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Relative Deprivation as a Cause of Risky Behaviors

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ABSTRACT

Relative Deprivation as a Cause of Risky Behaviors*

Combining a standard measure of concern about low relative wealth and a standard measure of relative risk aversion leads to a novel explanation of variation in risk-taking behavior identified and documented by social psychologists and economists. We obtain two results: (1) Holding individual i’s wealth and his rank in the wealth distribution constant, the individual’s relative risk aversion decreases when he becomes more relatively deprived as a result of an increase in the average wealth of the individuals who are wealthier than he is. (2) If relative deprivation enters the individual’s utility function approximately linearly then, holding constant individual i’s wealth and the average wealth of the individuals who are wealthier than he is, the individual’s relative risk aversion decreases when he becomes more relatively deprived as a result of a decline in his rank. Our findings provide a theoretical support for evidence about the propensity of relatively deprived individuals to gamble and resort to other risky behaviors.

JEL Classification: D01, D81, D91, I12
Keywords: social preferences, relative deprivation, concern about low relative wealth, risk aversion

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1. Introduction

Research in social psychology links relative deprivation with gambling, viewing relative deprivation as a factor that may account for the tendency of some people to gamble (Callan et al., 2008; Haisley et al., 2008). Social psychologists have sought to explain this link, conjecturing that because relative deprivation is an aversive state, people are motivated to reduce it by engaging in various behaviors that will enable them to achieve the outcomes they feel they merit: gambling offers a means by which people attempt to allay their relative deprivation. Callan et al. (2008) reason that because gambling offers the prospect of improving one’s financial situation swiftly and dramatically, it may be perceived by people as an avenue - and perhaps the only avenue - to achieve the outcomes (status, money) they feel they deserve in life. It is as if demand for the burst of intense pleasure that arises from winning at gambling rises with relative deprivation. Specifically, Callan et al. report that higher relative deprivation is significantly associated with stronger urges to gamble. In one experiment, they find that higher relative deprivation leads to increased gambling. In another experiment, they observe that participants who are led to believe that they have less discretionary income than their comparators opt to engage in real gambling more frequently than do participants who believe that they have a similar level of discretionary income as their comparators. Callan et al. (2011) refine the intervening variable between relative deprivation and the inclination to gamble, arguing that relative deprivation increases the desire for immediate rewards which, in turn, leads to increased gambling. This line of reasoning is consistent with evidence of a link between “financial deprivation” and increased gambling (Cross, 2000; Wheeler et al., 2006; Blalock et al., 2007; Callan et al., 2008; Beckert and Lutter, 2009). Beckert and Lutter (2013) who study lottery gambling by poor people, do not refer explicitly to relative deprivation, but present a reasoning that comes quite close to drawing on this concept. Beckert and Lutter write (pp. 2-3): “we test deprivation . . . that explains lottery gambling through . . . disadvantaged social position and assume that lottery participation serves as compensation for and release of tensions arising from social inequalities and feelings of deprivation,” adding that “to people of a lower social status situations . . . lottery gambling has a greater attraction than to people of a higher social status.”
Research in economics provides additional and broader links between relative deprivation and risky behavior. Kawachi et al. (1999) present evidence, based on data taken from the US General Social Surveys, that relative deprivation leads to violent crimes, which can be perceived as risky choices. Using data for males from the US National Health Interview Survey and from the US Behavioral Risk Factor Surveillance System, Eibner and Evans (2005) report that high relative deprivation is related to increased probability of risky behaviors such as smoking and not using seatbelts. Drawing on data from the US National Longitudinal Study of Adolescent Health, Balsa et al. (2014) find that relative deprivation is positively associated with substance abuse (alcohol consumption, drinking to intoxication levels, and smoking cigarettes) for adolescent males but, interestingly, not for adolescent females.

