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Uncertainty Shocks and Unemployment Dynamics

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ABSTRACT

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Recent events suggest that uncertainty changes play a major role in U.S. labor market fluctuations. This study analyzes the impact of uncertainty shocks on unemployment dynamics. Using a vector autoregression approach, we show that uncertainty shocks measured by stock market volatility have a significant impact on the U.S. unemployment rate. We then develop a quantitative version of the Diamond-Mortensen-Pissarides (DMP) model, in which uncertainty shocks hit the economy. Given the significant nonlinearities of the DMP model, we show that the introduction of uncertainty shocks not only allows this textbook model to account for observed characteristics of the U.S. labor market dynamics, with reasonable values for calibrated parameters, but also for the impact of rare episodes such as economic crises.

JEL Classification: E24, E32, J64

Keywords: uncertainty shocks, unemployment dynamics, search and matching, non-linearities

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1 Introduction

After the last recession, many economists emphasized the importance of understanding the role of uncertainty in shaping labor market outcomes, as it affects core expectations of future economic activity, which is important for investment and employment decisions. Large increases in uncertainty occur in periods of big changes. Economic uncertainty increased significantly during the 2008 financial crisis. The Brexit announcement is another important source of uncertainty for businesses in the UK (see Bloom et al. (2018)). Finally, the current coronavirus crisis also seems to suggest that significant uncertainty may lead to a deep recession. In sum, macro uncertainty seems to be countercyclical: episodes of high uncertainty are times of low economic activity. These rises in uncertainty can have persistent effects, by affecting expectations that drive long-term commitments, such as hiring decisions, and thus, unemployment dynamics.

We focus on the impact of uncertainty shocks on the aggregate unemployment rate. We show that lower vacancies and employment are characteristics of the economy in case of high variance of marginal returns, that is, when uncertainty increases. To understand this phenomenon, we use the Diamond (1982)-Mortensen (1982)-Pissarides (1985) matching model (DMP model). Although this model has become the dominant framework for analyzing labor market fluctuations, extensive analysis of its non-linearity—by Petrosky-Nadeau and Zhang (2017), Petrosky-Nadeau et al. (2018), Ferraro (2018), Ferraro (2020) and Adjemian et al. (2019)—is relatively recent. Moreover, these developments do not take into account that uncertainty can change over the business cycle.¹ This study bridges this gap.² More precisely, we show that a negative transitory shock has a larger effect when the variance is high. Therefore, time-varying uncertainty can magnify the impact of recessions; fixed costs lead decision makers to delay hiring decisions until they have clearer information about the future state of the economy. Moreover, in periods of large uncertainty, the optimal number of vacancies in recessions can be zero. This induces deep recessions, which are persistent because the matching model is nonlinear and asymmetrical. Indeed, if the small pool of vacancies for many unemployed workers gives firms incentives to post new vacancies just after the trough of a recession, the recovery implies that additional vacancies will not only cause a dribble in the probability of filling a vacancy (congestion effect) but also raise the marginal cost of hiring. This will slow down job creation and make recovery more gradual.³

Before presenting the model’s implications, we first identify some stylized facts about the relationship between unemployment and uncertainty. There is no objective measure of uncertainty. However, a large body of literature presents proxies of it, and on the relationship between recession and uncertainty. In fact, recession and uncertainty mutually affect each other. First, in recession,
activity slows down, and firms no longer trade actively, which reduces the flow of information and ultimately raises uncertainty.\textsuperscript{4} Second, when business conditions are unfamiliar, as they are in recessions, forecasting becomes very difficult.\textsuperscript{5} As stated in Baker et al. (2016), even policy uncertainty increases during recessions, as politicians use new tools that are beyond their comfort zones. Therefore, we need proxy measures of these changes in uncertainty. They can be based on mentions of uncertainty in newspaper articles or in the Federal Reserve’s Beige Book.\textsuperscript{6} They can also be based on the volatility of the stock market; this proxy is one of the most used, as it simply embodies the idea that when a data series becomes more volatile, it is more difficult to forecast, and hence, uncertainty increases. In this study, using a VAR approach based on U.S. monthly stock market volatility (S&P500 and VXO)\textsuperscript{7} and unemployment rate from 1962 until 2017, we show that an innovation on the uncertainty shocks, proxied by the volatility of the S&P500, leads to decreases in vacancies, and increases the U.S. unemployment rate.\textsuperscript{8}

