.*

¹⁸Earlier panels did not ask detailed questions about separations, so we can not identify displaced workers in those datasets.

¹⁹We winsorize monthly income between \$640 and \$14,500 in 2010 US dollars (deflated by the CPI). The upper limit drops top-coded earnings as recommended by the Center for Economic and Policy Research. Our results are robust to using the panel-specific top-codes (\$12,500 in nominal dollars for 1996 and 2001; and \$16,666 in nominal dollars for 2004 and 2008). The wage-growth statistic estimates are also robust to including individuals who are top-coded in one, but not both wage measurements (the case when wage comparisons can still be made).

wave of the panel to match the selection of the displacement sample. Descriptive statistics of the SIPP sample are reported in appendix table 9.

Description of CPS-DWS The Displaced Workers Survey (CPS-DWS) is a biennial supplement to the CPS taken during the January or February data collection.²⁰ Our CPS-DWS sample includes all surveys taken between 1984 and 2016. The CPS-DWS asks respondents if they had experienced a displacement in the last 3 years and what year they lost their job.²¹ The data have information on worker demographics; occupation, industry, and weekly earnings at the pre-displacement job; weeks without work after displacement; and occupation, industry, and weekly earnings at the current job.²² We also use data from the outgoing rotation group (CPS-ORG) supplement from January two years before each CPS-DWS survey in order to make comparisons between the displaced worker’s pre-displacement job and the jobs of workers from the cross-section. The Outgoing Rotation Group (CPS-ORG)—also called the earner study—asks about usual weekly hours and earnings in one fourth of the households surveyed each month.²³

We restrict the CPS-DWS sample to workers who were displaced from a full-time private-sector job by a plant closing. We keep the workers who report being displaced “two years” before the survey to minimize both selection bias and post-displacement wage growth bias.²⁴ This is the “JU” sample for the CPS-DWS. We then require that they are reemployed at the survey date at a different full-time private-sector job (*i.e.* they were not recalled). This is the “JUI” sample for the CPS-DWS. We restrict our analysis to full-time jobs because the CPS-DWS only

²⁰The CPS-DWS was given in February between 1994 and 2000 and was given in January all other years.

²¹For example, the 2016 supplement asked, “During the last 3 calendar years, that is, January 2013 through December 2015, did (name/you) lose a job, or leave one because: (your/his/her) plant or company closed or moved, (your/his/her) position or shift was abolished, insufficient work or another similar reason?” Before 1994, the CPS-DWS asked about layoffs in the last 5 years. To keep the sample consistent, we drop observations reporting layoffs more than 3 years in the past. If the worker experienced more than one layoff in the past three years, they ask about the job that the respondent held the longest.

²²We winsorize weekly income between \$160 and \$2,600 in 2010 US dollars (deflated by the CPI). The upper limit drops top-coded earnings in a similar way to the SIPP sample selection. Our results are robust to using the wave-specific top-codes (\$1,923 in nominal dollars before 1997; and \$2884.61 in nominal dollars afterwards). The wage-growth statistic estimates are also robust to including individuals who are top-coded in one, but not both wage measurements (the case when wage comparisons can still be made).

²³Households in the CPS are surveyed for four successive months, not surveyed for eight months, and then surveyed again for four successive months. Households are in the CPS-ORG in their fourth and eighth interview (fourth and sixteenth month).

²⁴Workers are asked if they last worked at the lost job either “last year,” “two years ago,” or “three years ago.” We found that workers who reported “last year” and had a job at the time of the interview are strongly selected compared to workers who report losing their job “two” or “three” years ago. The “last year” workers had shorter nonemployment durations and higher wages in the post-displacement job. We do not include the workers who report being displaced three years ago as they have been working at least a year longer and have higher post-displacement wages due to post-displacement wage growth. Estimated statistics for the “last year” and “three years ago” samples are reported in the appendix.

provides “usual” weekly earnings and the full/part time status of the worker’s old job before 1994, hence it is difficult to control for hours of work beyond requiring full-time status in a consistent way across the years. Finally, we use the CPS-ORG sample to construct a “CS” sample that can be compared to the CPS-DWS’s “JU” sample. To each CPS-DWS survey, we append the CPS-ORG sample from exactly two years earlier. The CPS CS sample includes data from 1996-2014.²⁵ Descriptive statistics of the sample are reported in appendix table 10.

3.2 Robustness of Identifying Assumptions

Representativeness of Displaced Workers One important threat to our identification strategy is that workers with certain wages may be more likely to be displaced. For example, workers at a low-paying plant may have a higher risk of displacement. Another example is that low quality worker-firm matches may be more sensitive to productivity shocks that lead to separations. These mechanisms will bias the wage-growth statistic downwards and also make the wage-dispersion statistic difficult to interpret. It is well-known in the displacement literature that wages at closing plants are low.

Table 1 shows a series of regressions comparing the pre-displacement wages of displaced workers to the wages of CS workers in both the CPS and the SIPP. In both the SIPP and CPS, the pre-displacement wages of displaced workers are significantly below the wages of the cross section of workers (column 1). Specifically, we find that pre-displacement wages are seven to nine log points less than the average worker in the US economy.

While our results are consistent with the displacement literature, we find that the differences in wages vary quite a bit if we separate the workers who find a job within a year and those that do not.²⁶ In columns 2-3 in table 1, we separate displaced workers who do not find a full-time, private-sector job within a year (JU) and those that did (JUU). In the SIPP in panel A, the JUU workers are much less selected compared to the CS workers (column 2) and the difference is not statistically significant. Once we control for four education indicators (column 3), the JUU-CS difference becomes even smaller. The similarity between JUU and the CS samples is robust to

²⁵The goal of constructing the CS sample is to be able to compare, along multiple dimensions, the displaced workers to workers in the cross section. One important dimension is job tenure. We do not include earlier years in the CS Sample as the Job Tenure Supplement was not given before 1996.

²⁶The requirement that workers find a job *within one year* does not appear to be important. The SIPP panels are too short to follow workers for much more than a year after displacement. We can compare workers in the CPS-DWS who were displaced “two years ago” vs. “three years ago.” Surprisingly we find that both the fraction of workers who find a job and the pre-displacement wage is almost identical in the two samples. For example, 52% of the “two years ago” sample and 54% of the “three years ago” had a full-time private-sector job at the time of the interview, even though the “three years ago” workers had an additional year to find a job. Only 42% of “last year” workers had a job at the time of the interview.

Table 1: Comparing Pre-displacement Wages of Displaced Workers to the Cross Section

Panel A: Log Wage Regressions						
	SIPP			CPS		
	(1)	(2)	(3)	(1)	(2)	(3)
Displaced	-0.0736*** (0.00937)			-0.0788*** (0.0125)		
JUJ		-0.0136 (0.0142)	0.0019 (0.0127)		-0.0076 (0.0170)	-0.0037 (0.0151)
JU		-0.117*** (0.0122)	-0.0703*** (0.0109)		-0.161*** (0.0181)	-0.103*** (0.0161)
Year FE	X	X	X	X	X	X
Education			X			X
R^2	0.005	0.005	0.207	0.002	0.003	0.212
Observations	103093	103093	103093	59999	59999	59999
Panel B: Kolmogorov-Smirnov Tests of Equality of JUJ and CS Wage Distributions						
	All	High School Dropout	High School Graduate	Some College	College Graduate	
SIPP p -value	0.047	0.128	0.146	0.405	0.468	
SIPP Observations	100554	9461	30941	34872	25280	
CPS p -value	0.350	0.429	0.109	0.278	0.795	
CPS Observations	59120	4639	18792	17454	18235	

Note: Standard errors are in parenthesis. Earnings of displaced workers are compared to a cross-section (CS) of workers. Earnings are measured as log of weekly (CPS-DWS) or monthly (SIPP) wage earnings. Samples consist of prime aged (25-54 years old), full-time, private-sector workers who are not working in agriculture or construction. Displaced is an indicator for workers who were displaced from their job (SIPP: layoff, employer bankruptcy, or slack work conditions; CPS: plant closing). JUJ is an indicator for displaced workers who found a full-time private-sector job within a year and JU is an indicator for displaced workers who did not find a full-time private-sector job within a year. Education includes four education indicators. CPS Data combines pre-displacement weekly earnings of workers from the 1998-2016 Displaced Worker Surveys (CPS-DWS) with weekly earnings of a cross-section of workers who were in the Outgoing Rotation Groups (CPS-ORG) two years before each Displaced Worker Survey. SIPP combines pre-displacement earnings of displaced workers with one randomly chosen wage observation for each cross-section worker from the 1996, 2001, 2004, and 2008 panels. See section 3.2 and appendix section C for more details. *** $p < 0.001$

adding additional controls, (*e.g.* occupation and experience). Likewise, we find similar results in the CPS. Thus, the wage differences of displaced workers are being driven by workers who do not find full-time private-sector jobs within a year after displacement and are not in the sample we use to calculate the statistics.

We interpret these findings on pre-displacement wages as indicating that there are two broad types of displaced workers. The first type have marketable skills and were paid similar wages to other workers in the economy. The first type finds a new job quickly (*i.e.* less than a year). The second type of displaced worker is employed using obsolete/unmarketable skills. The pre-displacement firm can pay low wages to these workers and not worry about losing them to other firms. Once they are displaced they have a difficult time finding a new full-time private-sector job. Our analysis compares pre-displacement wages with post-displacement wages, and hence focuses on the first type of worker.

Finally, since our statistics are not linear in the pre-displacement wages, we care about the full distribution and not just the mean. In panel B of table 1, we present results from a series of Kolmogorov-Smirnov tests of the equality of the JJJ and CS wage distributions. We find that in the CPS data we cannot reject unconditional equality, while in the SIPP we can at the 5% significance level. Once conditioning on education by testing the within education distributions, we cannot reject equality in any of the educational samples using SIPP or CPS. Not only are the means the same for the JJJ and CS samples, but, conditional on education, the distributions are also similar.

Independence of Pre- and Post-displacement Wages There are a number of economic mechanisms that could generate a correlation between the pre-displacement wage and the worker’s reservation wage, violating the independence assumption. Three examples are savings, loss aversion and unemployment benefits. First, higher wages may lead workers to have higher savings. Higher savings may lead to a higher reservation wage when unemployed due to a better ability to smooth consumption. Second, workers may have loss aversion, where there is a direct utility cost of accepting a lower wage relative to the wage prior to unemployment. Finally, unemployment benefits depend on a worker’s pre-displacement wage up to a maximum benefit that varies by state. All three of these mechanisms lead high-wage workers to have higher reservation wages and, conditional on worker type, longer unemployment durations.

We test the prediction that higher pre-displacement wages lead to longer unemployment durations in the SIPP and the CPS-DWS. In the CPS-DWS, displaced workers are asked how many weeks went by before they found work again. In the SIPP, we use the panel dimension to

calculate the number of months between the end of the pre-displacement job and the start of the post-displacement job. Table 2 shows a series of regressions of the unemployment duration measured in days on the pre-displacement log wage including different sets of controls. The top panel shows the results for the SIPP JUJ sample and the bottom panel shows the results for the CPS-DWS JUJ sample. In all cases, we do not find evidence of a relationship between unemployment duration and the pre-displacement log wage. We thus conclude that mechanisms that violate the independence assumption by generating a positive correlation between pre- and post-displacement wages through higher reservation wages are not a major concern.

Table 2: Relationship between Unemployment Duration and Pre-Displacement Wage

	Unemployment Duration (Days)		
	(1)	(2)	(3)
SIPP Log Wage	1.480 (3.408)	2.987 (3.638)	4.168 (4.117)
<i>N</i>	1838	1838	1838
CPS-DWS Log Wage	-6.688 (5.435)	-10.29 [†] (5.724)	-5.056 (6.304)
<i>N</i>	2062	2062	2062
Demographics		X	X
Education			X

Note: Standard errors are in parenthesis. Each element in the table is a regression of the unemployment duration measured in days on the pre-displacement log wage. Log wages are measured as log of weekly (CPS-DWS) or monthly (SIPP) wage earnings. The unemployment duration in the SIPP is measured as the number of months between the pre-displacement job and the post-displacement job. The unemployment duration in the CPS is the number of weeks without work reported by the worker. The number of observations for the CPS-DWS sample do not match the analysis sample as not everyone answers the unemployment duration question. Demographics controls include race and sex indicators. Education controls include high school graduate, some college, and college graduate indicators. [†] denotes statistical significance at the 10% level.