Using data on deaths by suicide in the US so as to identify the importance of interpersonal comparisons and “relative status,” Daly et al. (2013) found compelling evidence that individuals care not only about their own income but also about the income of others in their local area: Daly et al. showed that individual suicide risk rises with others’ income. This finding was obtained using two separate and independent data sets, suggesting that it is not an artifact of a particular sample design of either data set. The finding is robust to alternative specifications and cannot be explained by geographical variation in suicide classification, cost of living, or access to emergency medical care. Specifically, treating suicide as a choice variable regarding current life satisfaction and assessed value of future life, Daly et al. examined the relationship between suicide risk and one’s own and others’ income, using data from two independent sources: the National Longitudinal Mortality Study (NLMS) and data from publicly available death certificates combined with the 5% Public Use Micro Sample (PUMS) of the 1990 decennial census. Holding an individual’s income constant, they found that others’ income, measured by local area (county) median income, was positively and significantly correlated with suicide risk. The relative income association holds for individuals across the income distribution, suggesting that suicide risk rises with median county income for both high-income and low-income individuals. That the finding applies also to high-income individuals emphasizes that absolute income per se does not shield an individual from feeling relative deprivation. The finding is consistent with the idea that relative
deprivation, rather than one’s own absolute income, matters for wellbeing (happiness), and that the stress it causes can be severe enough to make people take their own life.

Mo (2014) inquires how vulnerability to being trafficked involves willingness to acquiesce to dangerous economic opportunities: specifically, Mo asks whether relative deprivation can lead individuals to be more risk-seeking, putting themselves and their children at risk of modern forms of slavery. Using a controlled survey experiment conducted in trafficking-prone areas of Nepal, she finds that relative deprivation induces more risk-seeking behavior with regard to economic opportunities.

What explains the relationship between relative deprivation and these behaviors? How do the findings reported above and similar ones relate to risk preferences, to people’s wealth, and to low income? Specifically, does a sense of relative deprivation cause people to become less relatively risk averse and, consequently, to exhibit riskier conduct?

The purpose of this paper is to outline a framework that can yield analytically the observations of social psychologists and economists. A common way of progressing in science is to construct a theory, derive testable propositions, and then pursue empirical inquiry. Here, we proceed in reverse order: we seek to substantiate the appeal and strengthen the validity of reported empirical insights by demonstrating that they can be derived theoretically.

In brief, what we do is as follows. By combining a standard measure of relative deprivation and a standard measure of relative risk aversion we are able to link variation in risk-taking behavior with changes in the level of relative deprivation. In order to investigate the pure effect of relative deprivation of an individual, we hold the individual’s wealth constant. We then identify two ways in which an individual can become more relatively deprived: an increase in the average wealth of individuals who are wealthier than he is, and a decline in his rank in the wealth distribution. It is easy to illustrate these two changes, given the measure of relative deprivation that we employ in this paper and present in equation (2) in Section 2, namely the aggregate of the excesses of the levels of wealth of others, divided by the size of the population (the reference group). Let the ascending distribution of five individuals, who are indexed by \( i \), be given
by the levels of their wealth: 1, 2, 3, 4, and 5. We also refer to these initial levels of
wealth as the names of the individuals. We consider individual \( i = 3 \) whose rank is third.
(Here, individual 5 holds the first rank, individual 4 holds the second rank, and so on.)
The relative deprivation of individual 3 is \( \frac{1}{5} \cdot (1 + 2) = \frac{3}{5} \). When the average wealth of the
wealthier individuals becomes higher than 4.5 then, holding the other levels of wealth
unchanged and, thus, holding the individual’s rank constant, the relative deprivation of
individual 3 increases because the aggregate of the excesses is bigger than \((1 + 2)\).
Conversely, let the average wealth of the wealthier individuals remain unchanged, and let
an individual who is positioned to the left of individual 3 in the wealth distribution
become richer than 3. For example, let individual 2 gain 2.5 units of wealth. Then,
individual 3 loses one rank position (in the new distribution 1, 3, 4, 4.5, and 5, individual
3 is now fourth), while at 4.5 the average wealth of the richer individuals remains
unchanged. Once again, the relative deprivation of individual 3 increases.

