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EUREQua, CNRS, University of Paris 1
and IZA Bonn

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IZA
P.O. Box 7240
53072 Bonn
Germany
Phone: +49-228-3894-0
Fax: +49-228-3894-180
Email: iza@iza.org

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ABSTRACT

Do Workers Really Benefit From Their Social Networks?*

This paper provides a simple matching model in which unemployed workers and employers in large firms can be matched together through social networks or through more "formal" methods of search. We show that networks do not necessarily add new externalities and that some results previously obtained in the literature are questionable. Nevertheless, social networks can, in some case, substitute for labor market and this crowding-out effect may be socially costly. We show that a policy increasing the number of workers embedded in the social networks can increase the unemployment rate and decrease workers welfare. Since it is mostly the firms which benefit from larger social networks, transfers from the firms to the workers are necessary to make larger access to the social networks efficient.

JEL Classification: E24, J64, J68, Z13

Keywords: economic policy, matching, social networks, unemployment

Corresponding author:

François Fontaine
EUREQua-CNRS
University of Paris 1
106-112 bv. de l'Hôpital
75013 Paris
France
Email: francois.fontaine@univ-paris1.fr

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1 Introduction

The existence of social networks can deeply change the aims of public policies. Since the lack of social capital of some groups induces inequalities in employment opportunities, some active labor market policies try to encourage the establishment or the improvement of personal networks (McClure, 2000, OECD, 2001). For example, the Australians Working Together program aims at providing people the incentives to stay involved with their communities even if they are economically disadvantaged (OECD, 2003). The McClure Report (2000) claims that "by building their social capital (through stronger networks, trust and shared values), communities can offer individuals more opportunities for economic and social participation. A key part of community capacity building is connecting individuals in ways that enable people to support each other".

Indeed, a large proportion of people (about 50% on average) hear about or obtain jobs through friends and relatives (see: Rees, 1966, Granovetter, 1995, Holzer, 1988, Montgomery, 1991, Topa, 2000, for the U.S., Gregg and Wadsworth, 1996, for the U.K. and Addison and Portugal, 2001, for Portugal). Moreover, to a large extent, employers also use social networks. For example, Holzer (1987) reports that 36 percent of firms interviewed filled their last opening with referred applicants. Campbell and Marsden (1990) find that about half of a sample of 52 Indiana establishments make regular use of referred applicants. Accordingly, this intensive use of networks means that disadvantaged people who do not have access to contact networks have fewer employment opportunities than others. More generally, do workers really benefit from the intensive use of the social networks? By using networks, firms reduce their search costs. However this could induce an underinvestment in job advertising. Consequently, who will benefit from a policy enlarging the access to social networks?

First we provide a simple matching model in which unemployed workers and employers in large firms can be matched together through social networks or through more "formal" methods of search. We show that, in some cases, networks can substitute for markets, firms only using acquaintances of their employees to fill their vacancies. Moreover, we argue that social networks
do not necessarily add a new matching externality. Some previous studies have argued that the
existence of the social networks can deeply change the nature of the inefficiency of the labor
market (Calvo-Armengol et Zenou, 2002). We show that these results mostly hinge on the use
of a nonhomogeneous matching function and that if the standard Hosios-Pissarides condition
(Hosios (1990), Pissarides (2000)) is respected, a decentralized equilibrium with networks can be
efficient. Second, we investigate the impact of new active labor market policies aiming at increas-
ing individuals’ social capital by enlarging the access to networks. We show that such policies
can increase the congestion externalities and induce firms to substitute employees referrals for
job advertising. Eventually unemployment can increase and workers’ welfare decrease while it is
mostly firms which benefit from larger social networks. Consequently, transfers from the firms
to the workers are necessary to make larger access to the social network efficient.

In economics, some recent contributions investigate the way transmission of job information
through contact networks influences the job-worker matching process and its consequences on
economic inequalities. To our knowledge, Boorman (1975) was the first to provide a formal
network model which described the information structure of finding a job. In Boorman’s model,
networks are endogenous: contacts are developed by individuals who maximize their probability
of getting a new job in the event that they lose their present job. Boorman only focuses on the
supply side of the labor market, whereas we take into account both sides of the market. Accord-
ingly, for simplicity’s sake, we assume that the network structure is exogenous. Calvó-Armengol
and Zenou (2002) use Boorman’s framework, but without an endogenous network formation,
to provide a matching model in discrete time with contact networks and an endogenous arrival
rate of job offers driven by free entry. Calvuc and Fontaine (2002) provide a matching model
in continuous time in which individuals (workers and employers) can explicitly choose between
different search methods (with or without contact networks). In this framework, they look at
the way conditional unemployment benefits can improve welfare and decrease unemployment
by coordinating the individuals on the efficient method. Our model is simpler and take into
account that there is heterogeneity in the efficiency of networks, inducing wages and job finding
inequalities. Moreover, by using a one worker per firm framework, previous papers providing
a matching model with contact networks (Calvo-Armengol and Zenou, 2002, and Cahuc and Fontaine, 2002) have bypassed the fact that employees produce applicant for the firm where they work. In this case, the employer has an incentive to take account of the social productivity of its employees during the wage bargaining or to reward them when they finds an applicant.

Calvó-Armengol and Jackson (2004) also look at the way job contact networks entail inequalities, but in a different context, in which the job arrival rate is exogenous and where all individuals have the same network structure. In comparison with this contribution, we do not model social networks explicitly but endogenize the job arrival rate and the number of advertised jobs. Indeed, networks can sometimes substitute for market and damage workers excluded from the most efficient networks. In a recent contribution, Bentolila et al. (2003) argue that in the presence of imperfect informations on jobs and workers’ characteristics, networks can induce a mismatch of talents. In our model, we show that, even in an homogenous framework without information asymmetries, networks can decrease workers’ welfare. Besides, larger networks can entail an higher unemployment rate.

The model and its properties are presented in Section 2. In Section 3 we extend the model in order to take into account the inequalities produced by differential access to networks. We assume that the labor force is divided between social groups which differ in the efficiency of their social network. We derive the decentralized equilibrium. In Section 4 we investigate the effect of new labor market policies. We show that an economy with greater access to social networks does not necessary lead to a lower unemployment rate or higher workers’ welfare.