These first empirical results encourage us to go further in our analysis via a more structured approach. We then propose a quantitative version of the DMP model, which considers uncertainty shocks. Following Petrosky-Nadeau and Zhang (2017), we solve the model using a global algorithm that allows for non-linearities and considers that vacancies can hit the zero bound. Following Bloom (2009), the changes in uncertainty are modeled by assuming that the volatility of the firm’s productivity follows a two-state Markov process. To the best of our knowledge, no previous study has presented such a perspective. Our structural model allows us to estimate that uncertainty shocks account for 25\% of unemployment variance and 20\% of the job find rate. We also show that uncertainty per se influences agents’ decisions, and consequently, magnifies productivity shocks. Finally, we show that the model’s impulse response function to an uncertainty shock is very close to the one based on the VAR estimation. These findings underline that the DMP model seems to be a promising framework to account for the U.S. labor market fluctuations, if it takes into account uncertainty shocks. Given this empirical success, we use the model to predict the impact

\textsuperscript{4}Micro-uncertainty such as dispersion of productivity, profits, and returns shocks to firms may increase during recession. Kehrig (2011) found that the dispersion of plant-level shocks to total factor productivity is higher in a recession than in an upturn. However, dispersion can change without any modifications in uncertainty if there is heterogeneity in loading on aggregate risk factors (see Jurado et al. (2010)).

\textsuperscript{5}When professional forecasters do not agree on their different predictions, this reflects high uncertainty. In fact, Bloom et al. (2014) have shown that between 1968 and 2012, the standard deviation across 50 different forecasts of US industrial production growth was 64\% higher during recession. Hence, those disagreements are more likely to happen in downturns. However, the disagreements on the forecasts more likely reflect differences in opinion than uncertainty (see Diether et al. (2002)).

\textsuperscript{6}Baker et al. (2016) used this proxy to measure the economic policy of uncertainty. This measure is based on counting the frequency of articles incorporating the words “economy,” “economics,” “uncertain” and “uncertainty” in ten leading US newspapers or counting the word “uncertain” in the Federal Reserve’s Beige Book. The results showed that this proxy is also highly countercyclical.

\textsuperscript{7}Compared to Bloom (2009)’s data, our data present more information about how and when the stock market volatility decreases after the 2008 credit crunch. It also includes two important economic crises that influence monthly stock market volatility: the Euro-zone Crisis in September 2011 and the Chinese stock market crash in 2015.

\textsuperscript{8}This empirical part of our work complements Bloom (2009) who estimates a range of VARs on monthly U.S. data from June 1962 to June 2008 to produce the impulse responses of uncertainty shocks, defined as the high volatility spikes of the S&P500, on some macroeconomic aggregates, and shows that uncertainty shocks produce a rapid drop and rebound in aggregate output and employment.
of rare events such as economic crisis on the labor market. We show that a combination of shocks on firms’ marginal returns and on uncertainty allows the model to reproduce the unemployment dynamics after the 2008 crisis. Therefore, we propose a scenario that forecasts the impact of the 2020 coronavirus crisis on U.S. unemployment.

The remainder of this paper is as follow: Section 2 presents an empirical study that measures the effect of uncertainty on the U.S. vacancies and unemployment rate using a VAR approach. Section 3 presents the model, and Section 4 presents its results. We also conduct a sensitivity analysis to check the robustness of the model in section 5. Section 6 studies the capacity of our model to account for the impact of an economic crisis on U.S. labor market dynamics. Finally, Section 7 concludes this paper.

2 Stylized fact: impact of uncertainty shocks on the labor market

We now present a VAR estimation of the impact of uncertainty shocks on the U.S. labor market, which identifies uncertainty shocks using a volatility index of the U.S. stock market, as in Bloom (2009).

Uncertainty shocks. For U.S. monthly stock market volatility, we use the VXO index of percentage-implied volatility. However, pre-1986, the VXO index data is nonexistent; hence, the actual monthly returns volatilities are given by the monthly standard deviation of the daily S&P500 index normalized to the same mean and variance as the VXO index when they overlap from 1986 onward (see Bloom (2009)). The timing is from 1962 until 2017. Therefore, the data used is S&P500 and VXO, both found on the CBOE website.9

To ensure that identification comes from large and exogenous volatility shocks rather than from the smaller ongoing fluctuations, we use an indicator function.10 It is constructed as follows: the events chosen as stock-market volatility jumps are those with a stock-market volatility bigger than 1.65 standard deviation above the mean value of the de-trended stock market volatility series.11 Figure 1 shows that all the volatility peaks are related to some bad event; in 1987, Black Monday was an economic crunch event. The 9/11 terrorist attack in September 2001 was a terror event or the Gulf War I in October 1990, which was a war event. None of these events could have been predicted, and therefore, they induce uncertainty in the stock market (see Appendix A for more details).