It is worth adding that our approach to the social context in which preferences
towards risk taking are formed differs from an approach taken in the psychological
literature with regard to the influence of group affiliation on risk-taking behavior. That
literature presents approaches such as a “risky shift” (Stoner, 1961), where the risk
tolerance in the decision-making of a group is higher than the average risk tolerance of
the individual members of the group, and a “cautious shift” (Wallach et al., 1963), where
a decreased propensity to take risks is attributed to responsibility for the wellbeing of the
group. In our approach, it is not group membership per se but, rather, the position in the
group’s wealth hierarchy that shapes risk preferences. This distinction becomes apparent
when we show that it is a change of an individual’s position in a group, rather than a
change of the individual’s group of affiliation or of the size of the group, which affects
the individual’s risk-taking behavior.

Our modeling framework yields two results: (1) Holding individual \( i \)’s wealth and
his rank in the wealth distribution constant, the individual’s relative risk aversion
decreases when he becomes more relatively deprived as a result of an increase in the
average wealth of the individuals who are wealthier than he is. (2) If relative deprivation
enters the individual’s utility function approximately linearly, then holding constant
individual $i$’s wealth and the average wealth of the individuals who are wealthier than he is, the individual’s relative risk aversion decreases when he becomes more relatively deprived as a result of a decline in his rank.

In Section 2, we show that an individual is less relatively risk averse when he is more relatively deprived. In Section 3, we conclude. In the Appendix we show that the claims presented in Section 2 can reside in a utility representation that is more general than the one drawn upon in Section 2.

2. The relative risk aversion of an individual declines with (a measure of) his relative deprivation

How does relative deprivation affect relative risk aversion? In this section, we show that, holding the wealth of an individual constant, the individual’s relative risk aversion decreases when he becomes more relatively deprived. The events that cause the rise in relative deprivation are an increase in the average wealth of the individuals wealthier than him (Claim 1), and a loss of rank in the wealth distribution (Claim 2). In this paper, we use relative deprivation as a convenient abbreviation of relative wealth deprivation.

Consider a population of $n$ individuals with wealth levels $w_1, \ldots, w_n$, such that $0 < w_1 < w_2 < \ldots < w_n$. Let the utility function of individual $i$ be $u_i(w_i, RD_i)$ where $w_i$ is the wealth level of individual $i$, and $RD_i$ is the relative deprivation (as defined in (2) below) of individual $i$. We assume that $u_i(w_i, RD_i)$ is twice continuously differentiable and strictly increasing with respect to $w_i$, and continuously differentiable and strictly decreasing with respect to $RD_i$. Specifically, we let $u_i(w_i, RD_i)$ be

$$u_i(w_i, RD_i) = (1 - \beta_i)v(w_i) - \beta_i(RD_i)^\alpha,$$  

(1)

where $v(\cdot)$ is a twice continuously differentiable, strictly increasing, and strictly concave function that describes the preferences towards one’s own wealth; $\beta_i \in (0,1)$ is the intensity of the concern of individual $i$ about having low relative wealth (being relatively deprived); $RD_i$ is the index of relative deprivation defined as
\[ RD_i \equiv \frac{1}{n} \sum_{j=i+1}^{n} (w_j - w_i) \text{ for } i = 1, 2, ..., n-1; \quad RD_n \equiv 0 \]  

and \( \alpha \in (1, 2) \). By assuming this range for the parameter \( \alpha \), we introduce a (small) degree of concavity of the utility function with respect to relative deprivation. In choosing \( \alpha > 1 \), we assume that the individual’s utility decreases faster when his wealth falls farther below the wealth of others.\(^1\) Unless specified otherwise, we will refer henceforth to relatively deprived individuals, namely we will employ the first part of (2).

The index of relative deprivation presented in (2) is essentially provided by the received literature. In Stark (2013), we provide a brief account of the history of relative deprivation in economics as well as of the rationale that has guided the construction of the widely used index (2) as a measure of (a means of quantifying) the repercussions of the individuals’ engagement in social comparisons. An axiomatic foundation for (2) is provided in Stark et al. (2017).