2 A simple model of social networks

We provide a matching model where employers take account of the fact that their employees produce applicants. We derive two possible equilibria: one where social networks substitute for the market, one where the firms use networks but also standard job advertising to fill vacancies. Eventually, we study the efficiency of the decentralized equilibrium.

2.1 The framework
In this subsection, we present the main feature of our model. The basic environment borrows from Pissarides’ (2000) matching model of the labor market. We build a matching function in which information about job offers can be transmitted to unemployed workers by standard job advertising or by current employees. We derive wages from a Nash bargaining rule and show that it does not hinge on the number of employees in the firm. This last point is important since the matching rate hinges on the number of employees in a firm.

It is worth noting that we only take into account the effects of social networks on the job arrival rate\(^1\). To a large extent, previous literature has focused on the informational effect while considering networks (Montgomery, 1991, Mortensen and Vishwanath, 1994). However, some empirical studies (Holzer, 1988, Margolis and Simonet, 2002) show that networks could mainly affect labor market outcomes by increasing the job arrival rate of workers and decreasing search costs. We aim at shedding some new light on these mechanisms.

*The matching function*

Sociological studies promotes the idea that the economic theory of job search bypasses the specific role of social networks in recruitment. Particularly, employees in a firm generate applicants for jobs that are not advertised by that firm. For instance Waldinger (1997) claims that “social networks produce applicants for employers who don’t yet have vacancies to fill”. In the same way, Granovetter, in his survey of the literature (1995), argues that “if employers do not advertise vacancies, this may be in part because they know they can be filled by friends and relatives of existing employees”. If job advertising is used by employers to produce matches, the existing employees can also find applicants. Accordingly, we distinguish in our model vacancies publicly advertised by employers (denoted by \(V\)) and offers transmitted by employees (the number of which depends upon the number of employees).

We consider a large number of firms producing a numeraire output and a large labor force whose size is denoted by \(N\). In our framework, a firm \(i\) produces \(F(L_i) = yL_i\), where \(L_i\) denotes the number of employees in firm \(i\). Hiring a worker and searching for a job are costly activities.

\(^1\)Calvó-Armengol and Zenou (2002) and Calhuc and Fontaine (2002) also provide theoretical models focusing on the effects of networks on the job arrival rate and search costs.
Employers and unemployed workers — the only job seekers, by assumption — are brought together in pairs through an imperfect matching process. However, we take into account the fact that the firms use different search methods. One is a high cost search method where each offer is sent and advertised at a cost \( h \) per unit of time. For example, the firm puts advertisements in newspapers and uses public or private agencies. We assume that each employee can forward job offers to their unemployed friends. Accordingly, the matching rate is

\[
M(\lambda L + V, N - L)
\]

Total number
of job offers

with \( V = \sum_i V_i \), the total number of publicly advertised jobs and \( L = \sum_i L_i \). As usual, the matching function is assumed concave, increasing in both its arguments and homogeneous of degree one\(^2\). \( \lambda \) denote the intensity at which each employee contacts unemployed workers. More generally, it also represent the efficiency of the social network: a high \( \lambda \) corresponds to a network where the information about job offers is easily transmitted to unemployed workers. Accordingly, in our framework, \( \lambda L + V \) is the total number of job offers.

Let us remark that this type of matching function could be based on an urn-ball process (see in Appendix A.1), in which each unemployed worker has an urn and each job offer is a ball that is sent into urns (i.e. an employer has one ball for each advertised job and each employee can also send a ball into the urns). In such a framework, \( \lambda \) denote the probability that an employee forwards a job offer towards the unemployed worker (to send a ball)\(^3\). Accordingly, at each period, the number of offers transmitted through the networks amounts to \( \lambda L \). For a large number of employees, unemployed and vacancies, this micro scenario leads to an homogenous matching function.

In our model, (1) implies that each firm \( i \) meets unemployed workers at a rate:

\[
(\lambda L_i + V_i) \frac{M(\lambda L + V, N - L)}{\lambda L + V} = (\lambda L_i + V_i)m(\theta)
\]

\(^2\)Let us remark that most of our results hold with a nonhomogenous matching function. Nevertheless this will increase the complexity of our framework without changing its qualitative properties.

\(^3\)We thus assume, for sake of simplicity, that an employee never forwards an offer toward another employee.
with $\theta = \frac{\lambda L + V}{N - L}$ and $m'(\theta) < 0$. $\theta$ denotes labor market tightness and $m(\theta)$ the probability that a match occurs thanks to the formal or informal channel. To be consistent with our model it also includes the offers sent by employees. If jobs are destroyed at a rate $q$, the law of motion for jobs is

$$\dot{L} = M(\lambda L + V, N - L) - qL$$  \hspace{1cm} (3)$$

**Expected utilities and profits**

Let us denote by $r$ the exogenous discount rate and $w(L_i)$ the wage of a firm using $L_i$ units of labor. The firm chooses the number of vacancies publicly advertised ($V_i$) and takes into account that its employees also produce applicants. The value function for the problem of the firm solves the Bellman equation:

$$\Pi(L_i(t)) = \max_{V_i(t) \geq 0}\left\{ \frac{1}{1 + r}\left\{ [gL_i(t) - w(L_i(t))L_i(t) - hV_i(t)] dt + \Pi(L_i(t + dt)) \right\} \right\}$$  \hspace{1cm} (4)$$

subject to

$$L_i(t + dt) - L_i(t) = ((\lambda L_i(t) + V_i(t))m(\theta(t)) - qL_i(t)) dt$$  \hspace{1cm} (5)$$

The firm chooses its optimal number of employees, knowing that the positions can be filled using two methods of search. On the one hand, jobs can be advertised by the firm. On the other hand, social networks of employees produce applicants at a rate $\lambda L_i m(\theta)$. Since the firm chooses the number of advertised vacancies at each period ($V_i(t)$), she controls the increase in the number of employees and, consequently, the subsequent number of applicants the social network will produce. Moreover, wages are the subject of bargaining and a firm can refuse to hire an applicant\(^4\). Besides, the firm could manipulate the wage by exploiting the fact that the hiring rate depends upon the number of employees\(^5\). Using (4), (5) and the Kuhn-Tucker conditions,

\(^4\)However, the Nash bargaining rule ensures that it is always profitable for a firm to hire an applicant (see below).