3.3 Measurement Error in Wages

If wages are measured with classical measurement error, the wage-dispersion statistic will be biased towards zero and the wage-growth statistic will be biased towards 1/2. In this section, we describe simple measurement-error corrections for each statistic.

A correction for the wage-dispersion statistic can be done by dividing the correlation between pre-displacement and post-displacement wages by the reliability (or signal-to-total-variance)

ratio of wages (λ_w^{rel}). Assume that wages have classical measurement error

$$w = \eta_w + \nu_w,$$

where η_w is the true wage and ν_w is the measurement error in wages. Let measurement error be independent between pre-displacement and post-displacement measurements and also independent of true wages. The correlation between pre-displacement and post-displacement wages is then

$$\begin{aligned} \text{corr}(w^{pre}, w^{post}) &= \frac{\text{cov}(\eta_w^{pre}, \eta_w^{post})}{\sigma_{w^{pre}} \sigma_{w^{post}}} \\ &= \lambda_w^{rel} \text{corr}(\eta_w^{pre}, \eta_w^{post}), \end{aligned}$$

where we assume that the reliability ratio ($\lambda_w^{rel} = \frac{\sigma_{\eta_w}^2}{\sigma_{\eta_w}^2 + \sigma_{\nu_w}^2}$) is the same for pre- and post-displacement wages ($\lambda_{w^{pre}}^{rel} = \lambda_{w^{post}}^{rel}$). Hence, a simple measurement-error correction is to divide the measured wage dispersion statistic by the reliability ratio.

The wage-growth statistic can be corrected for measurement error using a simple deconvolution exercise. Let the difference in pre-displacement and post-displacement log-wages be measured with error:

$$\Delta w = \eta_{\Delta w} + \nu_{\Delta w},$$

where $\eta_{\Delta w}$ is the true difference in log-wages and $\nu_{\Delta w}$ is classical measurement error. Assume that $\eta_{\Delta w}$ and $\nu_{\Delta w}$ are independent and normally distributed, where the measurement error has mean zero ($\mu_{\nu_{\Delta w}} = 0$). Hence, Δw is also normally distributed with mean $\mu_{\Delta w} = \mu_{\eta_{\Delta w}}$ and variance $\sigma_{\Delta w}^2 = \sigma_{\eta_{\Delta w}}^2 + \sigma_{\nu_{\Delta w}}^2$. We can express the true fraction earning lower wages ($\Pr(\eta_{\Delta w} < 0)$) in terms of the fraction measured (with classical measurement error) in the data ($\Pr(\Delta w < 0)$):

$$\begin{aligned} \Pr(\eta_{\Delta w} < 0) &= \Phi \left[\frac{-\mu_{\eta_{\Delta w}}}{\sigma_{\eta_{\Delta w}}} \right] = \Phi \left[\frac{-\mu_{\Delta w}}{\sigma_{\Delta w} \sqrt{\lambda_{\Delta w}^{rel}}} \right] \\ &= \Phi \left[\frac{\Phi^{-1}[\Pr(\Delta w < 0)]}{\sqrt{\lambda_{\Delta w}^{rel}}} \right], \end{aligned} \tag{6}$$

where $\lambda_{\Delta w}^{rel}$ is the reliability statistic for first-differences in wages ($\lambda_{\Delta w}^{rel} = \frac{\sigma_{\eta_{\Delta w}}^2}{\sigma_{\eta_{\Delta w}}^2 + \sigma_{\nu_{\Delta w}}^2} = \frac{\sigma_{\eta_{\Delta w}}^2}{\sigma_{\Delta w}^2}$) and Φ is the CDF of the standard normal distribution.

Thus, we need the reliability ratio for both the level of wages and the first-difference in order to correct the two statistics. The reliability ratio of wages is available from the literature on measurement error in survey data and, given certain assumptions, can also be measured in our data. For example, in the SIPP data we can compare wages across two observations of the same worker-firm match separated by four months. We do not look at adjacent months, because the measurement error is likely to be correlated within the same interview. If we assume that the true wage is constant across the four months, we can use the correlation of the within-match wages to estimate the reliability ratio in levels

$$\begin{aligned} \text{corr}(w_{i,t}, w_{i,t+1}) &= \frac{\sigma_{\eta_w}^2}{\sigma_{\eta_w}^2 + \sigma_{\nu_w}^2} \\ &= \lambda_w^{rel}. \end{aligned}$$

Our estimate using the within-match correlation in the SIPP ($\hat{\lambda}_w^{rel} \approx 0.855$) is very close to [Bound and Krueger \(1991\)](#) who estimate reliability statistics for men and women by comparing reported income in the CPS to social security records ($\hat{\lambda}_w^{rel} \approx 0.864$). [Bound, Brown, and Mathiowetz \(2001\)](#) review the literature on the correlations between worker and employer earnings and find that correlations in studies using weekly, monthly, and annual earnings are similar (see section 6.1.2 therein). While SIPP measures monthly earnings and CPS measures weekly earnings, we will use our SIPP estimates of $\hat{\lambda}_w^{rel}$ as [Bound and Krueger \(1991\)](#) do not estimate reliability ratios by education groups, which we need for our results conditional on education.

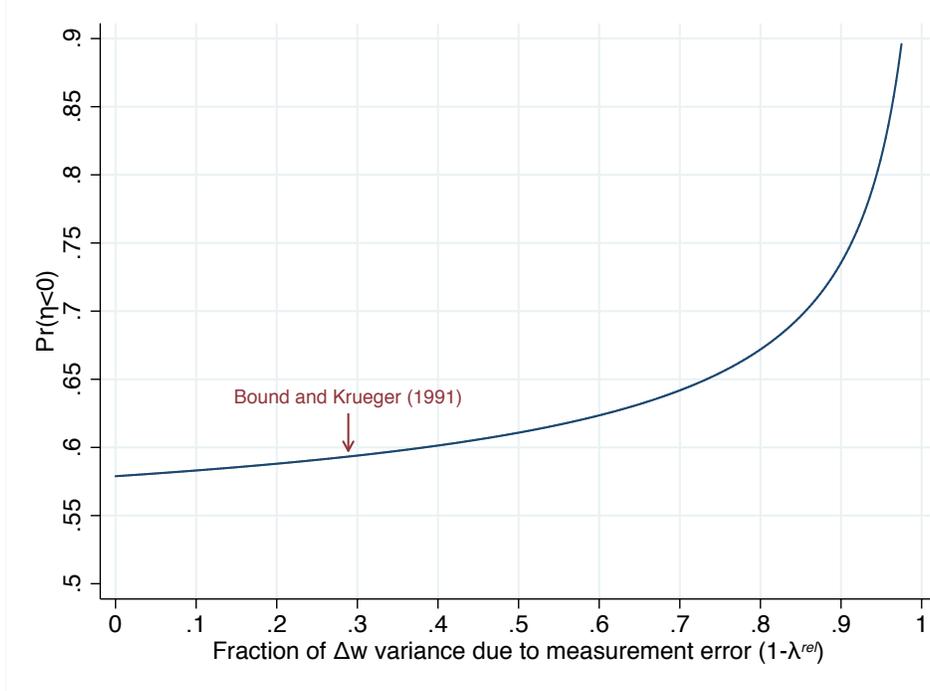
It is not as straightforward to measure the reliability ratio of the first-difference in wages ($\lambda_{\Delta w}^{rel}$) from survey data alone. We rely on [Bound and Krueger \(1991\)](#) who estimate reliability statistics for men and women by comparing reported income in the CPS to social security records. We calculate a weighted average of the reliability ratios of wage differences for men and women in CPS data of $\lambda_{\Delta w}^{rel} = 0.711$.²⁷ In other words, about 71% of the variance in wage differences is signal in CPS data.

It may not be clear from equation 6 if measurement error leads to important downward biases for the wage-growth statistic. In figure 2 we show how, given a measured fraction of 0.579 (our preferred estimate), the true fraction depends on different amounts of measurement error. The x-axis is the fraction of variance due to measurement error, $(1 - \lambda_{\Delta w}^{rel})$. Measurement error would have to be unreasonably large to have a substantive effect on the wage growth statistic.

²⁷See Table 6 on page 16 in [Bound and Krueger \(1991\)](#), where we use the estimates for classical measurement error for income differences and the gender composition of our baseline sample (57% male). While there are differences in the measure the SIPP uses (weekly earnings) with the measure used in [Bound and Krueger \(1991\)](#) (annual earnings), we prefer using their estimates as they directly estimate the measurement error in earnings changes in CPS data.

Given $\lambda_{\Delta w}^{rel} = 0.711$ from [Bound and Krueger \(1991\)](#), we calculate the corrected statistic to be $\hat{Pr}_{corr} = 0.593$.

Figure 2: Correcting the Wage-Growth Statistic for Measurement Error ($\hat{Pr}(\Delta w < 0) = 0.579$)



Notes: This figure shows how the corrected wage-growth statistic depends on different amounts of measurement error assuming the measured statistic of $\hat{Pr}(\Delta w < 0) = 0.579$ from SIPP (see section 3.4). Using $\lambda_{\Delta w}^{rel} = 0.711$ ([Bound and Krueger \(1991\)](#)), the corrected statistic is

$$\hat{Pr}_{corr} = 0.593.$$

3.4 Results

In this section we present our estimates of the wage-dispersion statistic and wage-growth statistic, with and without the measurement error-corrections presented above.

The results for the wage-dispersion statistics are presented in table 3. We find that the correlation between the pre- and post earnings are 0.68 and 0.72 in the SIPP and CPS, respectively. Correcting for measurement error increase the correlations to 0.79 and 0.84. Recall, that one minus the correlation is an upper bound on the relative importance of search frictions. This suggests that search frictions play a fairly minor role in generating wage dispersion compared to worker differences. Looking at educational sub-groups decreases the correlation some. This is only natural, since the variance of wages is lower in the sub-samples (*i.e.* we now only look at

Table 3: Results on Wage Dispersion Statistic

	N	$corr(w^{pre}, w^{post})$	$\frac{corr(w^{pre}, w^{post})}{\lambda_w^{rel}}$
Displaced (SIPP)	1838	0.677 (0.015)	0.792 (0.018)
High School or less	750	0.571 (0.032)	0.716 (0.040)
Some College	650	0.573 (0.030)	0.700 (0.037)
College Graduate	438	0.607 (0.034)	0.714 (0.040)
Plant closure (CPS-DWS)	2241	0.715 (0.012)	0.837 (0.014)
High School or less	813	0.632 (0.023)	0.793 (0.029)
Some College	916	0.681 (0.019)	0.832 (0.023)
College Graduate	512	0.673 (0.034)	0.792 (0.040)

Notes: Standard errors are in parenthesis. Samples selection: prime aged (25-54 years old), full-time, private-sector workers, not working in agriculture or construction, who made a full-time private sector to full-time private sector transition. Displaced workers in the SIPP sample includes workers who were displaced due to a layoff, employer bankruptcy, or slack work conditions. The CPS-DWS sample includes workers who were displaced due to a plant closing. λ_w^{rel} is the reliability ratio for measurement error in wages. Education-specific reliability ratios are calculated using the SIPP cross-sectional dataset. We estimate λ_w^{rel} to be 0.855, 0.797, 0.818, and 0.850 for the full sample, high school or less, some college, and college graduates, respectively. See section 3.3 for more details. Standard errors are estimated via 10,000 bootstrap samples. *Sources: SIPP: Survey of Income and Program Participation 1996, 2001, 2003, and 2008 panels. CPS-DWS Current Population Survey - Displaced Workers Survey 1984-2016.*

within group variation). The overall result is that search frictions play a minor role in explaining wage dispersion.