It is helpful to rewrite the index of relative deprivation in a slightly different form

\[
RD_i = \frac{n-i}{n} \left[ \frac{1}{n-i} \sum_{j=i+1}^{n} (w_j - w_i) \right] = \frac{n-i}{n} \left( \frac{\sum_{j=i+1}^{n} w_j}{n-i} - w_i \right) = \frac{n-i}{n} (\bar{w}_i - w_i),
\]

where \( \bar{w}_i = \frac{1}{n-i} \sum_{j=i+1}^{n} w_j \) is the average wealth of the individuals who are richer than \( i \) (these individuals are positioned to his right in the wealth distribution). The measure of individual \( i \)'s low relative wealth assumes that the manner in which placement in the wealth distribution measured is cardinal, that is, the \textit{magnitude} of the wealth of those placed higher up in the wealth hierarchy matters to an individual rather than merely their position (and thereby his own rank) in the wealth hierarchy.

For brevity’s sake, we henceforth refer to \( u_i(w_i, RD_i) \) as \( u_i(w_i) \). Drawing on (3), we rewrite the utility function in (1) as

\(^1\) From (2), it follows that the index of relative deprivation of individual \( i \) is linear with respect to \( i \)'s wealth. Therefore, if not for the incorporation of \( \alpha \), the relative deprivation term will vanish in the differentiation undertaken below in the calculation of \( i \)'s relative risk aversion.
The derivatives of this utility function have the following forms:

\[ u'_i(w_i) = (1 - \beta_i)v'(w_i) + \beta_i\alpha \left( \frac{n-i}{n} \right)^\alpha (\tilde{w}_i - w_i)^{\alpha-1} , \quad (4) \]

and

\[ u''_i(w_i) = (1 - \beta_i)v''(w_i) - \beta_i\alpha(\alpha - 1) \left( \frac{n-i}{n} \right)^\alpha (\tilde{w}_i - w_i)^{\alpha-2} . \quad (5) \]

As per Pratt (1964) and Arrow (1965, 1970), two measures of risk aversion, namely the coefficient of relative risk aversion (RRA), \( R_i(w_i) \), and the coefficient of absolute risk aversion (ARA), \( A_i(w_i) \), of individual \( i \), taken while holding the wealth levels of all the other individuals \( (w_1, \ldots, w_{i-1}, w_{i+1}, \ldots, w_n) \) constant, are defined, respectively, by

\[ R_i(w_i) = \frac{-w_i u''_i(w_i)}{u'_i(w_i)} \]

and

\[ A_i(w_i) = \frac{-u''_i(w_i)}{u'_i(w_i)} . \]

The measures are well-defined in some neighborhood of \( w_i \) such that \( w_{i-1} < w_i < w_{i+1} \).

Because the results reported below apply equally to these two measures of risk aversion (the proofs regarding ARA are analogous to the proofs regarding RRA), we limit our attention to the coefficient of relative risk aversion.

Using (4) and (5), we get that
We now have in place the components needed to formulate and prove two claims that forge a link between relative deprivation and relative risk aversion.

**Claim 1.** Holding individual $i$’s wealth and his rank constant, the individual’s relative risk aversion decreases when he becomes more relatively deprived because the average wealth of the individuals wealthier than him increases.

**Proof.** Because an increase of $\tilde{w}_i$ decreases the numerator of (6) and increases the denominator of (6), it follows that

$$\left(\frac{\partial}{\partial \tilde{w}_i} r_i(w_i)\right) < 0.$$  \hspace{1cm} (6)

Q.E.D.

The observed link between “financial deprivation” and increased gambling (Cross, 2000; Wheeler et al., 2006; Blalock et al., 2007; Callan et al., 2008; Beckert and Lutter, 2009) provides empirical support for Claim 1: higher relative deprivation induces riskier pursuits.