\(^5\)Stole and Zwiebel (1996) have shown that in some cases, intrafirm bargaining can lead to wages manipulation. In their framework it is due to the diminishing returns in the productivity of labor. In our model, the marginal productivity is constant and prevents this mechanism. However, the choice of the number of employees have an effect on the hiring rate. This element introduces a new possibility of manipulation.
ones gets, at the steady state,

$$\frac{h}{m(\theta)} = \frac{y - w(L_i) - w'(L_i)L_i}{r + q - \lambda m(\theta)} \leftrightarrow V_i > 0$$  \hspace{1cm} (6)$$

$$\frac{h}{m(\theta)} > \frac{y - w(L_i) - w'(L_i)L_i}{r + q - \lambda m(\theta)} \leftrightarrow V_i = 0$$  \hspace{1cm} (7)$$

(6) defines the optimal employment level when the optimal number of advertised vacancies is positive. In the second case (equation (7)) the expected cost of a job advertisement is higher than the expected profit. Consequently, the optimal number of advertised vacancies amounts to zero. Before presenting these two types of equilibria, we have to consider workers’ value functions and wages.

An unemployed worker benefits from an income flow $z$ and the expected exit rate from unemployment amounts to $\frac{M(\theta)}{N-\theta} = \theta m(\theta)$. All firms are assumed identical. Let us denote by $U$ the value function of an unemployed worker and by $E$ the value function of an employee in firm $i$. $U$ and $E$ satisfy at the symmetric steady state:

$$rU = z + \theta m(\theta)(E(L_i) - U)$$  \hspace{1cm} (8)$$

$$rE(L_i) = w(L_i) + q(U - E(L_i))$$  \hspace{1cm} (9)$$

Wage bargaining

“The employers view workers’ social connections as resources in which they can invest, and which might yield economic returns in form of better hiring outcomes” claim Fernandez et al.(2000). In our model, social connections are valuable since they increase the matching rate of the firm. This changes the value of jobs and influences wages through bargaining. Let us remark that we present in the Appendix A.2 another equilibrium where employers give a bonus when an employee finds an unemployed worker to fill the position. We show that this equilibrium is equivalent as regards the employment level of the economy and the number of job vacancies sent by the firms.
We consider an economy at the steady state. Wages are subject to bargaining between the firm and the worker and can be renegotiated each period at no cost. Then the surplus gotten by an employee paid wage \( w(L_i) \) is

\[
E(L_i) - U = \frac{w(L_i) - rU}{r + q}
\]

(10)

Besides, using (4) and (5), one gets the value of a marginal job:

\[
J(L_i) = \frac{y - w(L_i) - w'(L_i)L_i}{r + q - \lambda m(\theta)}
\]

(11)

Let us remark that \( J(L_i) \) is increasing with \( \lambda \), the intensity at which each employee tries to contact an unemployed worker. Indeed, a rise in \( \lambda \) increase the marginal productivity of an employee as regards the matching technology of a firm and, consequently, the hiring rate. The surplus of each match is shared according to the Nash solution of the bargaining problem

\[
\beta J(L_i) = (1 - \beta)(E(L_i) - U)
\]

(12)

with \( \beta \in [0, 1] \) the share that accrues to the worker. (12) leads to the following differential equation:

\[
w(L_i) = (1 - \beta) \left[ \frac{r + q - \lambda m(\theta)}{r + q - (1 - \beta)\lambda m(\theta)} \right] rU
\]

\[
+ \beta \frac{r + q}{r + q - (1 - \beta)\lambda m(\theta)} [y - w'(L_i)L_i]
\]

(13)

By integration the wage simplifies:

\[
w = (1 - \beta) \left[ \frac{r + q - \lambda m(\theta)}{r + q - (1 - \beta)\lambda m(\theta)} \right] rU + \beta \frac{r + q}{r + q - (1 - \beta)\lambda m(\theta)} y
\]

(14)

Consequently, the wage does not hinge on the number of employee in the firm. It is identical across firms. Indeed, in our framework, non-diminishing returns in the productivity of labor and the fact that the matching rate of a firm \( i \), \( (\lambda L_i + V_i)m(\theta) \), is a constant return function in \( L_i \) prevents the employer from manipulating wages as in Stole and Zwiebel (1996a, 1996b). Nevertheless, the firm takes into account the social productivity of its employees in the discount rate of the value of a marginal job \( (r + q - \lambda m(\theta)) \).
2.2 The decentralized equilibrium

In this subsection, we first define the circumstances under which firms do not advertise for vacancies and only use on their employees to find applicants. Then we study the case where firms use both methods of search.

*The decentralized equilibrium without job advertising*

We first define the conditions under which firms only use their employees to find applicants. In this case, the level of job offers depends only on the number of employees. While it is always possible for a firm to refuse an applicant, the Nash bargaining rule ensures that it is never profitable. When \( V = 0 \), (3) reads

\[
\dot{u} = (1 - u)q - M(\lambda(1 - u), u) \tag{15}
\]

At the steady state, the unemployment rate satisfies

\[
(1 - u)q = M(\lambda(1 - u), u) \tag{16}
\]

Consider an homogeneous matching function, concave in the unemployment rate. Since the matching rate is increasing in both its arguments, the right hand side of (16) is non monotonic. An increase in \( u \) induces more job seekers. However, it also induces a decrease in the number of employees and thus in the number of job offers. Eventually, the effect depends on the value of the unemployment rate. Since, the left hand side of (16) is a decreasing function in \( u \), multiple equilibria can occur (see Figure 1).

Let us remark that \( u_2^* = 1 \) is always a solution. No workers have a job and consequently there is no job offers in the economy. However, it is obvious from (15) that this equilibrium is unstable. For a large number of unemployed, the matching rate is higher than the destruction rate and a small deviation from \( u_2^* \) moves the unemployment rate away from this steady state. Besides, since \( M(\lambda(1 - u), u) \) is non monotonic, goes to zero when the unemployment rate goes to zero there is also an interior equilibrium \( u_1^* \) which is stable (Figure 1). Consequently, in an economy where networks are the only method of search, a non degenerate steady state (neither full employment, nor full unemployment) exists and is the only stable equilibrium.
Figure 1: The multiplicity and the stability of the equilibria in an economy without job advertising. The arrows reveal the direction in which \( u \) moves over time if it lies between 0 and \( u_1^* \) or between \( u_1^* \) and 1.