The results for the wage-growth statistics are presented in table 4. We measure the fraction $\Pr(\Delta w < 0) = 0.579$ using the SIPP data and $\Pr(\Delta w < 0) = 0.576$ using the CPS data. Correcting the fraction for measurement error increases it by about 0.014 in both samples to $\tilde{Pr}(\Delta w < 0) = 0.593$ and $\tilde{Pr}(\Delta w < 0) = 0.590$, respectively for SIPP and CPS. We use the corrected statistics to calculate the bounds on receiving no job offers while employed (see equation 4). The results suggest that many workers only receive the one job offer out of unemployment and no further job offers during their employment spell. Finally, we report the measurement error corrected implied value of κ , which is denoted by $\hat{\kappa}_{corr}$. Our results imply that workers are more likely to receive a job destruction shock than a job offer shock when employed. The results for the education sub-samples reveal important heterogeneity. While workers with less than a college degree are approximately equally likely to get a job destruction shock as a job offer, our results indicate that workers with a college degree experience almost no frictional wage growth. Our estimates imply a lower bound of about 74% and 100% of workers receive no voluntary job offers during an employment spell, in the SIPP and CPS respectively. Put another way, we estimate $\hat{\kappa}_{corr}$ to be about 0.3 and close to zero for workers with a college degree, in the SIPP and CPS respectively.

In this section, we computed the wage-growth and wage-dispersion statistics using two different samples. There are two overall conclusions based on these findings. First, the estimated wage-dispersion statistics (correlation of pre- and post-displacement wages) show that search frictions are not a main determinate of wage-dispersion. Second, the job offer and job destruction rates implied by the wage-growth statistic (fraction earning less after unemployment) are different than those implied by looking at job-to-job transition rates. In the next section, we structurally estimate two versions of a search model in order to show how the wage-growth statistic can be reconciled with observed the job-to-job transition rates within a search model.

Table 4: Results on Wage Growth Statistic

	N	$\hat{P}r(\Delta w < 0)$	$\hat{P}r_{corr}(\Delta w < 0)$	$\hat{P}r_{corr}(n = 1)$	$\hat{\kappa}_{corr}$
Displaced (SIPP)	1838	0.579 (0.011)	0.593 (0.013)	[0.440, 0.813] (0.081, 0.027)	0.761 (0.148)
High School or less	750	0.584 (0.018)	0.599 (0.021)	[0.404, 0.801] (0.128, 0.043)	0.828 (0.246)
Some College	650	0.602 (0.019)	0.620 (0.022)	[0.281, 0.760] (0.134, 0.045)	1.080 (0.300)
College Graduate	438	0.537 (0.024)	0.543 (0.028)	[0.740, 0.913] (0.168, 0.056)	0.297 (0.224)
Plant closure (CPS-DWS)	2241	0.576 (0.010)	0.590 (0.012)	[0.460, 0.820] (0.074, 0.025)	0.725 (0.132)
High School or less	813	0.605 (0.017)	0.624 (0.020)	[0.255, 0.751] (0.120, 0.040)	1.136 (0.275)
Some College	916	0.597 (0.016)	0.615 (0.019)	[0.311, 0.770] (0.113, 0.038)	1.013 (0.242)
College Graduate	512	0.492 (0.022)	0.491 (0.026)	[1.000, 1.000] (0.072, 0.024)	0.000 (0.082)

Notes: Standard errors in parenthesis. Sample selection: prime aged (25-54 years old), full-time, private-sector workers, not working in agriculture or construction, who made a full-time private sector to full-time private sector transition. Displaced workers in the SIPP sample includes workers who were displaced due to a layoff, employer bankruptcy, or slack work conditions. The CPS-DWS sample includes workers who were displaced due to a plant closing. $\lambda_{\Delta w}^{rel}$ is the average reliability ratio for men and women from [Bound and Krueger \(1991\)](#) ($\lambda_{\Delta w}^{rel} = 0.711$). $\hat{P}r_{corr}(n = 1)$ shows the bounds on the fraction of workers receiving zero job offers during the last employment spell after correcting for measurement error. $\hat{\kappa}_{corr}$ is the implied κ after correcting for measurement error. See section 3.3 for more details. Standard errors are estimated via 10,000 bootstrap samples. *Sources: SIPP: Survey of Income and Program Participation 1996, 2001, 2003, and 2008 panels. CPS-DWS Current Population Survey - Displaced Workers Survey 1984-2016.*

4 Using the Statistics in Structural Estimation

In this section we estimate two on-the-job search models with human capital accumulation. The purpose of this is three-fold. First, we show how search models can be consistent with both our frictional wage-growth statistic and the observed mobility in the data. At least two mechanisms can bridge the gap: involuntary job-to-job transitions and compensating differentials. Second, we demonstrate how our two proposed statistics can be used to estimate on-the-job search models. Third, we can use the estimated models to decompose cross-sectional wage dispersion and life-cycle wage growth into frictional and learning-by-doing human capital components. Although the fraction of displaced workers experiencing a wage loss identifies κ , a model is needed to *quantify* the relative importance of the mechanisms driving life-cycle wage growth.

4.1 Model

We develop a continuous-time, infinite-horizon model of the labor market, where agents discount the future at rate ρ . Workers have *ex ante* heterogeneous levels of permanent ability (α). Furthermore, they accumulate human capital (k) via learning-by-doing when employed, where k is discrete and has finite support ($k \in (0, \dots, K)$). Workers enter the labor market with $k = 0$ and draw their permanent ability from the ability distribution $H(\alpha)$. Firms are heterogeneous in their productivity (p) and in the non-pecuniary aspect of the job (z). They produce log output $\alpha + p + f(k)$. Workers draw job offers from the bivariate offer distribution $F(p, z)$.

Unemployed workers receive a flow utility that is a function of their ability and learning-by-doing human capital ($u_0(\alpha, k)$). While unemployed, they receive job offers at rate λ^u and do not accumulate human capital. Once they receive a job offer they must choose either to accept it and become employed or reject it and remain unemployed.²⁸

An employed worker with ability α and human capital k receives flow utility $u_1(\alpha, p, k, z)$ when working at a (p, z) firm.²⁹ Employed workers receive job offers at rate λ^e , which they can accept or decline. They also receive involuntary job offers at rate λ^d , which can only be rejected by entering unemployment.³⁰ At rate λ_h , their human capital increases by one unit via learning-by-doing for $k < K$. Finally, they receive job separation shocks at rate δ , and become unemployed.³¹

²⁸In our empirical specification we assume that unemployed workers choose to accept all job offers.

²⁹Note, that workers simply receive their full productivity, $\alpha + p + f(k)$. Thus, firms take a passive role in this model. In section 4.4, we will explore how sequential bargaining affects the two proposed statistics.

³⁰Again, note that in our specification unemployed workers will choose to accept all job offers, so employed workers will always accept involuntary job offers over unemployment.

³¹See appendix section B.1 for the value functions.

4.2 Estimation

In this section we describe the parametrization and estimation of the model.

Model Specification and Parameterization The flow utility of an employed worker is additively separable in $\log(\text{wage})$ and the non-pecuniary aspect,

$$u_1(\alpha, p, k) = \log(\text{wage}) + z = \alpha + f(k) + p + z.$$

Worker ability is normally distributed following $\alpha \sim N(\mu, \sigma_\alpha)$. We assume that p and z are independently distributed.³² We let p and z be exponentially distributed, where $p \sim \exp(\gamma_p)$ and $z \sim \exp(\gamma_z)$. We let $f(k) = \beta_1 k + \beta_2 k^2$ and set $K = 20$. Finally, wages are observed with measurement error $\epsilon \sim N(0, \sigma_\epsilon)$.

For reasons of tractability, we assume that unemployed workers accept all job offers.³³ Thus, we implicitly assume that the value of staying unemployed and taking a job at the worst firm is the same for all k .³⁴

Poisson rates are measured in annual terms. We fix $\rho = 0.05$ and normalize $\lambda_h = 1$, which leaves eleven free parameters $(\delta, \lambda^u, \lambda^e, \lambda^d, \mu, \gamma_p, \sigma_\alpha, \beta_1, \beta_2, \sigma_\epsilon, \gamma_z)$.

We estimate two versions of the model to show how our statistics discipline wage dispersion and wage growth in different settings. In the first version, the "wage-ladder" model, we fix $\gamma_z = \infty$ and allow for involuntary job offers. In the second version, the "compensating differential" model, we fix $\lambda^d = 0$ and include non-pecuniary aspects of the job. Each model has ten free parameters.

All workers enter the model as unemployed and with zero human capital ($k = 0$). We simulate workers for 25 years and auxiliary parameters are calculated using data from year 2 to year 25.³⁵

As the solution and simulation of the model are not a novel aspect of this paper, we provide a full description in Appendix Section B.2.

³²Taber and Vejlin (2016) find that this is approximately the case in a model without any restrictions on the covariance between p and z .

³³Since entering employment carries the option value of increasing human capital the flow utility of an unemployed worker will be higher than or equal to that of working for the firm which offers the lowest utility. However, calculating exactly how the flow utility depends on the level of human capital such that this is the case complicates the solution.

³⁴We are not the first to assume this. Our model is closely related to that of Bagger, Fontaine, Postel-Vinay, and Robin (2014), which introduce human capital into a sequential auction model with bargaining. In order to avoid dealing with reservation wages they implicitly make the same assumption.

³⁵Ignoring the first years after labor market entry is also done in Bagger, Fontaine, Postel-Vinay, and Robin (2014) in order to avoid noise in the real data from the early part of a labor market, where the transition from school to work can take some time and this is not modelled.

Auxiliary Parameters We estimate the model by indirect inference (Gourieroux, Monfort, and Renault 1993) using the bootstrap covariance matrix for the auxiliary parameters as a weighting matrix. We use two datasets to measure the auxiliary parameters. The 1979 National Longitudinal Survey of Youth (NLSY79) is a nationally representative panel that follows respondents starting at ages 14-22 when first interviewed in 1979. NLSY79 respondents have been interviewed either annually or biennially since 1979. We calculate transition rates and wage statistics using the NLSY79 and use the SIPP (see Section 3.2) to calculate the wage-growth and wage-dispersion statistics and measurement error moments.³⁶ All auxiliary parameters are calculating for workers who hold full-time private-sector employment. Here we give a brief description of the auxiliary parameters used for estimation.³⁷

The auxiliary parameters targeting the transitions parameters are calculated as the yearly probabilities of being displaced ($\Pr(E \rightarrow U)$), taking a job out of unemployment ($\Pr(U \rightarrow E)$), and making a job-to-job transition ($\Pr(E \rightarrow E)$) in the NLSY79. These transition rates help identify the job offer and job destruction rates in the models ($\lambda^u, \lambda^e, \delta$).

Frictional wage dispersion and wage growth are disciplined by the statistics studied in this paper. We use the SIPP to calculate the frictional wage dispersion ($\text{corr}(w_i^{pre}, w_i^{post})$) and the frictional wage growth statistics ($\Pr(w_i^{post} < w_i^{pre})$). These auxiliary parameters discipline the amount of frictional wage dispersion and frictional wage growth in the model. The wage-dispersion statistics identifies the variation in firm types, γ_p . In the wage-ladder model, the wage-growth statistic helps identify the involuntary job offer arrival rate (λ^d). While in the compensating differentials model, the wage-growth statistic helps identify the non-pecuniary parameter (γ_z). The auxiliary parameter for the measurement error, σ_ϵ is the within-job wage correlation ($\text{Corr}(w_t, w_{t+0.33} | \text{within match})$). It uses wages measured from two consecutive waves (four months apart) in the CS SIPP sample (see Section 3.2).