Var estimation on the impact of stock market volatility shocks. As stated, to study the effect of uncertainty shocks on some real economic outcomes, we follow Bloom (2009) closely. However, we estimate a more concise Vector Auto-Regression (VAR) model using extended data

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10For more details, see Bloom (2009).
11The series are de-trended using the Hodrick-Prescott (HP) with a smoothing parameter λ = 250,000
from June 1962 through December 2016. In addition to the stock market volatility time-series, we introduce uncertainty through a stock-market volatility indicator; we allocate each uncertainty event to the month with the largest volatility spike (highest volatility). The main measure of the stock-market volatility is an indicator that takes a value of 1 for each of the events and 0 otherwise. The set of VAR variables in the estimations is log(S&P500 level), the stock-market volatility indicator (i.e. uncertainty shocks), log(unemployment) and log(vacancies). This ordering is based on the assumptions that shocks instantaneously influence the stock market (levels and volatility), followed by unemployment and vacancies. When we introduce the stock-market levels as the first variable in the VAR, we identify uncertainty shocks as the innovations of this time series.

The impulse response function of unemployment shows that uncertainty shocks generate a rapid spike, undershoot, and finally a rebound. In fact, Figure 2 displays an increase of around 2.8% within 5 months followed by a recovery and rebound starting from six months after the shock. The one standard-error bands highlight that the result is statistically significant at the 5% level. The increase in unemployment is due to the real option value of inaction. Panel (b) of Figure 2 shows that vacancies decrease by 4.8% when uncertainty hits the economy, as decision makers choose to postpone their employment decisions rather than incur some non-refundable fixed cost. Employing is a form of labor-investment and firms pay hiring costs when employing. However, as it is difficult to forecast the future state of the economic activity because of uncertainty, firms choose to neither employ nor fire workers, and by not opening vacancies, hold their investment. However, exogenous labor attrition increases unemployment, leading to a recession in the labor market.

\footnote{See, the Appendix B for more details on the VAR estimation.} \footnote{The data for unemployment rate are described in Section 3.5.2, whereas the data used as proxy for vacancies are the Help-wanted index build by Barnichon (2010) over the period 1951m01 to 2016m12 (see https://sites.google.com/site/regisbarnichon/data). To obtain stationary data, we use the cyclical component of the HP filter.}
Figure 2: The impact of uncertainty shock. Dashed lines are 95% confidence bands of the response to a volatility shock.

3 Model

The environment includes workers and a representative firm whose only productive input is labor. Workers can be either employed or unemployed. The total mass of individuals is a unit mass. Workers are risk neutral with a time discount factor $\beta$.

3.1 Matching

The representative firm opens new vacancies $V_t$ to hire new workers $U_t$. They are filled with constant returns to scale matching function:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\tau + V_t^\tau)^{1/\tau}}$$  \hspace{1cm} (1)

with $\tau$ being a constant and positive parameter. The matching probabilities lie between 0 and 1. Using Equation (1), the probability for an unemployed worker to find a job per unit of time is

$$f(\theta_t) = \frac{G(U_t, V_t)}{U_t} = (1 + \theta_t^{-\tau})^{-1/\tau}$$

and the vacancy filling rate is defined as follows:

$$q(\theta_t) = \frac{G(U_t, V_t)}{V_t} = (1 + \theta_t^\tau)^{-1/\tau}$$

Hence, $f(\theta_t) = \theta_t q(\theta_t)$ with $\theta_t = \frac{V_t}{U_t}$ the labor market tightness. When the market is tight, unemployed workers are able to find jobs easily. However, employers take more time to fill vacancies. This could also be explained by the average duration of a vacancy $(1/q(\theta_t))$. In fact, given that $q'(\theta_t) < 0$, the average duration of a vacancy is such that it becomes harder to fill a vacancy when the number of vacancies increases more than the number of unemployed workers. The law of motion
of employment is

\[ N_{t+1} = (1 - s)N_t + q(\theta_t)V_t \]  \hspace{1cm} (2)

where, \( 0 < s < 1 \) is the exogenous probability of job destruction. Finally, as stated, there is a unit mass of agent, we have \( U_t + N_t = 1 \).

### 3.2 Firms

Firms produce \( Y_t \) using constant returns to scale production technology as follows:

\[ Y_t = A_t N_t \]  \hspace{1cm} (3)

The firm’s productivity is defined by an AR(1) process:

\[ a_t = \rho a_{t-1} + \sigma_t \epsilon_t \]  \hspace{1cm} (4)

where, \( a_t \equiv \log(A_t) \) and \( \epsilon_t \) is an i.i.d normal shock drawn independently across firms, \( 0 < \rho < 1 \) and \( \sigma_t > 0 \) are the auto-correlation and conditional volatility of the production process, respectively. We assume that the firm’s stochastic volatility follows a two-state Markovian process with \( \sigma_t \in \{ \sigma_L, \sigma_H \} \) and \( \Pr(\sigma_{t+1} = \sigma_j | \sigma_t = \sigma_k) = \pi_{j,k} \), where \( \sigma_L \) and \( \sigma_H \) are the low and high volatility, respectively. Hence, we can have two different regimes in the economy: one with a normal volatility and the other with a higher volatility. We expect the more volatile regime to react differently, as it represents higher uncertainty. However, at each period, the probability is \( \pi_{j,k} \) to move from a state \( k \) to another state \( j \).