An equilibrium without job advertising arises when, for a value of the unemployment rate defined by (16), and with \( \theta = \lambda \frac{1-u}{u} \), we have

\[
\frac{h}{m(\theta)} > \frac{(1 - \beta)(y - z)}{r + q + \beta \theta m(\theta) - (1 - \beta)\lambda m(\theta)}
\]  

(17)

Accordingly, there is no job advertising at the decentralized equilibrium when advertising cost \( h \) is high and networks are very efficient (high \( \lambda \)). In the same way, large values of the bargaining power, \( \beta \), and the destruction rate, \( q \), the parameters that decrease the profit of a filled position (right hand side of (17)), can lead to an equilibrium with no advertising.

This type of equilibrium is only a special case of our model but explains why in some submarkets firms do not advertised for job offers. For example, Waldinger (1997) argues that “network recruitment offers the opportunity to detach the hiring process from the open market, allowing insiders to ration openings to their referrals”. Our model formally proves the validity of this claim and gives analytical conditions to such equilibrium. In a sense, our model show how networks could substitute for the market.\(^6\)

\(^6\)Brown et al. (2004), in an experimental study, show social interactions can entail that trading relations that
The decentralized equilibrium

We now study the case where firms use both methods of search. Using (14) and (8), the wage is

\[ w = z + \beta \frac{r + q + \theta m(\theta)}{r + q + \beta \theta m(\theta) - (1 - \beta)\lambda m(\theta)} (y - z) \]  

Equation (18) is very close to that of standard matching models (see for example Pissarides (2000)). Nevertheless, in our framework, wages are increasing functions of the efficiency of networks \( \lambda \) because an higher efficiency induces an higher arrival rate but also because firms directly take into account during wage bargaining that their employees produce applicants (represented by \(-(1 - \beta)\lambda m(\theta)\) in (18)).

It is worth noting that empirical evidence suggests that networks have an ambiguous effect on wages (e.g. Simon and Warner, 1992, Granovetter, 1995, Marmaros and Sacerdote, 2002, Kugler 2003, Bentolila et al., 2003). In our simple model with homogenous firms and workers, the individuals have the same wage regardless of the channel by which they have found their jobs. However, the existence of networks has an effect on wages by defining workers’ outside opportunities and by reducing firms’ search costs. Remember that we focus on the impact of networks on the job arrival rate and search costs, and not on the allocation of workers accross occupation.

We now consider the equilibrium value of the labor market tightness. (6) together with (18) imply that labor market tightness satisfies the following condition:

\[ \frac{h}{m(\theta)} = \frac{(1 - \beta)(y - z)}{r + q + \beta \theta m(\theta) - (1 - \beta)\lambda m(\theta)} \]  

In equilibrium, the expected cost of an advertised vacancy, represented by the left–hand side, equates to the expected profit of a filled position, represented by the right–hand side. It can easily be checked that (19) defines a unique equilibrium value of labor market tightness. The higher the number of employees, the higher the number of job offers transmitted by networks. However, this induces a substitution effect and the number of advertised jobs decreases. This are subject to outside competition ex-ante become insulated ex-post. However, few theoretical works investigate this possibility (see references therein).
substitution effect ensures the uniqueness of equilibrium (displayed on Figure 2).

![Figure 2: The decentralized equilibrium with both methods of search](image)

Formally, the discontinuity in the value of a marginal job is due to the fact that for \( \bar{\theta} \), one gets \( r + q + \beta \bar{\theta} m(\bar{\theta}) = (1 - \beta) \lambda m(\bar{\theta}) \). For \( \theta < \bar{\theta} \) the value of a marginal job becomes negative: firms will not advertise for a job. Consequently, the equilibrium value of tightness \( \textit{when both methods are used} \) is always greater that \( \bar{\theta} \) and thus unique.

At the steady state, the law of motion for jobs is

\[
u = \frac{q}{q + \bar{\theta} m(\bar{\theta})}
\]

(20)

Accordingly, (19), together with (20), defines an unique value of the unemployment rate.

It is also worth noting that the unemployment rate decreases with \( \lambda \). Indeed, an increase in the effectiveness of social networks leads to an improvement in the matching function. On the one hand, it entails a rise in the wage (see (14)) since the expected exit rate from unemployment increases. However, the expected cost of a publicly advertised vacancy decreases and offsets the first effect. Eventually, the number of employees increases.
2.3 Efficiency

This subsection is devoted to the analysis of the efficient allocation and its comparison with the decentralized equilibrium. We begin by defining the efficient allocation. Then, we determine the values of the parameters such that the decentralized equilibrium is efficient\(^7\).

The social planner chooses the number of vacancies publicly advertised (i.e. \(V\)) at each date \(t\) that maximizes the discounted value of the stream of production. Consequently, the value function of the social planner, denoted by \(W\) satisfies:

\[
W(t) = \max_{V(t)} \left( \frac{1}{1 + rd_t} \right) \left\{ [yL(t) + (N - L(t))z - V(t)h] dt + W(t + dt) \right\}
\]

subject to \(L(t + dt) - L(t) = (M(\lambda L(t) + V(t), N - L(t)) - qL(t)) dt\)

The first order and envelope conditions imply that labor market tightness satisfies \(\theta^*\):

\[
\frac{h}{m(\theta^*)} = \frac{(1 - \eta(\theta^*))(y - z)}{r + q + \eta(\theta^*)\theta^* m(\theta^*) - (1 - \eta(\theta^*))\lambda m(\theta^*)}
\]  \(21\)

with \(\eta(\theta)\) the elasticity of the matching function with respect to the unemployment rate.

The comparison of the decentralized equilibrium, defined by (19), with the efficient allocation, defined by (21), shows that the decentralized equilibrium is efficient if and only if the share of surplus that accrues to the worker, \(\beta\), is equal to the elasticity of the matching function with respect to the number of unemployed workers. At first glance, it looks surprising that we get the standard Hosios-Pissarides condition (Hosios (1990), Pissarides (2000)). This result shows that taking into account the existence of social networks does not necessary change the relation between the decentralized equilibrium and the social optimum. Accordingly, some of the results obtained by previous papers (Calvo-Armengol and Zenou, 2002, and Cahuc and Fontaine, 2002) are questionable and are due to the use of a nonhomogeneous matching framework. Social networks do not add necessarily a new externality.

3 Social networks and inequalities

\(^7\) Since an economy with or without bonuses leads to the same equilibrium, we do not have to distinguish between the different types of equilibria.
The model we have provided can be easily extended for various applications. In this section, we aim at providing a model which take into account the inequalities which might be created by job contact networks. We extend our model in a very simple way by assuming that the labor force is divided between groups of individuals which differ as regards the effectiveness of their social networks.