Lifecycle wage growth parameters are measured by estimating a Mincer regression on experience and experience squared in the NLSY79. The intercept (ζ_0), linear coefficient (ζ_1), and quadratic coefficient (ζ_2) are used as auxiliary parameters to identify μ, β_1, β_2 , respectively. The standard deviation (σ_w) of wages are also measured in the NLSY79 and helps to identify σ_α .

Fit and Estimates Table 5 shows the fit of the auxiliary parameters for each model.

Both estimated models fit the data to almost four significant digits. This is not surprising

³⁶Notice that the sample selection for the structural estimation is different than in section 3.2 and the values differ slightly for that reason. The main difference between the two calculations is that the model estimation conditions on 2-25 years of experience.

³⁷The full details on how the auxiliary parameters are constructed can be found in appendix section C.

Table 5: Auxiliary Parameter Fit

	Wage Ladder Model	Compensating Differential Model	Data
$\Pr(\text{E} \rightarrow \text{U})$	0.109	0.109	0.109
$\Pr(\text{U} \rightarrow \text{E})$	0.639	0.639	0.639
$\Pr(\text{E} \rightarrow \text{E})$	0.109	0.109	0.109
$\Pr(w_i^{post} < w_i^{pre})$	0.560	0.558	0.559
σ_w	0.574	0.574	0.574
$corr(w_i^{pre}, w_i^{post})$	0.668	0.668	0.667
ζ_0	2.292	2.290	2.291
ζ_1	0.028	0.028	0.028
$\zeta_2 \times 100$	-0.033	-0.035	-0.034
$Corr(w_t, w_{t+0.33} \text{within match})$	0.853	0.853	0.853

The model is simulated using 5.000.000 worker histories from year 0 to year 25.

given that we have just as many structural parameters as auxiliary parameters, but it is not guaranteed in non-linear models.

In table 6 we show the resulting parameter estimates. Almost all are statistically significant.³⁸

³⁸Standard errors are computed using the formula in [Gourieroux, Monfort, and Renault \(1993\)](#). Because of simulation error the auxiliary parameters are not continuous in the parameters. Thus, taking numerical derivatives cannot be done in the usual way. In order to overcome this problem we do a Taylor expansion around the estimated parameter values. Specifically, we simulate the auxiliary parameters for different values of the parameters using a uniform grid of parameter values around the estimated parameter (25 percent deviation) and then fit the binding function by a linear function in the parameter values. We then use this function to take the derivative at the estimated parameter values. The numerical derivatives are robust to using upto a third order polynomial and using different values of the deviations.

Table 6: Structural Parameter Estimates

	Wage-Ladder Model		Compensating Differential Model	
	Parameter Value	Std. Err.	Parameter Value	Std. Err.
δ	0.116	0.002	0.116	0.002
λ^u	1.108	0.029	1.108	0.029
λ^e	0.159	0.035	0.425	0.010
λ^d	0.056	0.011	-	-
σ_α	0.444	0.008	0.439	0.008
γ_p	4.363	0.214	4.702	0.241
μ	2.077	0.014	2.084	0.014
β_1	0.017	0.003	0.018	0.003
$\beta_2 \times 100$	0.018	0.012	0.014	0.012
σ_ϵ	0.220	0.003	0.220	0.003
γ_z	-	-	2.238	0.678

Standard errors are computed using the formula in [Gourieroux, Monfort, and Renault \(1993\)](#). The exponential distribution has two common parametrizations. In this paper we use $f(x; \gamma) = \gamma x \exp^{-x\gamma x}$. The standard deviation of the firm effect (p) is 0.229 and 0.213 in the wage-ladder model and the compensating differential model, respectively. The standard deviation of the non-pecuniary aspect of a job (z) is 0.447 in the compensating differential model.

We find that the variance of worker ability is much higher than the variance of firm productivity and measurement error. This is expected given the high correlation of pre- and post-displacement wages, which we show in section 2 is closely related to the fraction of the total variance explained by between worker differences. Also notice, that we find λ^d to 0.058 in the wage-ladder model, implying that workers receive an involuntary job offer shock on average once every seventeen years. It is the relatively high value of λ^d that allows us to reconcile the large job-to-job flows with the wage growth statistic.³⁹ On average an involuntary job-to-job shock throws the worker down the job ladder. This makes future job-to-job transitions more likely, since the worker is now at a lower rung in the firm productivity distribution and thereby accepts more of the offers that he receives.

In the compensating differential model, we find $\gamma_z = 2.068$, which implies that the standard deviation of the non-pecuniary aspect is 0.447. Thus, the non-pecuniary aspect is slightly more important than the differences in worker ability. Because the non-pecuniary aspect of job is so important, workers often select jobs primarily based on that. The compensating differential model matches the large job-to-job flows in the data by estimating a high λ^e , but does so without much frictional wage growth, consistent with the wage growth statistic.

The parameter estimates of δ , λ^u , σ_α , γ_p , μ , β_1 , β_2 , and σ_ϵ are virtually the same across the two models.

If we calculate κ from the wage-ladder and compensating differential model, we get 0.88 and

³⁹The estimate is fairly close to [Bagger and Lentz \(2018\)](#), who finds a value of 0.078 using Danish data.

3.66, respectively. The inferred κ from the wage-growth statistic, which we target as an auxiliary parameter, is 0.76. The small difference between the implied κ and the κ in the wage-ladder model are most likely due to differences in the sample compositions. κ in the compensating differential model is not consistent with the wage-growth statistics calculation. This is not unexpected either, since in the compensating differential model the wage is not an order statistic of the job value. Regardless, we will see that the inference on wage growth is consistent across both models. In section 4.4 we will illustrate how the two statistics behave in compensating differential models more generally.

4.3 Results

We use the model to make two kinds of decompositions. First, we show how much of the variance of wages is caused by frictional wage dispersion and secondly, how much of life-cycle wage growth is caused by frictional wage growth compared to human capital induced wage growth.

Frictional Wage Dispersion In order to show the importance of frictional wage dispersion, we decompose the variance of wages in table 7 for the wage-ladder model and the compensating differential model.⁴⁰ The first thing to note, is that even though the two models substantially differ, the estimated variance decompositions are very similar. First, we take out variance caused by measurement error, since this is not relevant for workers. In both models we estimate that around 15 percent of the variance in wages is due to measurement error. This decreases the variance from 0.331 to 0.282 and 0.283 in the two models. We then decompose the true wage variance into four components

$$var(\tilde{w}) = var(\alpha) + var(p) + var(f(k)) + 2cov(p, f(k))$$

where \tilde{w} is the true wage without measurement error. The co-variances between the worker type, α , and the the firm component, p , and the human capital component, $f(k)$, are zero by construction.

From the results it is clear that the main part of the variance is explained by worker heterogeneity as also suggested by the wage dispersion statistic, which showed that the correlation of pre- and post-displacement wages were 0.667. Differences in firm productivity account for about

⁴⁰This is the variance of wages measured in the annual repeated cross-sections from year 1 to year 25 in the labor market.

Table 7: Wage Variance Decomposition

	Wage-Ladder Model		Compensating Differential Model	
	Variance	Share	Variance	Share
Wage Var	0.331		0.331	
Var(ϵ)	0.048		0.048	
Wage Var Without Meas Err (ϵ)	0.283		0.282	
Var(α)	0.195	0.688	0.191	0.676
Var(p)	0.065	0.229	0.068	0.241
Var($f(k)$)	0.018	0.065	0.019	0.068
2 Cov($p, f(k)$)	0.005	0.017	0.004	0.015

Notes: The table shows a linear decomposition of the wage variance taking out measurement error for each model. It decomposes the true variance into a worker component ($\text{Var}(\alpha)$), a firm component ($\text{Var}(p)$), a human capital component ($\text{Var}(f(k))$) and the covariance between the worker and firm components ($2 \text{Cov}(p, f(k))$).

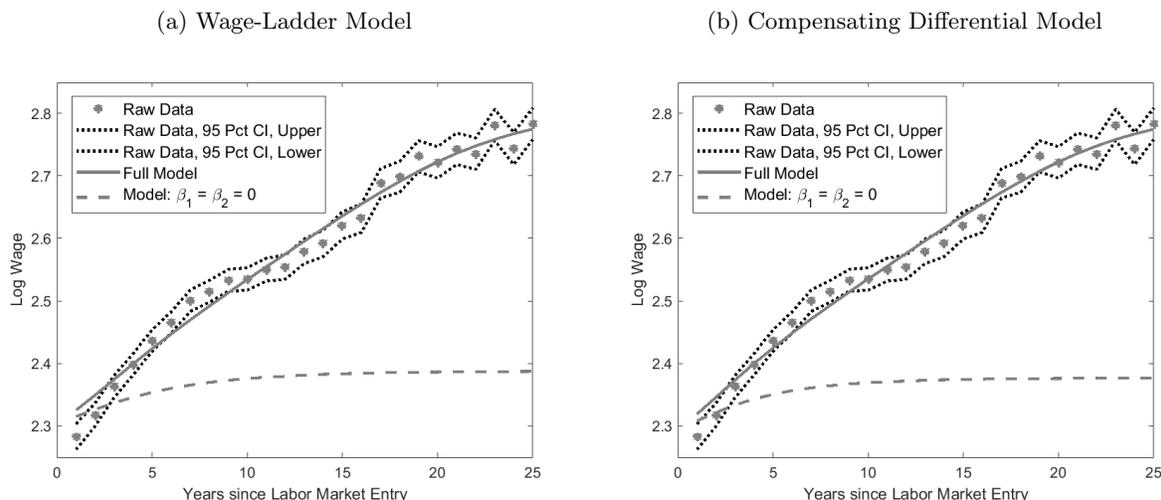
23%-24% across the two models, while human capital accumulation matters to a smaller extent, about 7%. This is to be expected as a simple Mincer regression of wages on experience typically do not explain much more than 5-7 percent of wage variation.

Life-Cycle Wage Growth We now move to the second question of how much of life-cycle wage growth is caused by frictional wage growth. Figure 3 contains two sub-figures, one for each model. In each sub-figure we plot three graphs. First, we plot the average wage by years since labor market entry from the real data together with the 95 percent confidence interval. Second, we plot the average wage from the model and finally, we plot the average wage from the model imposing that there is no human capital accumulation ($\beta_1 = \beta_2 = 0$), so the only reason wages go up over the life-cycle is frictional wage growth due to workers climbing the productivity ladder.

Figure 3 shows that the model fit is excellent for both models in terms of matching the life-cycle wage profile from the data. As was the case for the variance decomposition of wages, the results from the two models are strikingly similar. Comparing the full models with the models without any return to human capital accumulation, it is clear that early in the life cycle both human capital accumulation and frictional wage growth play important roles for the overall wage growth. However, after the first 5-10 years frictional wage growth is negligible.

Table 3 decomposes total wage growth from labor market entry to 25 years into human capital wage growth and frictional wage growth. Search frictions account for about 15-16 percent of the

Figure 3: Life-Cycle Wages



Notes: The plot of the real data ("Raw Data") uses the same NLSY data as we used in the estimation except that here we also plot data after the first year in the labor market. The simulated data is from the model, where we simulate workers from labor market entry (at time zero), where they enter as unemployed, to 25 years after. Frictional wage growth is measured by simulating the model setting $\beta_1 = \beta_2 = 0$, while human capital wage growth is the difference in growth between total wage growth and frictional wage growth.

total wage growth over the first 25 years of the life cycle in both models.