The representative firm has to pay a unit cost per vacancy, given by

\[ \kappa_t = \kappa_0 + \kappa_1 q(\theta_t) \]

where \( \kappa_0 \) is the constant part of the proportional costs (the constant part of the marginal costs) and \( \kappa_1 \) is the variable part of these proportional costs because \( q(\theta_t) \) is given at the level of the firm (variable component of the marginal costs).

Hence, the dividend of the firm’s shareholders are \( D_t = Y_t - W_t N_t - \kappa_t V_t \). Taking the wage \( W_t \) and \( q(\theta_t) \) as given and using Equation (3), the firm posts the optimal number of vacancies to maximize the market value of equity \( S_t \):

\[
S_t = \max_{V_{t+\tau}, N_{t+\tau+1}} \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \beta^\tau \left( A_{t+\tau} N_{t+\tau} + q(\theta_{t+\tau})V_{t+\tau} \right) \right] \hspace{1cm} (5)
\]

s.t

\[
\begin{align*}
N_{t+1} &= (1 - s)N_t + q(\theta_t)V_t \\
V_t &\geq 0
\end{align*}
\]

The first-order conditions of the firm’s program lead to the following inter-temporal job creation
condition:

$$\frac{\kappa_t}{q(\theta_t)} - \lambda_t = \beta E_t \left[ A_{t+1} - W_{t+1} + (1 - s) \left( \frac{\kappa_t}{q(\theta_{t+1})} - \lambda_{t+1} \right) \right]$$  (6)

The inter-temporal job creation condition is equalizing the marginal costs of hiring at time \( t \) to the marginal value of hiring to the firm, which is represented by its marginal benefits of hiring at time \( t + 1 \) discounted to \( t \) with the stochastic discount factor \( \beta \). Intuitively, this marginal cost should be higher in the economy with higher variance as the marginal benefits include the marginal product of labor net of wages. The Kuhn-Tucker conditions are given by

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t)V_t = 0$$  (7)

When \( \lambda_t = 0 \), the equilibrium paths are the same as in the DMP model. When \( \lambda_t > 0 \), we have \( V_t = 0 \) and the solution is constrained with \( \theta_t = 0 \) and \( N_{t+1} = (1 - s)N_t \) until \( N_t > 0 \).

### 3.3 Wage Bargaining

To find the equilibrium wage endogenously, we use a simple sharing rule of a generalized Nash bargaining process between the worker and the firm, which is given by

$$W_t = \eta(A_t + \kappa_t\theta_t) + (1 - \eta)b$$  (8)

where \( \eta \in (0, 1) \) is the workers’ relative bargaining weight, \( b \) is the workers’ flow value of unemployment activities. For a given bargaining weight, the higher the costs of filling a vacancy and the more productive the worker, the higher the wages earned.

### 3.4 Equilibrium

We assume that all labor incomes are consumed as follows: \( C_t = W_tN_t + b(1 - N_t) \), \( \forall t \), where the wage is given by Equation (8) and unemployment benefits \( b(1 - N_t) \) are financed by a lump-sum tax paid by all workers (employed and unemployed). The competitive equilibrium is defined by i) the hiring decisions given by Equations (6) and (7), and ii) the employment dynamics (Equation 2). Using Equation (3), the goods market clears and implies \( C_t + \kappa_tV_t = A_tN_t \), \( \forall t \).

### 3.5 Solving Method and Calibration

#### 3.5.1 Projection Algorithm

We discretize the states in the economy. The productivity process (Equation (4)) is approximated with the discrete space method of Rouwenhorst (1995): \( A_t \) can take a finite number of values \( A_x \in \{ A_n \}_{x=1}^{n_x} \) with \( n_x \) the number of grid points.\(^{14}\) The grid is even-spaced. The distance between any two adjacent grid points is \( d_x = \frac{2\sigma_i}{\sqrt{(1 - \rho^2)(n_x - 1)}} \), where \( 0 < \rho < 1 \) is the persistence, \( \sigma_i \), for \( i = L, H \), is the conditional volatility of \( A_t \). For the numerical implementation, we choose \( n_x = 17 \).