*When the return of the social capital differ across workers*

Empirical evidences suggests that different outcomes in job search hinge on the inequality of social networks. For instance, Petersen et al. (2000) argue that recruitment “need not be discriminatory in intent or design, but women and ethnic minorities may have lower access to social networks having higher rates of success in hiring”. We provide an extension of our model that takes into account that social networks differ in efficiency. Homophily, that is the tendency of socially similar people to band together, implies that an employee of a social group forwards job offers only to unemployed workers belonging to the same group. In our framework, workers do not differ with respect to their productivity since we want to study the effects of social networks which only depend on inequality with respect to efficiency of the networks.

Let us assume that the labor force is divided between two groups $j$ with $j = \{1, 2\}^8$. Let us denote by $\gamma$ the proportion of individuals in the labor force from group 1. Accordingly, we have $\gamma N = N_1$ and $(1 - \gamma)N = N_2$ with $N_1$ (respectively $N_2$) denoting the size of the labor force of the first group (respectively of the second group). The key assumption is that these two groups do not have the same effectiveness (denoted by $\lambda_1$ and $\lambda_2$) in generating employment opportunities.

As before, the firm can use formal methods and advertise for a vacancy or use its employees to find new workers. We denote by $L_1$ the number of employees who belong to the first group and $L_2$ the number of employees who belong to the second group. In the same way, we denote by $u_j$ the unemployment rate defined by $U_j/N_j$, with $U_j$ the number of unemployed workers in group $j$ and $v \equiv V/N$. Labor market tightness is defined by $\theta = (\lambda_1 L_1 + \lambda_2 L_2 + V)/(N - L_1 - L_2)$

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8Let us remark that it is straightforward to extend this framework to $n$ groups of workers. However, for sake of simplicity, and because it does not affect our results, we assume that there are only two types.
since employees of the two groups forward job offers at different rates. Eventually, the number of job matches taking place per unit of time is given by the following homogenous matching function \( M(\lambda_1 L_1 + \lambda_2 L_2 + V, N - L_1 - L_2) \).

**Expected utilities and profits**

When firms decide their optimal employment policies, they have to distinguish between the different types of employees. Thus, we rewrite the value functions in order to take into account the heterogeneity of the labor force. Firm \( i \) solves the following Bellman equation:

\[
\Pi(L^i_1(t), L^i_2(t)) = \max_{V_i(t)} \left\{ \frac{1}{1 + rd} \left\{ [y(L^i_1(t) + L^i_2(t)) - w(L^i_1(t)L^i_1(t) - w(L^i_2(t)L^i_2(t))] \right. \\
- hV_i(t)]dt + \Pi(L^i_1(t + dt) + L^i_2(t + dt)) \right\} 
\]

subject to

\[
L^i_1(t + dt) - L^i_1(t) = (\lambda_1 L^i_1(t) + pV_i(t))m(\theta(t)) - q L^i_1(t)) dt
\]

\[
L^i_2(t + dt) - L^i_2(t) = (\lambda_2 L^i_2(t) + (1 - p)V_i(t))m(\theta(t)) - q L^i_2(t)) dt
\]

with \( p \) denoting the probability that an offer, sent randomly through the formal channel, is received by an unemployed worker who belongs to the first group. This probability amounts to \( p = U_1 / (U_1 + U_2) = \gamma u_1 / (\gamma u_1 + (1 - \gamma) u_2) \). Let us remark that offers forwarded by an employee only reach unemployed within his contact network, i.e., by assumption, within his group. \( m(\theta) \) denotes the probability to find an applicant for a job offer and \( (\lambda_1 L_1 + pV) \) the total number of job offers which reaches the first group unemployed workers. Accordingly, the rate at which an unemployed individual belonging to group 1 gets an offer is

\[
\frac{(\lambda_1 L_1 + pV)m(\theta)}{U_1} \cdot \frac{1}{U_1} \cdot \frac{1}{U_1}
\]

Job offers arrival rate to the first group unemployed workers

Probability that an unemployed worker \( j \) gets the offer

\[
= \left\{ \frac{1}{\psi_1} \left( \frac{1 - u_1}{u_1} V + \frac{v}{u_1 + (1 - \gamma) u_2} \right) m(\theta) \right\}
\]

(25)
In the same way, $\psi_2$ amounts to
\[
\left\{ \lambda_2 \frac{1-u_2}{u_2} + \frac{v}{\gamma u_1 + (1-\gamma)u_2} \right\}
\]
and $\psi_2 m(\theta)$ is the probability for an unemployed worker from the second group to receive a job offer.

The value functions of the workers can be rewritten as follows, with $w^j$ the wage rate for an employee of the group $j$:

\[
\begin{align*}
 rU &= z + \psi_1 m(\theta)(E - U) \quad (26) \\
 r\bar{U} &= z + \psi_2 m(\theta)(\bar{E} - \bar{U}) \quad (27) \\
 rE &= w^1 + q(U - E) \quad (28) \\
 r\bar{E} &= w^2 + q(\bar{U} - \bar{E}) \quad (29)
\end{align*}
\]

with $U$ and $E$ the value functions for an unemployed worker and an employee of the first group, and $\bar{U}$, $\bar{E}$ the value functions for the second group.

The decentralized equilibrium

Using (22), (26), (27), (28) and (29), simple replication of previous reasoning implies that the wages amounts to:

\[
w^j = z + \beta \left( \frac{r + q + \psi_j m(\theta)}{r + q + \beta \psi_j m(\theta) - (1-\beta)\lambda_j m(\theta)} \right) (y - z) \quad (30)
\]

Moreover, using the first order condition of equation (22), the envelope condition for an optimal choice of $v$ and the wage equations, labor market tightness must satisfy the following condition:

\[
\frac{h}{m(\theta)} = p \frac{(1-\beta)(y - z)}{r + q + \beta \psi_1 m(\theta) - (1-\beta)\lambda_1 m(\theta)} + (1-p) \frac{(1-\beta)(y - z)}{r + q + \beta \psi_2 m(\theta) - (1-\beta)\lambda_2 m(\theta)}
\]  

(31)

As usual, in equilibrium, the expected cost of an advertised vacancy, represented by the left-hand side of (31), equates to the expected profit of a filled position, represented by the right-hand
side. In our framework, the expected profit of a marginal job is a weighted average of the profit if the advertised job offer is received by an unemployed worker of the first group which occurs with a probability \( p \), and of the profit if the vacancy is received by a unemployed worker of the second group, which occurs with a probability \( 1-p \).