**Claim 2.** If relative deprivation enters the individual’s utility function approximately linearly, then holding individual’s $i$ wealth and the average wealth of the individuals wealthier than him constant, the individual’s relative risk aversion decreases when he becomes more relatively deprived because he loses rank in the wealth distribution.

**Proof.** We say that individual $n$ has the highest (top) rank in the population, namely the first rank, that individual $n - 1$ has the second rank in the population, and so on. Because “$i$” represents individual $i$’s position in the wealth hierarchy, that same “$i$” can be linked to individual $i$’s rank. Even though we cannot differentiate $r_i(w_i)$ in (6) with respect to $i$ because $i$ is a discrete variable, we can investigate the sign of the
derivative of $r_i(w_i)$ with respect to $\frac{n-i}{n}$ which for a large $n$ approaches a continuous variable. Here, a loss of rank in the wealth hierarchy increases $\frac{n-i}{n}$. A loss of rank can arise when one (or more) of the individuals to the left of individual $i$ in the wealth distribution becomes richer than $i$, that is, moves in the wealth hierarchy to the right of $i$. We have that

$$\frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} = \frac{w_i\beta \alpha^2 \left(\frac{n-i}{n}\right)^{\alpha-1} (\tilde{w}_i - w_i)^{\alpha-1}}{(1-\beta_i)v'(w_i) + \beta_i \alpha \left(\frac{n-i}{n}\right)^\alpha (\tilde{w}_i - w_i)^{\alpha-1}} \times \left[ \frac{\alpha-1}{\tilde{w}_i - w_i} \left(1-\beta_i\right)v'(w_i) + \beta_i \alpha \left(\frac{n-i}{n}\right)^\alpha (\tilde{w}_i - w_i)^{\alpha-1} \right] + (1-\beta_i)v''(w_i) - \beta_i \alpha (\alpha-1) \left(\frac{n-i}{n}\right)^\alpha (\tilde{w}_i - w_i)^{\alpha-2} \right]$$

$$= \frac{w_i(1-\beta_i)\beta \alpha^2 \left(\frac{n-i}{n}\right)^{\alpha-1} (\tilde{w}_i - w_i)^{\alpha-1}}{(1-\beta_i)v'(w_i) + \beta_i \alpha \left(\frac{n-i}{n}\right)^\alpha (\tilde{w}_i - w_i)^{\alpha-1}} \times \left[ \frac{\alpha-1}{\tilde{w}_i - w_i} v'(w_i) + v''(w_i) \right].$$

Because the fraction preceding $\frac{\alpha-1}{\tilde{w}_i - w_i} v'(w_i) + v''(w_i)$ is positive, the sign of $\frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}}$ depends on the sign of $\frac{\alpha-1}{\tilde{w}_i - w_i} v'(w_i) + v''(w_i)$. We then have that $\frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} < 0$ if

$$\frac{\alpha-1}{\tilde{w}_i - w_i} v'(w_i) + v''(w_i) < 0,$$

which in turn can be rewritten as

$$\alpha < 1 - \frac{v''(w_i)}{v'(w_i)} (\tilde{w}_i - w_i).$$

Given that $v(w_i)$ is strictly concave, the right-hand side of this last inequality is strictly larger than 1. Therefore, if $\alpha$ is close enough (from above) to 1 (recalling that $\alpha \in (1,2)$) then, indeed, $\frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} < 0$. With $\frac{n-i}{n}$ depending inversely on rank $i$, we conclude that a
rise in rank leads to an increase in relative risk aversion or, for that matter, that a fall in rank decreases relative risk aversion. Q.E.D.