\(^{14}\)This method presents high accuracy when approximating a persistent first-order auto-regressive process
The competitive equilibrium is solved using a projection algorithm that accounts for transitions between the two regimes of the economy, as well as for fluctuations within each regime. On the state space $A_x \in \{A_n\}_{x=1}^n$, we find the optimal values for vacancy $V(A_n)$ and the multiplier $\lambda(A_n)$ using Equations (6) and (7). To do so, the first step is to estimate the right hand side of the inter-temporal job creation condition as follows:

$$E_t(A_t,\sigma_j) = \beta \left( \sum_{j=H,L} \pi_{jk} E_{t,\sigma_j} \left[ A_{t+1} - W_{t+1} + (1-s) \left( \frac{\kappa_t}{q(\theta_{t+1})} - \lambda(A_{t+1}) \right) \right] \right) \quad \text{for} \quad k \in \{H, L\}$$

where $E_{t,\sigma_j}$ denotes the conditional expectations when $\sigma_t = \sigma_j$. We compute $E_t(A_t,\sigma_j)$, for $j = H, L$, using a collocation approach with Chebyshev polynomials of order three. With one state variables ($A_t$), we identify the policy rule by setting the Euler equation residuals equal to zero on a grid with seventeen nodes, and we approximate the expectations using the Gauss-Hermite quadrature.

In the second step, under the restriction that the lowest value for $A$ ensures that $E(A_1) > 0$, we use these conditional expectation functions $E_t(A_t,\sigma_j)$, for $j = H, L$, to compute $V_t,\sigma_j$ and $\lambda_t,\sigma_j$ as well as $q_t(\theta_t) = \frac{\pi_t}{E_t(A_t,\sigma_j)}$ for $j = H, L$. If $\bar{q}_j(\theta_t) < 1$. Then, the $V_t,\sigma_j \geq 0$ constraint is not binding. Therefore, we set $\lambda_t,\sigma_j = 0$ and $q(\theta_t) = \bar{q}_j(\theta_t)$. Hence, we can solve $\theta_t,\sigma_j = q^{-1}(\bar{q}_j(\theta_t))$ and $V_t,\sigma_j = \theta_t,\sigma_j (1 - N_t)$. However, if $\bar{q}_j(\theta_t) \geq 1$, then the $V_t,\sigma_j \geq 0$ constraint is binding; we can set $V_t,\sigma_j = 0, \theta_t,\sigma_j = 0, q(\theta_t) = 1$ and $\lambda_t,\sigma_j = \kappa_t - E_t(A_t,\sigma_j)$. Once solving optimal vacancy, multiplier, and market tightness given each state $A_n$, we deduce the employment dynamics using equation (2).

3.5.2 Calibration

We aim to test the ability of our augmented DMP model to reproduce the usual moments of the U.S. labor market. We first set a subset of parameters using external information. Second, we find the reminder parameters by minimizing the distance between model’s implications and observed data.

**Parameters coming from external information** The subset of parameter restricted to be equal to those found in the literature is

$$\chi = \{\beta, \rho, s, \pi_{HH}, \pi_{LH}\}$$

where the time discount factor $\beta$ is equal to 0.9954 to match the mean discount rate in international data 5.73% per annum. The persistence of the aggregate productivity $\rho$ is set to 0.951/3 following Petrosky-Nadeau et al. (2018). The exogenous separation rate is 0.025, which is the mean value of the US job separation rate data constructed by Adjemian et al. (2019). The probability matrix of transition is set following Bloom (2009): $\pi_{L,H} = \frac{1}{12}$ as the uncertainty shock is to be expected every three years and $\pi_{H,H} = 0.71$, which represent the average two-month half-life of an uncertainty

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15 With our calibration, this constraint is binding for 17.5% of the grid points.
shock. The data also provides information about the relative standard deviation; we set $\sigma_H = 2 \times L$ as in Bloom (2009).

**Parameters chosen to match selected moments** The vector of structural parameters chosen to minimize the distance between the model’s implications and observed data is

$$\Theta = \{\eta, b, \tau, \kappa_0, \kappa_1, \sigma_L\}$$

which are the worker’s bargaining power $\eta$ and their flow value of unemployment benefits $b$, elasticity of labor matching function $\tau$, proportional costs of opening a vacancy $\kappa_0$, and the fixed costs $\kappa_1$. The solution for $\Theta$ is chosen as follows:

$$\Theta = \arg \min [m(\Theta, \chi) - m_{US,T}]'Id[m(\Theta, \chi) - m_{US,T}]$$

where $m(\Theta, \chi)$ denotes the vector of simulated moments

$$m(\Theta, \chi) = \{E[ur], E[jfr], V[ur], V[jfr], E[ur \cdot ur - 1], E[jsr \cdot jsr - 1]\}.$$

with $E_x$ and $V_x$ the mean and the variance of $x = ur, jfr$ where $ur$ denotes the unemployment rate and $jfr$ the job finding rate, and $m_{US,T}$ the corresponding moments from the data on a sample size equals to $T$. The moments $E[x \cdot x - 1]$ for $x = ur, jfr$ represent the auto-correlation. As we do not perform an estimation, the weight matrix is the identity matrix ($Id$).