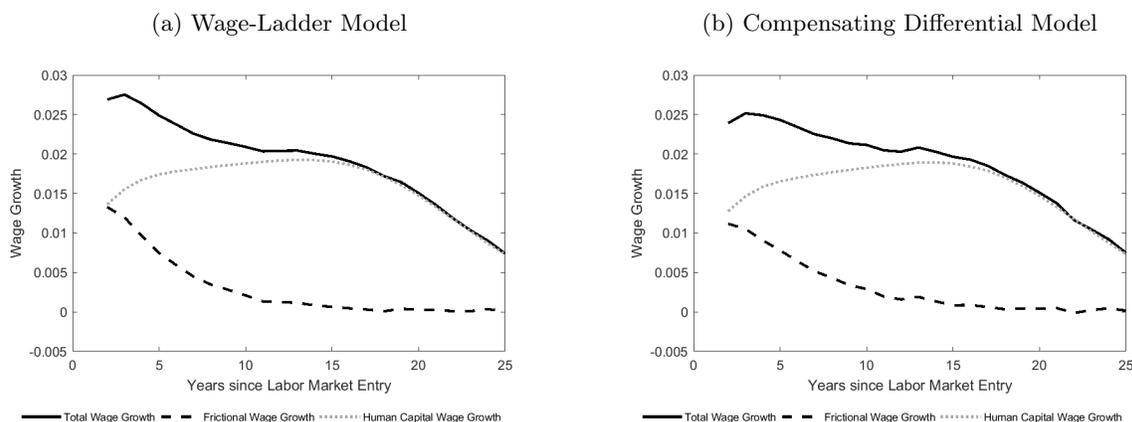
Table 8: Decomposition of Wage Growth At 25 Years of Experience

	Wage-Ladder Model		Compensating Differential Model	
	Wage Growth	Share	Wage Growth	Share
Total Wage Growth	0.449		0.455	
Human Capital Wage Growth	0.377	0.839	0.387	0.849
Frictional Wage Growth	0.072	0.161	0.069	0.151

Notes: The table decompose total wage growth from labor market entry to 25 years after into human capital wage growth and frictional wage growth. Frictional wage growth is measured by simulating the model setting $\beta_1 = \beta_2 = 0$, while human capital wage growth is the difference in growth between total wage growth and frictional wage growth.

Finally, as indicated by figure 3 the relative roles of human capital and frictions in generating wage growth varies over the life-cycle. In figure 4 we show the differences in yearly average wages in the full model (labeled Total Wage Growth) and in the model without any human capital accumulation (labeled Frictional Wage Growth). Finally, we show the difference between the

Figure 4: Wage Growth Decomposition



two (labeled Human Capital Wage Growth).⁴¹

It is clear from figure 4 that in the early part of a worker’s career frictional induced wage growth is actually around the same level as human capital wage growth. After about 10 years, the average frictional wage growth is zero.

We draw the following conclusions based on the results presented from the structural model. First, the different κ ’s implied by the two models suggest that one should be careful by estimating involuntary job offer shocks from transition rates, since a model with compensating differentials would imply a very different value. Second, even though the κ ’s were very different, the amount of frictional wage growth in the two models were surprisingly similar. This shows how the wage-growth statistic can discipline the frictional wage growth across different types of models.

4.4 Models Where Wage is Not an Order Statistic of Job Value

In this section, we consider how the wage-growth statistic relates to wage growth when the wage is not an order statistic of the job value, which is the underlying assumption in section 2.2. In section 4.3, we used the wage-growth statistic to estimate a compensating differentials model, which does not satisfy the aforementioned assumption. While the estimated offer ratio (κ) was different compared to equation 5, the estimated frictional wage growth was quite consistent with the wage-ladder model. We explore further whether the wage-growth statistic is informative of both κ and frictional wage growth in other classes of models. In other words, if we infer frictional

⁴¹Notice that the reason that human capital wage growth has an inverse U-shape is that the return to human capital is convex ($\beta_2 > 0$), which causes the small increase in the growth rate until around year 15. Human capital wage growth begins to decline as workers start to obtain the maximum human capital level, K . Recall, that learning-by-doing human capital arrives stochastically.

wage growth from our statistic using the wage-ladder model, is that informative about frictional wage growth in a broader class of models? Of course, the wage-growth statistic can be used in structural estimation even if the above relationships differ and is likely useful in disciplining the estimated frictional wage growth regardless.

We study two classes of models where the wage is not an order statistic of the job value. The first class of models are models with within-job frictional wage growth. One of the most prominent examples is the sequential auction model pioneered by [Postel-Vinay and Robin \(2002\)](#).⁴² The second class of models are models with compensating wage differentials.⁴³ In these models workers care about non-pecuniary aspects of the job. While the non-pecuniary aspect differs across models, it is always the case that the wage is not necessarily an order statistic of the job value.

Models with Within-job Frictional Wage Growth The sequential auction model of [Postel-Vinay and Robin \(2002\)](#) has been used extensively and been extended in various directions.⁴⁴ In these models wages are not an order statistic of job value. Workers are willing to accept a lower wage for the promise of faster wage growth in the future both within and between firms. Wage growth is generated by outside offers from other firms, where firms engage in Bertrand competition over the worker. Within the class of models using the sequential auction wage setting, the fraction experiencing a wage loss after displacement is higher, since the wage after displacement is going to be lower, because the worker values the better outside option of employment compared to unemployment. Thus, we will tend to over-estimate κ using Equation 5. In general, we view this as a minor problem, since our main empirical result is that κ is quite low. In the context of the sequential-auction model, the inferred κ from equation 5 can be interpreted as an upper bound.

In the wage-tenure contract model of [Burdett and Coles 2003](#) workers trade-off the initial firm wage against how much tenure they have at the current firm. We follow [Hornstein, Krusell, and Violante \(2011\)](#) in grouping the sequential auction model and the wage-tenure model together. [Hornstein, Krusell, and Violante \(2011\)](#) argue that the wage path arising from a standard wage-tenure contract model is observationally equivalent to the wage path arising in the [Postel-Vinay and Robin \(2002\)](#). For this reason, we will only consider the sequential auction model in what

⁴²Another example of within-job frictional wage growth models is the wage-tenure contract models pioneered by [Burdett and Coles \(2003\)](#). We will not consider this model explicitly here, but we conjecture that we will get similar results.

⁴³see *e.g.* [Dey and Flinn \(2005\)](#), [Becker \(2009\)](#), [Jarosch \(2015\)](#), [Taber and Vejlín \(2016\)](#), and [Sullivan and To \(2014\)](#).

⁴⁴see *e.g.* [Cahuc, Postel-Vinay, and Robin \(2006\)](#), [Lentz \(2010\)](#), [Robin \(2011\)](#), [Bagger, Fontaine, Postel-Vinay, and Robin \(2014\)](#), [Taber and Vejlín \(2016\)](#), [Lise and Robin \(2017\)](#).

follows.

Models with Compensating Differentials If workers value non-pecuniary aspects of a job, the wage may not fully represent the value of a job. In a model with compensating differentials, the fraction of displaced workers receiving lower *utility* identifies κ . The fraction of displaced workers earning lower wages will no longer identify κ as workers will sometimes climb the job ladder by accepting lower wages in order to receive higher non-pecuniary benefits. On one hand, if non-pecuniary benefits represent an important component of the value of a job, then the κ could be much larger than the κ implied by Equation 5. On the other hand, if non-pecuniary benefits are important then a high κ can be consistent with little frictional wage growth. So although the fraction of displaced workers earning lower wages may not be informative about κ in a model with non-pecuniary benefits, it may still be informative about frictional wage growth.

Numerical Examples For the sequential auction model, we follow the extension by Cahuc, Postel-Vinay, and Robin (2006), which introduces bargaining power (χ) into the Postel-Vinay and Robin (2002) model. One extreme case is that the worker has no bargaining power ($\chi = 0$) in which case the model is equivalent to Postel-Vinay and Robin (2002). When the worker has all the bargaining power ($\chi = 1$), the model is equivalent to a wage-ladder model, where the wage is an order statistic of the job value.⁴⁵ For the compensating differential model, we consider the model presented in Section 4.1.⁴⁶

The model specification and parameter values used for both models in these simulations are the ones we estimated earlier in section 4. In all simulations in this section, we set $\lambda_d = 0$ (no involuntary job-to-job transitions), $\beta_1 = \beta_2 = 0$ (no human capital accumulation), and $\sigma_\epsilon = 0$ (no measurement error). When we simulate the sequential auction model we set the variance of the non-pecuniary aspect to zero and when we simulate the compensating differential model, we set the bargaining power of the worker to one.

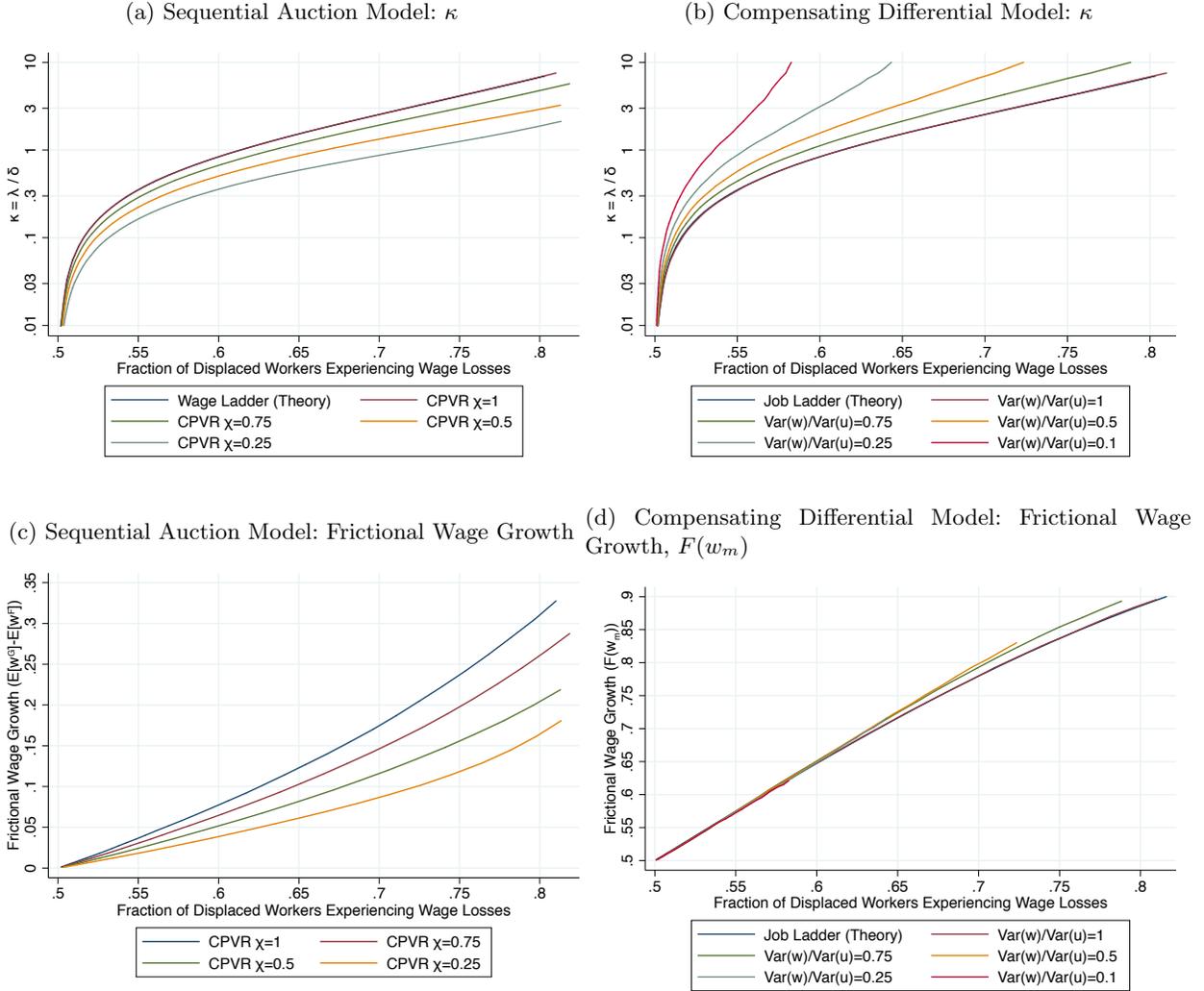
In figure 5 (a) and (b), we show the relationship between the wage-growth statistic and κ for different values of χ and the variance of the non-pecuniary aspect, respectively. For the sequential auction model, our frictional wage growth measure, $F[G^{-1}(0.5)]$, is not very informative, since F and G in that model are not wage distributions, but productivity distributions. Instead in (c), we plot the difference in the average cross-sectional wage and the average wage coming out

⁴⁵We simulate the model in steady-state by drawing values from the steady state distributions derived by Cahuc, Postel-Vinay, and Robin (2006). We then simulate the model for different values of κ and χ .