In order to define the symmetric decentralized equilibrium at the steady state, we must use the flow equations (23) and (24) for \( \dot{L}_1 = \dot{L}_2 = \dot{L} = 0 \). One gets:

\[
(\lambda_1 \gamma(1 - u_1) + pv) m(\theta) = q \gamma(1 - u_1)
\]

\[
(\lambda_2 \gamma(1 - u_2) + (1 - p)v)m(\theta) = q(1 - \gamma)(1 - u_2)
\]

For each type of employee the number of job destructions must equal the number of new created jobs. Consider a CRS matching function \( M(\lambda_1 L_1 + \lambda_2 L_2 + V, N - L_1 - L_2) = (\lambda_1 L_1 + \lambda_2 L_2 + V)^\alpha (N - L_1 - L_2)^{1-\alpha} \). Using the job creation condition and the flow equations, one gets that the unemployment rate at the steady state must satisfy

\[
\frac{h}{f(u_1, u_2)} = p \frac{(1 - \beta)(y - z)}{r + q + \beta q \gamma \frac{1-u_1}{u_1} - (1 - \beta) \lambda_1 f(u_1, u_2)} + (1 - p) \frac{(1 - \beta)(y - z)}{r + q + \beta q (1 - \gamma) \frac{1-u_2}{u_2} - (1 - \beta) \lambda_2 f(u_1, u_2)}
\]

(32)

and

\[
\frac{1 - u_1}{u_1} (q - \lambda_1 f(u_1, u_2)) = \frac{1 - u_2}{u_2} (q - \lambda_2 f(u_1, u_2))
\]

(33)

with

\[
f(u_1, u_2) = \left( \frac{\gamma u_1 + (1 - \gamma) u_2}{q (1 - \gamma u_1 - (1 - \gamma) u_2)} \right)^{(1-\alpha)/\alpha}
\]

Neither (32), nor (33) define an unambiguous relation between \( u_1 \) and \( u_2 \). Accordingly, this could entail multiple equilibria for some parameters values. However, it is impossible to find simple restrictions on the parameter space which guarantee the uniqueness of the equilibrium. Consequently we check in the next section whether multiple equilibria appear during our numerical simulations for realistic parameters values. They do not.
4 Is it always efficient to increase the social capital of individuals?

For some disadvantaged workers, matching is generally poor because their information about job offers is not sufficient. These workers, unable to use the informal channel, have a lower exit rate from unemployment (Hansen and Pratt, 1991, Petersen et al., 2000). Accordingly, some active labor market policies try to encourage the establishment or the improvement of personal networks (McClure, 2000, OECD, 2001). Sociologists have already discussed various schemes for getting disfavored groups back to work (see Granovetter (1995) and the references therein). These policies rely on preexisting social network or construct referral networks artificially. In our model, a policy which extends the proportion of individuals that have a contact network could correspond to a policy inducing a rise in \( \gamma \). We investigate the effect of this policy on unemployment and welfare.

Parameterization

We use the model of the previous section and, for the sake of simplicity, we assume that the second group of workers has no job contact networks: \( \lambda_2 = 0 \) and \( \lambda_1 = \lambda > 0 \). This assumption represents the fact that some individuals are excluded from some social activities, or, more precisely, from networks which have relevant informations about job offers\(^9\). We show that, surprisingly, an economy where more individuals have access to social networks does not necessary lead to a lower unemployment rate and to a higher level of welfare for the workers. Indeed, larger networks induce a non-neutral substitution between network and market. To a large extent, it is the firms which benefit from larger social networks. Consequently, transfers from firms to workers could be necessary to make active labor market policies efficient.

We parametrize our model to match the french labor market in the late 90’s. This example is only illustrative and the results we get are robust to a large range of values. We take the period to be one year and therefore set the discount rate to \( r = 0.05 \). We assume that the

\(^9\) Moreover, we get exactly the same results in a framework where \( 0 < \lambda_2 < \lambda_1 \).
matching function is Cobb-Douglas\textsuperscript{10} : 

\[ M(\lambda_1 L_1 + V, N - L_1 - L_2) = A(\lambda_1 L_1 + V)\alpha(N - L_1 - L_2)^{1-\alpha}. \]

A represent the \textit{ex-ante} efficiency of the matching technology and is calibrated to get a unemployment consistent with the observed one (about 9\%). The productivity of a new job is normalized at unity and, as it is usually do in the matching literature, \( \alpha \) and \( \beta \) are set to satisfy the Hosios condition for social efficiency and the estimations of the elasticity of the matching function (see Mortensen and Pissarides, 1999).

Cahuc et al. (2003) have estimated on french data that the yearly destruction rate ranges between .03 and .1. To be consistent with these findings, we set the destruction rate to the typical value of .06. Besides, we calibrate the search cost of a firm for an advertised job to represent 30\% of the yearly productivity \( y \) of an employee to be consistent with survey results reported by Hamermesh (1993). Eventually, the value of leisure amounts to 0.45. Interpreted as unemployment benefit, and given that the simulated average wage is 0.9, this induces a replacement ratio of 50\%, consistent with the french replacement ratio.

Descriptive statistics on the french labor force survey\textsuperscript{11} (\textit{Enquête emploi}) show that about 80\% of the individuals \textit{use} their friends and relatives to find a job whereas one third \textit{find} their job through friends and relative. Accordingly, we set \( \lambda \) in order to get, when \( \gamma = .8\textsuperscript{12} \),

\[ \frac{\lambda_1 L_1}{\lambda_1 L_1 + V} \sim \frac{1}{3} \]

Let us remark that we control for the possibility of multiple equilibria. In parameters values we adopt for the calibration the equilibrium is always unique. Our calibration is reported in Table 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( y )</th>
<th>( A )</th>
<th>( h )</th>
<th>( z )</th>
<th>( \beta )</th>
<th>( \alpha )</th>
<th>( r )</th>
<th>( q )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>1</td>
<td>.5</td>
<td>.3</td>
<td>.45</td>
<td>.5</td>
<td>.5</td>
<td>.05</td>
<td>.06</td>
<td>.06</td>
</tr>
</tbody>
</table>

Tab. 1 - Parameters values (french economy)

\textsuperscript{10} Most empirical applications of the matching theory assume a Cobb-Douglas form with constant returns to scale (for a survey of the matching functions see Petrongolo and Pissarides (2001)).