**Data.** We use Adjemian et al. (2019)’s data, who extend the data set of Lise and Robin (2017) to the current period. The data are from the BLS and cover the period from 1951m1 to 2018m12. These monthly employment and unemployment levels for all people aged 16 and over are seasonally adjusted. To construct worker flows, Adjemian et al. (2019) use the number of unemployed workers with unemployment durations of more than five weeks. After dividing the unemployment levels in each month by the sum of unemployment and employment, they obtain monthly series for $U_m$ and $U^5_m$ ($m$ refers to the monthly frequency), which correspond to the proportion of unemployed individuals and the proportion of individuals unemployed for more than five weeks. The worker flows are given by $JSR_m = \frac{U_{m+1} - U^5_{m+1}}{E_m}$ and $JFR_m = \frac{U_m - U^5_m}{U_m}$. All data are stationarized using a HP filter with parameter $2.5 \times 10^5$.

**Estimated parameters and targeted moments.** In Table 1, we report the solution for parameters allowing the model to match these selected moments. We also report the targeted moments

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16We only focus on moments of order one and two to identify the parameters of the model in order to have information that is homogeneous to that usually retained in the literature during the calibration of the DMP model. Remark that Petrosky-Nadeau et al. (2018) or Ferraro (2018) retain only first order moments for the calibration their models. Only Adjemian et al. (2019) provide an estimation of structural parameters that take into account all nonlinearities of the DMP model.

17We use series LNS12000000, LNS13000000, LNS13008396, LNS13008756, LNS13008516, and LNS13008636.
and the simulated moments to check the accuracy of the model’s fit.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow value of unemployment benefits ( b )</td>
<td>0.39</td>
<td>( E[jfr] )</td>
<td>40.89%</td>
<td>41.98%</td>
</tr>
<tr>
<td>Worker’s bargaining power ( \eta )</td>
<td>0.19</td>
<td>( E[ur] )</td>
<td>5.8%</td>
<td>6.13%</td>
</tr>
<tr>
<td>Elasticity of matching function ( \tau )</td>
<td>0.51</td>
<td>( V[ur] )</td>
<td>( 1.7527 \times 10^{-4} )</td>
<td>( 1.7523 \times 10^{-4} )</td>
</tr>
<tr>
<td>Proportional cost of posting a vacancy ( \kappa_0 )</td>
<td>0.65</td>
<td>( V[jfr] )</td>
<td>( 3.46 \times 10^{-3} )</td>
<td>( 3.27 \times 10^{-3} )</td>
</tr>
<tr>
<td>Fixed cost of posting a vacancy ( \kappa_1 )</td>
<td>0.75</td>
<td>( E[ur \cdot ur_{-1}] )</td>
<td>0.9992</td>
<td>0.9838</td>
</tr>
<tr>
<td>Low standard deviation ( \sigma_L )</td>
<td>0.08</td>
<td>( E[jfr \cdot jfr_{-1}] )</td>
<td>0.9974</td>
<td>0.9124</td>
</tr>
</tbody>
</table>

Table 1: Model’s Calibration and Target and Simulated Moments

First, it appears that the gaps between targeted and simulated moments are very small. Therefore, we can take the solution of the parameter vector seriously. The value of the opportunity cost of employment \( b \) (Flow value of unemployment benefits) is equal to 0.39. This value is close to the calibration of Shimer (2005) used to show that the DMP model does not match the U.S. data. However, this value is much lower than the calibration chosen by Robin (2011) (close to 0.86), and it is also lower than the estimates of Hall and Milgrom (2008) (close to 0.70), Christiano et al. (2016) or Petrosky-Nadeau et al. (2018) (0.88 - 0.85) and the extreme calibration proposed by Hagendorn and Manovskii (2008) (close to 0.95). Our value of bargaining power \( \eta = 0.19 \) is close to the long-run average of this wage bargaining power computed by Langot and Pizzo (2019) using OECD data and larger than the one used by Hagendorn and Manovskii (2008) or Petrosky-Nadeau et al. (2018). The elasticity of the matching function \( \tau \) is equal to 0.51, which is slightly larger than the value used by Petrosky-Nadeau and Zhang (2017) (0.407); however, it is significantly lower than the value obtained by Denhaan et al. (2000) (1.27). This result is not surprising because Denhaan et al. (2000) do not target the same set of moments than in this study. The parameters of the function cost of posting a vacancy \( \kappa_0, \kappa_1 \) are pinned down to 0.65 and 0.75, two values close to the ones used by Petrosky-Nadeau et al. (2018)of 0.5 and 0.5, respectively, which result in hiring costs to be in the range of the estimates provided by Merz and Yashiv (2007). Finally, the standard deviation of the uncertainty shock in the regime of low variance \( \sigma_L \) is equal to 0.08, a largely lower value than the ones estimated by Bloom (2009), which is between 0.1 and 0.44. Section 5 provides a sensitivity analysis on these parameter choices.