⁴⁶We simulate the model in steady-state by letting the simulation run for 100 years before we start to simulate the data for the figures. We do this for different values of κ and variances of the non-pecuniary benefit.

of unemployment. Finally, panel (d) shows the relationship between our frictional wage growth measure, $F[G^{-1}(0.5)]$, and the wage-growth statistic for the compensating differential model.⁴⁷

Figure 5: Alternative Models and the Frictional Wage-Growth Statistics



Notes: CPVR refers to [Cahuc, Postel-Vinay, and Robin \(2006\)](#) and χ is the bargaining power of the worker. The "Wage Ladder (Theory)" graph in each figure shows the values derived in section 2.2. In panel (a) and (c) we vary the bargaining power. In panel (b) and (d) we change the variance of the non-pecuniary aspect in order to change the total variance of utility, $Var(u)$.

Figure 5 (a) shows the relationship between κ and the wage-growth statistic for the sequential auction model. As expected for the sequential auction model, we find that for a given κ with full bargaining power to the worker ($\beta = 1$) the fraction is the same as in the wage-ladder model.

⁴⁷We prefer the wage growth measure as it does not depend on the wage offer distribution in the wage-ladder model. In the sequential-auction model, unemployed and employed workers do not draw wages from the same wage distribution, so the measure $F[G^{-1}(0.5)]$ is less relevant.

Decreasing the bargaining power implies that the implied κ using equation (5) is an upper bound for the κ implied by the sequential model.

Figure 5 (b) shows the relationship between κ and the wage-growth statistic for the compensating differentials model. As expected, the implied κ using equation (5) is a lower bound for a compensating differentials model. For a given level of κ , the more the compensating differential part of utility matters, the less wages are informative about the job value and the position on the job ladder. Hence, fewer displaced workers will experience wage losses. Taber and Vejlin (2016) find that the variance of utility is around 2.5-3 times the variance of wages, while Hall and Mueller (2018) finds that the standard deviation of the non-wage value the a job offer is almost 1.5 times the standard deviation of the offered wage. Both results suggest that non-pecuniary aspects of a job play a substantial role in the choices of workers.

Lastly, figures 5 (c) and (d) provide evidence that the wage-growth statistic is informative of frictional wage growth in both classes of models. In figure 5 (c), the wage growth in the sequential auction model is lower than the wage-ladder model. In other words, the frictional wage growth inferred using the wage-ladder model can be considered an upper bound with respect to the sequential-auction model. In figure 5 (d), we show how our wage growth measure varies by the relative importance of the non-pecuniary benefit. While there are important differences in the relationship between κ and the wage-growth statistic between the wage-ladder model and the compensating differential model, the relationship between wage growth and the wage growth statistic is almost identical between the wage-ladder model and the compensating differential model for empirically relevant values of the wage-growth statistic.

In summary, the wage growth inferred from the wage-growth statistic is a good approximation for wage growth in sequential auction and compensating differential models, while a researcher has be more careful when inferring job offer rates (κ) using equation 5.

5 Conclusions

In this paper, we present two sufficient statistics. The first is the correlation of pre- and post unemployment wages. These statistics is informative of frictional wage dispersion. The second is the fraction of workers earning less after an unemployment spell, which is informative about frictional wage growth. We show how these statistics are informative across a large class of search models independent of wage offer distributions and other labor market parameters. We estimate the statistics using displaced workers in both CPS and SIPP data. We find that frictional wage dispersion explains less than 20 percent of the variation in wages and likewise, workers

receive few wage improving offers during an employment spell. In other words, the wage-growth statistic implies modest frictional wage growth. While there is not much heterogeneity in the wage-dispersion statistic, we find that college-educated workers experience almost no frictional wage growth. We use the statistics to estimate two different models of the labor market: an involuntary job offers model and a compensating differential model. While the implied job offer rates depend on the details of the model, the importance of frictional wage dispersion and frictional wage growth is estimated to be almost identical across the two models. Frictional wage dispersion is around 25 percent of total wage dispersion and only around 15 percent of the life-cycle wage growth is due to frictional wage growth. Finally, we show that the wage-growth statistic is informative of frictional wage growth in models where wages are not an order statistic of the job value.

References

- ABOWD, J., F. KRAMARZ, AND D. MARGOLIS (1999): “High wage workers and high wage firms,” *Econometrica*, 67(2), 251–333.
- ALVAREZ, F. E., K. BOROVICKOVA, AND R. SHIMER (2014): “A nonparametric variance decomposition using panel data,” Discussion paper, Mimeo, University of Chicago.
- BAGGER, J., F. FONTAINE, F. POSTEL-VINAY, AND J.-M. ROBIN (2014): “Tenure, Experience, Human Capital, and Wages: A Tractable Equilibrium Search Model of Wage Dynamics,” .
- BAGGER, J., AND R. LENTZ (2018): “An Empirical Model of Wage Dispersion With Sorting,” *Review of Economic Studies*, forthcoming.
- BARLEVY, G. (2008): “Identification of Search Models using Record Statistics,” *Review of Economic Studies*, 75(1), 29–64.
- BECKER, D. (2009): “Non-Wage Job Characteristics and the Case of the Missing Margin,” Federal Trade Commission.
- BLANCHARD, O. (2018): “On the future of macroeconomic models,” *Oxford Review of Economic Policy*, 34(1-2), 43–54.
- BONTEMPS, C., J.-M. ROBIN, AND G. J. VAN DEN BERG (1999): “An Empirical Equilibrium Job Search Model with Search on the Job and Heterogeneous Workers and Firms,” *International Economic Review*, 40(4), 1039–1074.
- BOUND, J., C. BROWN, AND N. MATHIOWETZ (2001): “Measurement Error in Survey Data,” in *Handbook of Econometrics*, ed. by J. J. Heckman, and E. Leamer, vol. 5 of *Handbooks in Economics*, pp. 3705–3843. Elsevier Science, Amsterdam.
- BOUND, J., AND A. B. KRUEGER (1991): “The extent of measurement error in longitudinal earnings data: Do two wrongs make a right?,” *Journal of Labor Economics*, 9(1), 1–24.
- BOWLUS, A. J., AND H. LIU (2013): “The contributions of search and human capital to earnings growth over the life cycle,” *European Economic Review*, 64, 305–331.
- BURDETT, K., C. CARRILLO-TUDELA, AND M. G. COLES (2011): “Human capital accumulation and labor market equilibrium,” *International Economic Review*, 52(3), 657–677.

- BURDETT, K., AND M. COLES (2003): “Equilibrium wage-tenure contracts,” *Econometrica*, 71(5), 1377–1404.
- BURDETT, K., AND D. MORTENSEN (1998): “Wage differentials, employer size, and unemployment,” *International Economic Review*, pp. 257–273.
- CAHUC, P., F. POSTEL-VINAY, AND J. ROBIN (2006): “Wage bargaining with on-the-job search: Theory and evidence,” *Econometrica*, 74(2), 323–364.
- CHETTY, R. (2009): “Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods,” *Annu. Rev. Econ.*, 1(1), 451–488.
- DEY, M. S., AND C. J. FLINN (2005): “An Equilibrium Model of Health Insurance Provision and Wage Determination,” *Econometrica*, 73(2), 571–627.
- FALLICK, B., J. HALTIWANGER, AND E. MCENTARFER (2012): “Job-to-job flows and the consequences of job separations,” .
- GALLAGER, R. G. (2013): *Stochastic processes: theory for applications*. Cambridge University Press.
- GARIBALDI, P., E. R. MOEN, AND D. E. SOMMERVOLL (2016): “Competitive on-the-job search,” *Review of Economic Dynamics*, 19, 88–107.
- GOTTFRIES, A., AND C. N. TEULINGS (2017): “Returns to on-the-job search and the dispersion of wages,” .
- GOURIEROUX, C., A. MONFORT, AND E. RENAULT (1993): “Indirect inference,” *Journal of Applied Econometrics*, 8(S1), S85–S118.
- HALL, R. E., AND A. I. MUELLER (2018): “Wage Dispersion and Search Behavior: The Importance of Nonwage Job Values,” *Journal of Political Economy*, 126(4), 1594–1637.
- HORNSTEIN, A., P. KRUSELL, AND G. L. VIOLANTE (2011): “Frictional Wage Dispersion in Search Models: A Quantitative Assessment,” *THE AMERICAN ECONOMIC REVIEW*, 101, 2873–2898.
- HYATT, H., AND E. MCENTARFER (2012): “Job-to-job Flows and the Business Cycle,” .
- JAROSCH, G. (2015): “Searching for Job Security and the Consequences of Job Loss,” .

- LENTZ, R. (2010): “Sorting by search intensity,” *Journal of Economic Theory*, 145(4), 1436–1452.
- LISE, J., AND J.-M. ROBIN (2017): “The Macrodynamics of Sorting between Workers and Firms,” *American Economic Review*, 107(4), 1104–35.
- MENZIO, G., I. A. TELYUKOVA, AND L. VISSCHERS (2016): “Directed search over the life cycle,” *Review of Economic Dynamics*, 19, 38–62.
- MORTENSEN, D. (2005): *Wage dispersion: why are similar workers paid differently?* The MIT Press.
- MOSCARINI, G., AND F. POSTEL-VINAY (2019): “The job ladder: Inflation vs. reallocation,” *Unpublished draft, Yale University*.
- POSTEL-VINAY, F., AND J. ROBIN (2002): “Equilibrium wage dispersion with worker and employer heterogeneity,” *Econometrica*, 70(6), 2295–2350.
- ROBIN, J.-M. (2011): “On the Dynamics of Unemployment and Wage Distributions,” *Econometrica*, 79(5), 1327–1355.
- RUBINSTEIN, Y., AND Y. WEISS (2006): “Post Schooling Wage Growth: Investment, Search and Learning,” in *Handbook of the Economics of Education*, ed. by E. Hanushek, and F. Welch, vol. 1 of *Handbooks in Economics*, chap. 1, pp. 1–67. North-Holland, Amsterdam.
- SHIMER, R., AND L. SMITH (2000): “Assortative Matching and Search,” *Econometrica*, 68(2), 343–369.
- SULLIVAN, P., AND T. TO (2014): “Search and Nonwage Job Characteristics,” .
- TABER, C., AND R. VEJLIN (2016): “Estimation of a Roy/Search/Compensating Differential Model of the Labor Market,” Working Paper 22439, National Bureau of Economic Research.
- TJADEN, V., AND F. WELLSCHMIED (2014): “Quantifying the Contribution of Search to Wage Inequality,” *American Economic Journal: Macroeconomics*, 6(1), 134–161.
- TOPEL, R. H., AND M. P. WARD (1992): “Job Mobility and the Careers of Young Men,” *Quarterly Journal of Economics*, 107(2), 439–479.
- VAN DEN BERG, G. J., AND G. RIDDER (1998): “An Empirical Equilibrium Search Model of the Labor Market,” *Econometrica*, 66(5), 1183–1222.

YAMAGUCHI, S. (2010): "Job Search, Bargaining, and Wage Dynamics," *Journal of Labor Economics*, 28(3), 595–631.