\textsuperscript{11} We compute these statistics for 1998. Let us remark that they do not really depend on the level of education of the individuals.

\textsuperscript{12} It is worth noting that in our model there is no difference between the share of workers embedded in social networks and the share of individuals using their social networks. Indeed, there is no endogenous choice of the search method. See Cahuc and Fontaine (2002) for a model where workers and firms can choose between formal and informal method of search.
The unexpected effects of an increase in social capital

In our framework, the parameter $\gamma$ denotes the share of the labor force who have access to contact networks. At first glance, an increase in the number of workers embedded in a social network may improve the labor market by extending a better matching technology to more workers. However, as illustrated by Figure 3, an higher number of individuals embedded in networks can have negative impacts on workers, whatever their type.

![Graphs showing the effect of $\gamma$ on unemployment rates $u_1, u_2, u$ and wages inequality $w_1 - w_2$.]

Figure 3: The effect of an increase in $\gamma$ on the unemployment rates $(u_1, u_2, u)$ and wages inequality $w_1 - w_2$.

First, since $u_1 < u_2$, an increase in the proportion of workers with social networks ($\gamma$) induces mechanically a fall in the global unemployment rate ($u = \gamma u_1 + (1 - \gamma)u_2$), leading, taken all parameters as given, to an increase in the expected cost of a publicly advertised vacancy (congestion effect). Taking the optimal employment level as given, the rise in the number of the workers embedded in networks increases the matching rate and consequently decreases the number of vacancies that the firm has to advertise to find applicants (substitution effect).
Consequently, the firms publicly advertise less vacancies \((V)\) and the unemployment rate of each category of workers \((u_1\) and \(u_2)\) increases. For some value of \(\gamma\) (here around 0.7) the decrease in the vacancy rate \(v\) balances the rise in the number of workers with a contact network and the unemployment rate eventually increases. The rise in wages inequality comes from the rise in the unemployment rate of both types of individuals. For these values of the parameters, both wages decrease, but \(w^1\) more slowly than \(w^2\). Indeed, the unemployed workers embedded in a job contact network suffer less from the decrease in the number of vacancies advertised by the employers since the use of networks limits the fall in the job offer arrival rate. Hence an increase in the social capital of disadvantaged unemployed can entail a higher unemployment rate and wages inequality.

**Who benefit really from social networks?**

Eventually, Figure 4 shows that a larger access to networks leads to a fall in the value of both employment and unemployment. On the one hand, the expected values of a job for both types of workers \((rE, r\bar{E})\) decrease since the wages decrease. Moreover, since the rise in the unemployment rate induces a decrease in the probability of finding a job, the expected value of unemployed workers \((rU, r\bar{U})\) decreases.
Figure 4: The effect of an increase in $\gamma$ on welfare.

As regards total welfare $W$, it increases with the share of workers who belong to a social network (see Figure 5). First, firms advertise less vacancies and, accordingly, the total cost of the vacancies $hv$ decreases and offsets the negative effect of $\gamma$ on welfare. Moreover, some individuals who did not belong to a social network now belong to one and the welfare of these workers has increased. Nevertheless the increase in $\gamma$ harms the workers who stay in the same group as before (with or without network).

Figure 5: The effect of an increase in $\gamma$ on total welfare $W$ and on workers total welfare $Wt$. 

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Let us denote by $W$ total welfare and $Wt$ workers welfare:

$$\begin{align*}
W &= y \times (1 - \gamma u_1 + (1 - \gamma)(1 - u_2)) + z(\gamma u_1 + (1 - \gamma)u_2) - hv \\
Wt &= y \times (1 - \gamma u_1 + (1 - \gamma)(1 - u_2)) + z(\gamma u_1 + (1 - \gamma)u_2)
\end{align*}$$

If we only take into account total welfare, the increase in the number of individuals in social networks has an unambiguous effect. Indeed, this increase improves the matching technology. However, for some values on $\gamma$ (beyond the point $\gamma = 0.7$ in Figure 6) only firms benefit from this rise. Beyond some threshold, workers welfare $Wt$ decreases\(^1\) and the rise in total welfare is mostly due to the reduction of advertising cost : the firms advertise less vacancies, $hv$ falls. Hence, an economic policy which tries to improve access to social networks should set up transfers from firms to workers in order to lead to a rise in workers’ welfare.

Thus, the extension of the use of the informal channel (an increase in $\gamma$) have ambiguous outcomes. On the one hand, it betters the matching technology. On the other it entails congestion and substitution effects which induce a decrease in the number of advertised jobs. The unemployment rates of both types of workers can increase and workers’ welfare decrease.

Our results shed some light on the consequences of a larger use of social networks on welfare. Bentolila et al (2003) have shown that networks could induce mismatch of talents and thus could be inefficient. Our model show that even in a framework with no workers and firms heterogeneity, networks’ effect on workers welfare can be negative. It is, to a large extent, the firms which benefit from this change since it reduces their search costs, while workers can suffer from congestion effect and from the decrease of the number of advertised vacancies. Consequently firms should contribute to the establishment of the new active policies. Transfers from the firm to the workers are necessary since neither the wages, nor monetary bonuses to employees (see in appendix) are able to counteract the negative effect of the extension of the use of the informal channel.

\(^1\) Besides, before this threshold there is only a weak increase in $Wt$. 

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5 Conclusion

We provide a simple matching model with large firms in which unemployed workers and employers can be matched together through social networks or through more "formal" methods of search. We argue that, contrary to what have been assumed by previous works, social networks do not add necessarily a new externality. Besides, we show that networks can substitute for markets and define the conditions under which firms only use their employees to find applicants.

We use our model to investigate the impact of new active labor market policies relying on social networks. Surprisingly, our model show that even in a framework with no workers and firms heterogeneity, networks' effect on workers can be negative. Indeed, an increase in the number of workers embedded in social networks can decrease welfare for both types of workers. Even if firms benefit from this increase by reducing their search costs, workers suffer from congestion effect and from the decrease of the number of advertised vacancies. Hence, we argue that firms should contribute to the establishment of the new active policies and that firms to workers transfers could be implemented to increase the efficiency of such policy.