With these parameter values, which exclude extreme calibrations, the model is able to reproduce the observed volatility of the U.S. labor market. This result reconciles the DMP model with the stylized facts summarized by the second order moments. Figure 3 shows that the cyclical dynamics of unemployment rate can reach a value of 17.9\% and go down to a value of 1.74\%, these fluctuations being centered around a mean equal to 6.13\%. This basic simulation illustrates the capacity of the model to generate large fluctuations. Indeed, our model generates a negative impact of uncertainty on vacancies; the number of grid points where it is optimal to choose a vacancy equals to zero in the low variance regime is lower \( (4/17 \approx 23.5\% \) of the grid points) than in the high variance regime. \footnote{It is also lower than the estimation provided by Adjemian et al. (2019).}
Figure 3: Unemployment simulation in the basic framework. Time series of unemployment generated by the model using a draw for the sequences of $\epsilon$ and $\sigma$ over sample size of 2500 months.

regime ($5/17 \approx 29.4\%$ of the grid points). Hence, the comparison of the decision rules between frameworks with different degrees of uncertainty suggests that uncertainty plays a major role in coercing employers to stop recruiting. Consequently, a DMP model with uncertainty shocks induces higher unemployment; as decision-makers take into account the costs related to opening vacancies, they are more prudent when employing, as the risks are high.

The model also generates a negative correlation between unemployment and vacancies. It is equal to -0.4, whereas this correlation is equal to -0.7 in the U.S. data. This result is counter-intuitive, because when we shut off uncertainty shocks, the model’s correlation goes up to -0.5. Therefore, the model, regardless of an uncertainty shock, slightly underestimates the correlation between unemployment and job vacancies. This result is not very surprising, considering the difficulties experienced by the DMP model in matching this moment, since Merz (1994), Langot (1995), Andolfatto (1996) among others. Note also that the strong non-linearity of the Beveridge curve (stronger in the model with uncertainty shocks) decreases the relevance of this moment approaching a linear slope in the unemployment-job vacancy plan.

4 How Do Uncertainty Shocks Affect Unemployment Fluctuations?

To assess the impact of uncertainty shocks on U.S. labor market dynamics, we compare simulations from two different economies. The first is the baseline economy and the second is the same economy, except that the agents know that the variance of the shocks does not change and remains at its lowest level, here $\sigma_L$. This latter counterfactual economy is one without uncertainty shocks.
4.1 Impact of Uncertainty Shocks on Labor Market Moments

Table 2 compares moments generated by the baseline economy and the one without uncertainty. The averages of the aggregate are very close; the changes in uncertainty increase the average unemployment rate by only 0.02 pp, that is, an increase by 0.45% of the unemployment rate.

<table>
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<th></th>
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<tr>
<td>Baseline</td>
<td>6.13%</td>
<td>41.98%</td>
<td>$1.75 \times 10^{-4}$</td>
<td>$3.27 \times 10^{-3}$</td>
<td>0.9838</td>
<td>0.9124</td>
</tr>
<tr>
<td>Without uncertainty shocks</td>
<td>6.11%</td>
<td>41.47%</td>
<td>$1.3 \times 10^{-4}$</td>
<td>$2.73 \times 10^{-3}$</td>
<td>0.9829</td>
<td>0.9186</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>0.45%</td>
<td>1.25%</td>
<td>74%</td>
<td>83%</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2: Model’s Decomposition. “Baseline” refers to statistics based on simulations of the complete model, whereas “Without uncertainty shocks” refers to simulations of the model where $\sigma_t = \sigma_L, \forall t$, and thus, $\pi_{LH} = 0$. In the two first columns, $\Delta$ measures the percentage of change to go from the average aggregates ($E$) of the economy “without uncertainty shocks” to the ones of the “baseline” economy. In columns 3 and 4, $\Delta$ measures the share in the variance ($V$) of the “baseline” explained by the economy “without uncertainty shocks.”

Hairault et al. (2010) have already shown that the relation between the stochastic realizations of unemployment rate ($ur$) and the stochastic realizations of the job finding rate ($jfr$) is a convex function. This can be approximated by $Eur \approx ur + ur(1 - ur)^2 \left( \frac{V[jfr]}{E[jfr]} \right)^2$ using the steady state value of unemployment rate $Eur = \frac{s}{s + jfr}$ with $s$ a constant separation rate. Therefore, if the variance of the job-finding rate is higher in the baseline than in the economy without uncertainty shocks, the average unemployment rate will be larger. Table 2 shows that this is the case; the economy without uncertainty shocks can explain only 83% of the total variance of the job finding rate observed in the baseline. Table 2 also shows that $jfr$ is higher on average in the baseline than in the economy without uncertainty shocks. This suggests that the introduction of uncertainty shocks convexify the relation between the stochastic realization of the $jfr$ and aggregate shocks. This phenomena is moderate but reaches $jfr$ by 0.51pp, that is, an increase of 1.25%. Table 2 also shows that uncertainty shocks increase the variance of unemployment rate; this source of uncertainty explains 26% of the variance of unemployment rate. This underlines that uncertainty shocks play a significant role in U.S. labor market dynamics. Finally, Table 2 shows that the auto-correlation of unemployment and job finding rates are not sensitive to the introduction of uncertainty shocks.