A Mathematical Derivations

A.1 Derivation of Frictional Wage-Dispersion Statistic: $Cov(w^{post}, w^{pre})$

Let $\mu_i = E_{F_i}[w_i^{post}]$ be the conditional mean of the job offer distribution for worker of type i and $\Delta\mu_i = E_{G_i}[w_i^{pre}] - \mu_i$ be the difference in the conditional means of the $G_i(w)$ and the $F_i(w)$ distributions for worker of type i . If the post-displacement wage w_i^{post} is independent of the pre-displacement wage w_i^{pre} , then the population covariance is

$$Cov(w^{post}, w^{pre}) = Var(\mu) + Cov(\mu, \Delta\mu).$$

The covariance depends only on the variation in the means of the type-specific distributions and is independent of the shape of the distributions.

There are three distributions that need to be integrated over to calculate the covariance: (1) the expectation over types denoted as E_i , (2) the expectation over the wage offer distribution conditional on type i (E_{F_i}), and (3) the expectation over the pre-displacement wage distribution conditional on type i (E_{G_i}). Without loss of generality, we decompose wages into the type-specific mean μ_i and the demeaned wage.

$$\begin{aligned} w_i^{post} &= \mu_i + \varepsilon_i \\ w_i^{pre} &= \mu_i + \eta_i. \end{aligned}$$

Note that, by definition of μ_i , $E_{F_i}[\varepsilon_i] = 0$, while $E_{G_i}[\eta_i] = \Delta\mu_i$ has a non-zero mean.

The covariance is

$$cov(w^{post}, w^{pre}) = E[w^{post}w^{pre}] - E[w^{post}]E[w^{pre}]. \quad (7)$$

The first term on the RHS in equation 7 becomes

$$\begin{aligned} E[w^{post}w^{pre}] &= E_i \{ E_{F_i G_i} [w_i^{post} w_i^{pre}] \} \\ &= E_i \{ E_{F_i G_i} [\mu_i^2 + \varepsilon_i \mu_i + \mu_i \eta_i + \varepsilon_i \eta_i] \} \\ &= E_i \{ \mu_i^2 + \mu_i E_{F_i} [\varepsilon_i] + \mu_i E_{G_i} [\eta_i] + E_{F_i G_i} [\varepsilon_i \eta_i] \} \\ &= E_i [\mu_i^2 + \mu_i \Delta\mu_i], \end{aligned}$$

where we used $E_{F_i}[\varepsilon_i] = 0$ and $E_{F_i G_i}[\varepsilon_i \eta_i] = E_{G_i}[E_{F_i}[\varepsilon_i \eta_i]] = E_{G_i}[\eta_i E_{F_i}[\varepsilon_i]] = 0$, due to the independence of w_i^{post} and w_i^{pre} .

The second term on the RHS in equation 7 becomes

$$\begin{aligned} E[w^{post}]E[w^{pre}] &= E_i \{E_{F_i}[w_i^{post}]\} E_i \{E_{G_i}[w_i^{pre}]\} \\ &= E_i[\mu_i]E_i[\mu_i + \Delta\mu_i]. \end{aligned}$$

We finish the derivation by combining the two terms and putting them back into equation 7

$$\begin{aligned} Cov(w^{post}, w^{pre}) &= E_i[\mu_i^2] + E_i[\mu_i \Delta\mu_i] - E_i[\mu_i]E_i[\mu_i] - E_i[\mu_i]E_i[\Delta\mu_i] \\ &= E_i[\mu_i^2] - E_i[\mu_i]E_i[\mu_i] + E_i[\mu_i \Delta\mu_i] - E_i[\mu_i]E_i[\Delta\mu_i] \\ &= Var(\mu) + Cov(\mu, \Delta\mu). \end{aligned}$$

A.2 Derivation of Frictional Wage-Growth Statistic: $Pr_i(w_i^{post} < w_i^{pre})$

We would like to relate the fraction to fundamental parameters from on-the-job search models, namely the Poisson job separation rate (δ_i) and the Poisson on-the-job offer rate (λ_i^e). These two parameters determine the probability distribution of the number of job offers a worker receives during an employment spell. Specifically, the probability of receiving $n - 1$ additional offers is

$$Pr_i(n) = \left(\frac{\lambda_i^e}{\lambda_i^e + \delta_i} \right)^{n-1} \frac{\delta_i}{\lambda_i^e + \delta_i}$$

from the literature on Poisson processes (see *e.g.* [Gallager 2013](#)).⁴⁸

Recall that the probability of earning a lower wage after being displaced conditional on receiving n job offers is $Pr_i(w_i^{post} < w_i^{pre} | n) = \frac{n}{n+1}$.

The relationship between the wage growth statistic and the Poisson parameters of the on-

⁴⁸The probability that the k^{th} arrival of Poisson process 1 occurs before the j^{th} arrival of Poisson process 2 is

$$Pr(S_k^1 < S_j^2) = \sum_{i=k}^{k+j-1} \binom{k+j-1}{i} \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} \right)^i \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{k+j-1-i}.$$

the-job search model is then

$$\begin{aligned}
Pr_i(w_i^{post} < w_i^{pre}) &= \frac{\delta_i}{\lambda_i^e + \delta_i} \sum_{n=1}^{\infty} \frac{n}{n+1} \left(\frac{\lambda_i^e}{\lambda_i^e + \delta_i} \right)^{n-1} \\
&= \frac{1}{\kappa_i + 1} \sum_{n=1}^{\infty} \frac{n}{n+1} \left(\frac{\kappa_i}{\kappa_i + 1} \right)^{n-1} \\
&= \frac{1}{\kappa_i} \sum_{n=1}^{\infty} \frac{n}{n+1} \left(\frac{\kappa_i}{\kappa_i + 1} \right)^n \\
&= \frac{1}{\kappa_i} \sum_{n=0}^{\infty} \frac{n}{n+1} \left(\frac{\kappa_i}{\kappa_i + 1} \right)^n \\
&= \frac{1}{\kappa_i} \left[\sum_{n=0}^{\infty} \left(\frac{\kappa_i}{\kappa_i + 1} \right)^n - \sum_{n=0}^{\infty} \frac{1}{n+1} \left(\frac{\kappa_i}{\kappa_i + 1} \right)^n \right] \\
&= \frac{1}{\kappa_i} \left[\frac{1}{1 - \frac{\kappa_i}{\kappa_i + 1}} - \frac{\kappa_i + 1}{\kappa_i} \sum_{n=0}^{\infty} \frac{1}{n+1} \left(\frac{\kappa_i}{\kappa_i + 1} \right)^{n+1} \right] \\
&= \frac{1}{\kappa_i} \left[\kappa_i + 1 - \frac{\kappa_i + 1}{\kappa_i} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\kappa_i}{\kappa_i + 1} \right)^n \right] \\
&= \frac{1}{\kappa_i} \left[\kappa_i + 1 - \frac{\kappa_i + 1}{\kappa_i} \ln(\kappa_i + 1) \right] \\
&= 1 - \frac{(\kappa_i + 1) \ln(\kappa_i + 1) - \kappa_i}{\kappa_i^2},
\end{aligned}$$

where $\kappa_i = \lambda_i^e/\delta_i$, the geometric series $\sum_{n=0}^{\infty} \frac{1}{r^n} = \frac{1}{1-r}$, and the series expansion $\ln(x) = \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{x-1}{x} \right)^n$.

A.3 Alternate Derivation of Frictional Wage-Growth Statistic using Burdett-Mortensen Steady-State Accounting

Now consider an economy populated with heterogeneous, infinitely-lived workers. As before, let worker heterogeneity be described by discrete types, $i \in \mathcal{I}$. Unemployed workers receive job offers at rate λ_i^u . If a worker accepts a job, they receive job offers at rate λ_i^e while employed. Separations occur at rate δ_i . The wage of a job offer is drawn from a well-behaved job offer distribution function $w_i \sim F_i(w)$.⁴⁹ We assume that the wage is an order statistic of the worker's value of a job, hence a worker prefers any job that pays a higher wage.⁵⁰ We assume that the economy is in steady state.

We now show that the fraction of workers who suffer a wage loss after displacement identifies $\kappa_i = \lambda_i^e/\delta_i$, independent of the wage offer distribution. Let $G_i(w)$ be the wage-earned distribution

⁴⁹The fact that we assume $F_i(w)$ is exogenous here is not restrictive. We follow [Hornstein, Krusell, and Violante \(2011\)](#) in arguing that allowing the wage-offer distribution to be determined in equilibrium has no impact on our results. Their argument can easily be amended to allow for on-the-job search.

⁵⁰In Section 4.4, we consider various models from the literature that relax this assumption.

of employed workers of type i . In other words, the cross-sectional distribution of wages of workers of type i . We can relate the probability that a worker suffers a wage loss to the wage-offer and wage-earned distributions

$$Pr_i(w_i^{post} < w_i^{pre}) = \int F_i(w) dG_i(w),$$

where w_i^{pre} and w_i^{post} are the pre-displacement and post-displacement spell wage, respectively.

We use steady state flow accounting (à la [Burdett and Mortensen, 1998](#)) to find an analytical relationship between $F_i(w)$ and $G_i(w)$. In steady state, we can equate the flow of workers into and out of jobs with wage less than w and the flow of workers into and out of unemployment (u_i)

$$\begin{aligned} \lambda_i^u F_i(w) u_i &= (\delta_i + \lambda_i^e (1 - F_i(w))) G_i(w) (1 - u_i), \\ \lambda_i^u u_i &= \delta_i (1 - u_i). \end{aligned} \tag{8}$$

These equations can be used to solve for $F_i(w)$ in terms of $G_i(w)$

$$F_i(w) = \frac{(1 + \kappa_i) G_i(w)}{1 + \kappa_i G_i(w)}, \tag{9}$$

where $\kappa_i = \lambda_i^e / \delta_i$.

The probability of earning a lower wage after displacement is then

$$\begin{aligned} Pr_i(w_i^{post} < w_i^{pre}) &= \int F_i(w) dG_i(w) \\ &= \int \frac{(1 + \kappa_i) G_i(w)}{1 + \kappa_i G_i(w)} dG_i(w) \\ &= (1 + \kappa_i) \int_0^1 \frac{x_i}{1 + \kappa_i x_i} dx_i \\ &= 1 - \frac{(1 + \kappa_i) \ln(1 + \kappa_i) - \kappa_i}{\kappa_i^2}. \end{aligned}$$

The first equality is found by substituting $F_i(w)$ from equation 9 and the second equality by doing a change of variables $x_i = G_i(w)$. Importantly, notice that the fraction of workers earning lower wages after displacement depends only on κ_i and is independent of the wage offer distribution. This is because the probability $Pr_i(w_i^{post} < w_i^{pre})$ depends only on the order

statistic of w_i^{pre} and not the actual value of the wage.

Extension: Reallocation shocks Our results can be extended to the case where workers are exposed to a reallocation shock. A reallocation shock is a shock where employed workers receive a job offer while employed that they must accept or go into displacement. Adding a reallocation shock λ_i^d , equation 8 becomes

$$\lambda_i^u F_i(w) u_i + \lambda_i^d F_i(w) [1 - G_i(w)] (1 - u_i) = (\delta_i + \lambda_i^e [1 - F_i(w)] + \lambda_i^d [1 - F_i(w)]) (1 - u_i) G_i(w).$$

Solving for $F_i(w)$,

$$F_i(w) = \frac{(1 + \tilde{\kappa}_i) G_i(w)}{1 + \tilde{\kappa}_i G_i(w)},$$

where $\tilde{\kappa}_i = \frac{\lambda_i^e}{\delta_i + \lambda_i^d}$. In terms of the job wage ladder, a reallocation shock functions in a similar way to a job destruction shock, in that workers lose their search capital.

The probability of a wage loss is similar to the model without reallocation shocks

$$Pr_i(w_i^{post} < w_i^{pre}) = 1 - \frac{(1 + \tilde{\kappa}_i) \ln(1 + \tilde{\kappa}_i) - \tilde{\kappa}_i}{\tilde{\kappa}_i^2}.$$

A.4 A Measure of Frictional Wage Growth in Search Models $F(w_m)$

In section 4.4, we show that although the fraction of displaced workers earning lower wages does not identify κ in a model with non-pecuniary benefits, it is informative about the amount of frictional wage growth independent of non-pecuniary benefits. Our measure of frictional wage growth ($F(w_m)$) measures the fraction of the offer distribution that is below the median wage in the market. In other words, it is a measure of the distance between the F and G distributions ($F[G^{-1}(0.5)]$). Below, we show that this measure is a simple function of κ_i and is independent of the wage offer distribution.