Our results have been obtained with a very simple model where we treat the network structure as given. In comparison with Calvo-Armengol and Jackson (2004), this model particularly well-suited to the study of economic policy and economic inequality\textsuperscript{14}. However, it would be interesting to let the parameter which drives the effectiveness of the networks ($\lambda$) be the outcome of agent’s decisions. For example, agents could invest in order to improve it (increase $\lambda$). This strategic element could shed some new light on the efficiency of active labor market policies and on the evolution of social inequalities. In this paper, we investigate how networks can affect labor market outcomes by increasing the job arrival rate of workers. Consequently, we choose to not take into account that networks can generate a mismatch between heterogenous workers and heterogenous occupations. We have now to introduce this dimension. Theses issues are on our research agenda.

\textsuperscript{14}Calvo-Armengol and Jackson (2004) study employment inequality. However, in their model, inequality across agents arises because workers can “drop-out” of the networks and become inactive.
Appendix:
A.1. An urn-ball matching model

Consider an economy with $V$ advertised vacancies and $U$ unemployed workers. A job offers is represented by a ball which is sent into urns, i.e. the unemployed. Besides, at each period, each employee also contact unemployed friends (send a ball into the urns) with a probability $\lambda$. We assume that every worker has the same probability to get a job offer. According to this scenario, there are $\lambda L + V$ balls (job offers) and $U$ urns (unemployed). At each period, the number of job offers received by a given job seeker follows a binomial distribution $B(\lambda L + V, \frac{1}{U})$. The aggregate hires amounts to the number of job seekers who receive at least one offer, that is

$$M = U \left[ 1 - \left( 1 - \frac{1}{U} \right)^{\lambda L + V} \right]$$

For a large number of job offers and a large number of unemployed, this binomial distribution can be approximated by a Poisson distribution $P(\theta)$, with $\theta = \frac{\lambda L + V}{U}$ the labor market tightness. Thus, the probability a match for a job offer $M/(\lambda L + V)$ can be approximated by

$$m(\theta) = \frac{1}{\theta} (1 - \exp(-\theta))$$

Thus we have $M \approx (\lambda L + V) m(\theta)$, i.e. the matching function exhibits approximate constant return to scale.

Besides

$$\frac{\partial m(\theta)}{\partial \theta} = (\theta \exp(-\theta) - 1 + \exp(-\theta)) / \theta^2$$

which is negative as long as $1 - \exp(-\theta) > \theta \exp(-\theta)$. Remark that $1 - \exp(-x) - x \exp(-x) = 0$ when $x = 0$ and that the derivative of this function with respect to $x$ is positive for $x > 0$. Hence $\frac{\partial m(\theta)}{\partial \theta} < 0$. For recent urn-ball matching models see Cahuc and Fontaine (2002) and Albrecht et al. (2002).

A.2. The decentralized equilibrium with bonuses

Let us assume that, when an employee finds an unemployed worker to fill a position, the employer can give him a bonus $b$ to reward his effort\footnote{Monetary bonuses are a common organizational practice (see Fernandez et al., 2000).}. In this case, the equations (4) and (9), that define the value functions of the firm and of an employee, can be rewritten as follows:

$$\Pi(L_i(t)) = \max_{V_i(t)} \left\{ \frac{1}{1 + r \Delta t} \left[ \frac{1}{1 + r \Delta t} \right] \left[ y_1 L_i(t) - w(L_i(t)) L_i(t) - h V_i(t) - \lambda L_i(t)m(\theta(t)) b \right] \right\}$$

$$+ \Pi(L_i(t + \Delta t)) \right\} \right)$$

subject to the law motion of jobs:

$$L_i(t + \Delta t) - L_i(t) = (\lambda L_i(t) + V_i(t)) m(\theta(t)) - q L_i(t)$$

and

$$r E(L_i) = w(L_i) + \lambda m(\theta) b + q (U - E(L_i))$$

(35)
\( \lambda m(\theta) \) denotes the rate at which each employee contacts an unemployed friend to work in his firm, and consequently, \( \lambda L \Sigma m(\theta) \) denotes the rate at which an employer is matched with unemployed worker thanks to the informal channel. We assume that the bonus is bargained according to a Nash bargaining rule like the wages. The firm is matched with the employee’s contact only if this bargaining is successful. The Nash solution maximizes the weighted product of the worker’s and the firm’s net return from the information about this match opportunity. If the bargaining is successful, the employee gets \( b \), but if it is not his permanent income \( rE \) is unchanged. In the same way, if the firm receives the information it gets the value of a marginal job, which is, according to (34)

\[
J = \frac{y - w - \lambda m(\theta)b}{r + q - \lambda m(\theta)}
\]

but has to give \( b \) to the employee. Therefore the reward for the information \( b \) satisfies:

\[
b = \text{Arg max}_{b'} \left( b' \right)^{\beta'} \left( \frac{y - w - \lambda m(\theta)b'}{r + q - \lambda m(\theta)} - b' \right)^{1-\beta'}
\]

where \( \beta' \) may be interpreted as a relative measure of the employee’s bargaining strength. For the sake of simplicity, it is assumed that \( \beta' = \beta \). The first order condition reads:

\[
b = \frac{\beta(y - w)}{r + q - (1 - \beta)\lambda m(\theta)}
\]

(36)

Moreover, in a decentralized equilibrium with bonuses, the wage is derived as before and we get:

\[
w = (1 - \beta) \left[ \frac{r + q - \lambda m(\theta)}{r + q - (1 - \beta)\lambda m(\theta)} \right] rU + \beta \frac{r + q}{r + q - (1 - \beta)\lambda m(\theta)} y - \lambda m(\theta)b
\]

(37)

Eventually, the labor market tightness satisfies the following condition:

\[
\frac{h}{m(\theta)} = \frac{(1 - \beta)(y - z)}{r + q + \beta \theta m(\theta) - (1 - \beta)\lambda m(\theta)}
\]

(38)

This equations shows that the two types of equilibria are equivalent as regards the employment level of economy. By decreasing the wage rate (see (37)), the bonus keeps the value of the marginal job unchanged. Accordingly, the firm’s hiring policy does not change.

Besides, the equation (37), together with (36) implies:

\[
w = (1 - \beta)rU + \beta y
\]

(39)

which is the usual wage equation of the matching models (see for example Pissarides (2000)).
References


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