What is the intuition for the requirement that $\alpha$ needs to be small enough? Given the range of possible values of this parameter, that is, given that $\alpha \in (1, 2)$, this requirement implies that for relative risk aversion to decrease when rank declines, relative deprivation should enter the individual’s utility function approximately linearly. The farther away the effect of changes in relative deprivation on utility is from being linear (that is, when changes in relative deprivation affect utility in a concave manner), the less likely it is that relative risk aversion will decrease with a fall in rank. This consequence coincides with intuition. When the relative deprivation term enters the utility function concavely, the magnitude of the negative impact of relative deprivation on utility escalates with increases in relative deprivation. In other words, for an individual with a concave relative deprivation term in the utility function, high relative deprivation is more painful / more costly in terms of utility loss than for an individual with a more or less linear relative deprivation term in the utility function. Thus, it is not surprising that individuals whose utility is affected approximately linearly by relative deprivation will be more prone to increased risk taking when they fall in rank than individuals whose utility is affected concavely by relative deprivation.

3. Conclusion

Combining a standard measure of relative deprivation and a standard measure of relative risk aversion enabled us to link causally variation in risk-taking behavior with changes in the level of relative deprivation. The idea that concern about experiencing relative deprivation maps onto risk-taking behavior in a systematic and predictable manner received empirical attention in several studies, and deserves further empirical inquiry.

Additional development of the modeling framework presented in this paper could involve replacement of the “partial” comparative statics, where one variable is changed while all other variables are held constant, with a “general” comparative statics that will take into account the simultaneous effects of a change in one variable on all variables. Such an approach will draw on the consideration that a change in risk-taking behavior can determine the distribution of wealth which, in turn, affects preferences. Another
approach might involve replacing the current modeling framework, which is based on a finite number of individuals, with a continuous distribution of wealth.

The theme of this paper belongs to a research program that seeks to explain variation in risk-taking behavior by incorporating social preferences or, to use an eloquent sociological term, by incorporating “social value orientations” (SVO). The core idea is to focus on an individual’s position in the income or wealth distribution. For example, Stark (2019) studies the relative risk aversion of an individual with particular social preferences: his wellbeing is influenced by his relative wealth, and by how concerned he is about having low relative wealth. Holding constant the individual’s absolute wealth, Stark obtains two results. First, if the individual’s level of concern about low relative wealth does not change, the individual becomes more risk averse when he rises in the wealth hierarchy. Second, if the individual’s level of concern about low relative wealth intensifies when he rises in the wealth hierarchy and if, in a precise sense, this intensification is strong enough, then the individual becomes less risk averse: the individual’s desire to advance further in the wealth hierarchy is more important to him than possibly missing out on a higher rank. As yet another example, assuming that an individual’s rank in the wealth distribution is the only factor determining the individual’s wellbeing, Stark et al. (2019) analyze the individual’s risk preferences in relation to gaining or losing rank, rather than the individual’s risk preferences towards gaining or losing absolute wealth. Stark et al. show that in this characterization of preferences, a high-ranked individual is more willing than a low-ranked individual to take risks that can provide him with a rise in rank: relative risk aversion with respect to rank in the wealth distribution is a decreasing function of rank. This result is robust to incorporating (the level of) absolute wealth in the individual’s utility function. The risk-taking behavior of the poor, the relatively poor, the rich, and the relatively rich is a fertile topic for future inquiry, both in sociology and in economics.
Appendix: Robustness of Claims 1 and 2 to a generalization of the utility function

Can Claim 1 of an inverse relationship between an individual’s relative risk aversion and the average wealth of wealthier individuals reside in a utility representation that is more general than (1)? The answer is yes. To this end, we let the utility function of the individual with wealth level \( w_i \) be

\[
u_i(w_i) = (1 - \beta_i) v(w_i) - \beta_i z \left( \frac{n - i}{n} (\bar{w}_i - w_i) \right), \tag{A1}\]

where, as before, \( v(\cdot) \) is a twice differentiable, strictly increasing, and strictly concave function. As to \( z(\cdot) \), it is a continuous, thrice differentiable, strictly increasing, and strictly convex function.