First we show that the relationship between G and F can be derived using the same approach as equation 5. As shown in sections 2.2 and A.2, a worker of type i who keeps the maximum of n independent offers from $F_i(w)$ has a wage distribution $G_i(w|n) = F_i(w)^n$. If workers of type i receive job offers at Poisson rate λ_i^e and lose jobs at Poisson rate δ_i , then we can sum the probabilities of receiving $n = 1, \dots, \infty$ offers during an employment spell,

$$\begin{aligned}
G_i(w) &= \frac{\delta_i}{\lambda_i^e + \delta_i} \sum_{n=1}^{\infty} F_i(w)^n \left(\frac{\lambda_i^e}{\lambda_i^e + \delta_i} \right)^{n-1} \\
&= \frac{1}{\kappa_i} \sum_{n=1}^{\infty} F_i(w)^n \left(\frac{\kappa_i}{\kappa_i + 1} \right)^n \\
&= \frac{1}{\kappa_i} \sum_{n=0}^{\infty} F_i(w)^{n+1} \left(\frac{\kappa_i}{\kappa_i + 1} \right)^{n+1} \\
&= \frac{F_i(w)}{\kappa_i + 1} \sum_{n=0}^{\infty} \left(\frac{\kappa_i}{\kappa_i + 1} F(w) \right)^n \\
&= \frac{F_i(w)}{\kappa_i + 1} \frac{1}{1 - \frac{\kappa_i F(w)}{\kappa_i + 1}} \\
&= \frac{F_i(w)}{1 + \kappa_i [1 - F_i(w)]}.
\end{aligned}$$

This equation may look familiar as it is the equation for the steady state wage distribution in [Burdett and Mortensen \(1998\)](#). Note that although [Burdett and Mortensen \(1998\)](#) assume a steady state, we have shown that the relationship holds even outside of steady state. This relationship can be inverted to find $F_i(w)$ as a function of $G_i(w)$,

$$F_i(w) = \frac{(1 + \kappa_i)G_i(w)}{1 + \kappa_i G_i(w)}.$$

Our measure of wage growth in the canonical wage-ladder model can be calculated by substituting $G_i(w_{m,i}) = 0.5$, so that

$$F_i(w_{m,i}) = \frac{1 + \kappa_i}{2 + \kappa_i}.$$

B Model Appendix

B.1 Value Functions

The value function for a unemployed worker with human capital (α, k) is given by

$$\begin{aligned}
(\lambda^u + \rho)U(\alpha, k) &= u_0(\alpha, k) \\
&\quad + \lambda^u \int W(\alpha, x, k) - U(\alpha, k) dF(x),
\end{aligned}$$

where $W(\alpha, p, k)$ is the value function of a worker with human capital (α, k) working at a firm with productivity p .

The value function for an α -type employed worker with human capital $k < K$, who is working at a firm with productivity p and non-pecuniary aspect z is given by

$$\begin{aligned}
& (\lambda^d + \lambda_h + \delta + \rho)W(\alpha, p, k, z) \\
& = u_1(\alpha, p, k, z) + \lambda^d \int \int W(\alpha, x, k, y) dF(x, y) \\
& + \lambda^e \int \int \max[W(\alpha, p, k, z), W(\alpha, x, k, y)] - W(\alpha, p, k, z) dF(x, y) \\
& + \lambda_h W(\alpha, p, k + 1, z) \\
& + \delta U(\alpha, k).
\end{aligned}$$

When $k = K$ learning-by-doing human capital does not increase anymore, so $\lambda_h = 0$ in this state.

B.2 Solution of the Model

We estimate the model by indirect inference, [Gourieroux, Monfort, and Renault \(1993\)](#). In order to do this, we simulate 5.000.000 worker histories from time 0 to time 25 (recall that the model is cast in continuous time with the unit of time being a year). All the workers draw an ability $\alpha \sim N(\mu, \sigma_\alpha)$ and start out being unemployed. For an unemployed worker the current spell length is given by T_{λ^u} , where T_{λ^u} is drawn from an exponential distribution with parameter λ^u . In the next spell he is employed drawing a productivity level, $p \sim \Gamma(\gamma_p)$, and a non-pecuniary level, $z \sim \Gamma(\gamma_z)$. The spell length of the employed worker is given as $T_{\text{Empl.}} = \min(T_\delta, T_{\lambda_1}, T_{\lambda^d}, T_{\lambda_h})$, which are respectively the time until arrive of a job destruction shock, a job offer, an involuntary job offer shock, and a human capital shock. Again, these are all drawn from exponential distributions. If, *e.g.*, $T_{\text{Empl.}} = T_\delta$ then the worker becomes unemployed. If $T_{\text{Empl.}} = T_{\lambda_1}$ then the worker makes a JtJ transition if the new drawn productivity and non-pecuniary aspect yields a higher utility than the current job. If $T_{\text{Empl.}} = T_{\lambda^d}$ the worker is forced to move to a new firm with a random drawn productivity. If $T_{\text{Empl.}} = T_{\lambda_h}$ the human capital of the worker increases with 1 if $k < K$.

Simulated Data

Having simulated the worker histories, the wage is given by $w = \alpha + p + \beta_1 k + \beta_2 k^2 + \epsilon$ with $\epsilon \sim N(0, \sigma_\epsilon)$. We assume that each time we measure the wage we will draw a new measurement

error.

Discretization

In the solution we discretize worker ability, the non-pecuniary aspect, and productivity. We chose to the grid as equally spaced points on the CDF's. Thus, the grid size is determined $\frac{1}{I+1}$ where I is the number of intervals. In the simulation $I = 999$ such that the grid points ranges from 0.001 to 0.999. These grid types are mapped into ability, non-pecuniary aspect, and productivity space by the use of the inverse of the CDF's of the normal and and exponential distributions, respectively.

C Data Appendix

C.1 Descriptive Statistics

Table 9: Survey of Income and Program Participation
Descriptive Statistics

	JU	JUU		CS
		Pre-Disp	Post-Disp	
Age	38.82 (8.43)	38.45 (8.37)		38.51 (8.77)
Male	0.558 (0.497)	0.595 (0.491)		0.534 (0.499)
White	0.645 (0.479)	0.671 (0.470)		0.693 (0.461)
HS	0.296 (0.457)	0.289 (0.453)		0.308 (0.462)
Some college	0.347 (0.476)	0.354 (0.478)		0.347 (0.476)
College	0.220 (0.414)	0.238 (0.426)		0.252 (0.434)
Tenure	4.820 (5.864)	4.709 (5.737)	0 -	6.139 (7.146)
Wage	3255.55 (2221.13)	3424.85 (2268.88)	3136.74 (2178.46)	3495.36 (2310.92)
Observations	4,377	1,838		98,716

Note: The standard deviation is reported in parenthesis. The monthly wage is deflated to 2010 dollars using the CPI. JU = job to displacement transition, JUU = job to displacement to job transition, CS = cross-section of employed workers. The post-displacement wage is measured the first month the worker is observed working at the firm and, hence, the tenure is zero by construction. The SIPP sample includes the 1996, 2001, 2004, and 2008 panels. See section 3.2 for more details on the sample construction. *Source: Survey of Income and Program Participation*

Table 10: Current Population Survey Descriptive Statistics
Full DWS Sample (1984-2016) and DWS-ORG Comparison Sample (1996-2016)

	Full Sample		DWS-ORG Comparison Sample			
	DWS JUJ		DWS JU	DWS JUJ		ORG
	Pre-Disp	Post-Disp		Pre-Disp	Post-Disp	
Age	37.41		39.28	38.69		39.23
	(8.02)		(8.41)	(8.12)		(8.45)
Male	0.562		0.523	0.544		0.530
	(0.496)		(0.500)	(0.498)		(0.499)
White	0.866		0.827	0.844		0.830
	(0.341)		(0.379)	(0.363)		(0.375)
High School	0.274		0.349	0.353		0.317
	(0.446)		(0.477)	(0.478)		(0.465)
Some college	0.409		0.310	0.279		0.296
	(0.492)		(0.462)	(0.449)		(0.456)
College	0.228		0.248	0.290		0.309
	(0.420)		(0.432)	(0.454)		(0.462)
Tenure	5.411	1.685	6.264	5.863	1.425	18.645
	(7.349)	(7.456)	(9.214)	(7.992)	(5.606)	(31.561)
Wage	779.67	720.17	795.63	838.09	784.00	857.33
	(425.19)	(407.78)	(458.47)	(468.26)	(456.20)	(476.77)
Observations	2,241		1,886	987		58,113

Note: The standard deviation is reported in parenthesis. The weekly wage is deflated to 2010 dollars using the CPI. JU = job to displacement transition, JUJ = job to displacement to job transition. Full Sample includes all CPS-DWS surveys from 1984-2016. The DWS-ORG comparison sample is constructed to compare workers the CPS-DWS and the CPS-ORG. The comparison sample includes the CPS-DWS sample from 1998-2016 and the CPS-ORG surveys from two years earlier (1996-2014). See section 3.2 for more details on the sample construction. *Source: Current Population Survey*

C.2 Description of Auxiliary Parameters used for Estimation

Let w denote log real wages. The auxiliary parameters will be measured in two samples: The main data sample will be the 1979 version of the National Longitudinal Survey of Youth (NLSY79) and we will supplement this with data from the SIPP (see section 3.2 for a description of the SIPP dataset). The NLSY79 consists of a representative sample of individuals born in 1957-64, where each birthyear is sampled equally.

All auxiliary parameters are calculated for workers with between 2-25 years of potential experience (age - years-of-schooling - 6). For the auxiliary parameters, we do not make any age other requirements (*i.e.* we drop the prime-aged worker selection from section 3.2).

1. **Transition Rates:** Measured in the NLSY79. Weekly job histories have been constructed for each worker.
 - (a) **Pr(E→U):** For the population of workers that are employed in the first week of January in year X: What is the probability that they are observed in displacement in year X?
 - (b) **Pr(E→E):** For the population of workers that are employed in the first week of January in year X: What is the probability that they are observed working for a new employer in the first week of January in year X+1 without an intervening displacement spell. (Our definition of E→U and E→E are mutually exclusive. So $\Pr(E→U) + \Pr(E→E) + \Pr(\text{stayer}) = 1$).
 - (c) **Pr(U→E):** For the population of workers that are employed in the first week of January in year X-1 and then unemployed in the first week of January in year X: What is the probability that they are observed employed in the first week of January in year X+1?
2. **Wage distribution and regression:** Moments on wages are taken from the NLSY79 dataset. The wages are measured annually from 1979 through 1994 and bi-annually after that. In order to account for potential bias from the sampling, we weigh wage observations so that each experience year bin has the same weight in calculating the auxiliary parameters based on wages.
 - (a) **Mincer wage regression** ($\zeta_0, \zeta_1, \zeta_2$): Regress $w_{it} = \zeta_0 + \zeta_1 exp_{it} + \zeta_2 exp_{it}^2 + \varepsilon_{it}$.
 - (b) σ_w : Standard deviation of wages
3. **Wage dynamics across JUJ spells:** These auxiliary parameters will be calculated from the SIPP, which is a series of short monthly panels. See section 3.2 for more details on

sample selection.

- (a) $Pr(w_i^{post} < w_i^{pre})$: The fraction earning lower wages after displacement compared to before.
- (b) $corr(w_i^{pre}, w_i^{post})$: Correlation between the wages of the pre- and post-displacement spell jobs
- (c) $Corr(w_t, w_{t+0.33} | \text{within match})$: The within-match wage correlation for the SIPP cross-sectional worker sample. See sections [3.2](#) and [3.3](#).