Because

\[
u_i'(w_i) = (1 - \beta_i) v'(w_i) + \beta_i \frac{n - i}{n} z' \left( \frac{n - i}{n} (\bar{w}_i - w_i) \right),
\]

and

\[
u_i''(w_i) = (1 - \beta_i) v''(w_i) - \beta_i \left( \frac{n - i}{n} \right)^2 z'' \left( \frac{n - i}{n} (\bar{w}_i - w_i) \right),
\]

then the coefficient of relative risk aversion is

\[
\rho_i(w_i) = -\frac{w_i \left( (1 - \beta_i) v''(w_i) - \beta_i \left( \frac{n - i}{n} \right)^2 z'' \left( \frac{n - i}{n} (\bar{w}_i - w_i) \right) \right)}{(1 - \beta_i) v'(w_i) + \beta_i \frac{n - i}{n} z' \left( \frac{n - i}{n} (\bar{w}_i - w_i) \right)} \tag{A2}
\]

and, thus,
\[
\frac{dr_i(w_i)}{d\tilde{w}_i} = \frac{w_i\beta_i \left(\frac{n-i}{n}\right)^2}{\left[1 - \beta_i n \beta_i \left(\frac{n-i}{n}\right)z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right)\right]^2} \times \left\{ \frac{n-i}{n} z'\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right) \right\}
\]

We know that
\[
u'(w_i) = \left(1 - \beta_i\right) v'(w_i) + \beta_i \frac{n-i}{n} z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right) > 0 ,
\]
that
\[
u''(w_i) = \left(1 - \beta_i\right) v''(w_i) - \beta_i \left(\frac{n-i}{n}\right)^2 z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right) < 0,
\]
and that
\[
z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right) > 0 .
\]

Therefore, the sign of \(\frac{dr_i(w_i)}{d\tilde{w}_i}\) depends on the sign of \(z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right)\). If
\[
z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right) < 0 ,
\]
then \(\frac{dr_i(w_i)}{d\tilde{w}_i}\) is negative, and the claim is proven. However, even when
\[
z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right) > 0 ,
\]
the inequality \(\frac{dr_i(w_i)}{d\tilde{w}_i} < 0\) will hold true if
\[
\frac{z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right)}{z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right)} < -\frac{\nu''(w_i)}{\nu'(w_i)} \frac{n}{n-i}
\]
which, for example, will be the case when
\[
z\left(\frac{n-i}{n}\left(\tilde{w}_i - w_i\right)\right) is sufficiently small.
Can Claim 2, that the individual’s relative risk aversion decreases when the individual’s rank is lowered, reside in a utility representation that is more general than (1)? The answer is yes. Once again, we let the utility function of the individual with wealth level $w_i$ be as in (A1), with the properties of $v(\cdot)$ and $z(\cdot)$ as before. Then, the coefficient of relative risk aversion, $r_i(w_i)$, is as in (A2). In order to investigate how $r_i(w_i)$ depends on the individual’s rank, we evaluate the derivative

$$\frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} = \frac{w_i \beta_i}{\left[ (1 - \beta_i)v'(w_i) + \beta_i \frac{n-i}{n} z' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right) \right]^2} \times \left[ u'(w_i) \frac{n-i}{n} \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right) z'' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right) + 2 z'' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right) \right] + u''(w_i) \left[ \frac{n-i}{n} (\tilde{w}_i - w_i) z'' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right) + z' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right) \right].$$

Because $u''(w_i) < 0$, $u'(w_i) > 0$, $z'(\cdot) > 0$ and $z''(\cdot) > 0$, a sufficient condition for $\frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} < 0$ to hold is that

$$\frac{n-i}{n} (\tilde{w}_i - w_i) z'' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right) + 2 z'' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right) < 0,$$

which is equivalent to requiring that

$$\frac{z'' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right)}{z' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right)} < -\frac{2n}{(n-i)(\tilde{w}_i - w_i)}.$$

A necessary condition for this inequality to hold is that $z'' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right) < 0$; this requirement is the same as the requirement regarding the sign of $z'' \left( \frac{n-i}{n} (\tilde{w}_i - w_i) \right)$ that was identified earlier in this appendix on revisiting Claim 1 and introducing $z(\cdot)$ under the utility specification in (A1).
References