Using Impulse Response Functions (IFR), we describe the economic mechanisms at work in the following section.

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19To study the effect of different shocks on the economy, we start by simulating an artificial data in continuous state space 5000 times. For each draw of a sequence for $\epsilon_t$, a Markov chain gives the transitions from a regime to the other. To eliminate the initial conditions, which are arbitrary, we simulate the economy 8000 times, and do not consider the 3000 first observations, thereby, preventing the influence of the initial state on the results.

20Indeed, the Table 2 shows that if we normalize the unemployment variance in the baseline model at 100, the unemployment variance of a model without uncertainty is only 74. Thus, the share of unemployment variance explained by uncertainty shocks is $26 = 100 - 74$. 

13
4.2 Impact of TFP Shocks

In the baseline economy, two sets of IRF to an innovation $\epsilon$ must be distinguished: the first is conditional to an $\epsilon$-shock that occurs in the regime where $\sigma = \sigma_L$, and the second is conditional to a $\epsilon$-shock that occurs in the regime where $\sigma = \sigma_H$.\(^{21}\)

![Figure 4: Impact of $\epsilon$-shock conditionally to be initially in a regime where $\sigma = \sigma_L$. The size of the shock is $\sigma_L$. The red-solid lines represent the dynamics of the economy after a positive shock, whereas the blue-dotted lines display the opposite of the dynamics after a negative shock. We report only the median of the IRF distribution.](image)

Before analyzing the impact of uncertainty shocks on the IRF, let us first illustrate the substantial non-linearities of our model that generate asymmetric impacts of a $\epsilon$-shock. Panel (a) of Figure 4 shows that a negative shock has a larger impact on unemployment than a positive shock.\(^{22}\) This is largely explained by the decreasing returns of the matching function leading to a positive variation

\(^{21}\)To compute these IRFs in nonlinear economies, we draw $n \in N$ sequences for $\{\epsilon_{n,t}\}_{t=0}^{T}$, and we apply a shock of standard error magnitude to their first points. Using the difference with a simulation without shock on the first point, we deduce the impact of a shock for a particular draw $n \in N$. We then characterize the distribution of the IRF over the $N$ draws.

\(^{22}\)In the Appendix C we show that this result also occurs in an economy where the $\epsilon$-shock hits the economy while it is initially in a regime where $\sigma = \sigma_H$, and thus, where the size of the shock is $\sigma_H$.\(^{14}\)
in vacancies (panel (b) of Figure 4), which has a lower impact on the job finding rate (panel (c) of Figure 4) than a negative variations. This phenomena account for the congestion effects. More precisely, there are more vacancies than unemployed workers during expansions, and thus, an additional vacancy will not only cause a dribble in the probability of filling a vacancy but also a raise in the marginal cost of hiring, which slows down the job creation motion and makes expansion more gradual. Hence, during a boom, a small increase in vacancy posting is sufficient to largely decrease the job-finding rate, leading to small response of unemployment.

Beyond the nonlinear properties of the matching model, the model’s simulations provide a quantitative measure of the interaction between shocks and uncertainty. Figure 5 allows us to compare the IRFs conditionally to the “level” of uncertainty perceived by the agents. When agents know that there are no uncertainty shocks (the variance is constant over time and set at its lowest level), the magnitude of the impact of $\epsilon$ shock is the lowest; this is the usual IRFs of the DMP model.

Even if the initial condition is in the regime where the variance is low (and at the same level than in an economy without uncertainty shocks), when agents know that there are uncertainty shocks, that is, the economy can become more risky, they are more reactive to a productivity shock than in an economy without uncertainty shocks. The red lines ($\epsilon$-shock conditionally to be the regime where $\sigma = \sigma_L$) of all Figure 5’s panels display an IRF having a larger magnitude than the green lines (without uncertainty shocks). Hence, uncertainty per se magnifies the impact of shocks. Indeed, when uncertainty shocks matter, firm integrate in their expectations the possibility to switch in the regime where the variance is high. Therefore, without certainty equivalence (the model solution does not use linearization techniques), even if the variance is low, the decision rules take into account that the transition to a more risky economy is possible, which makes entrepreneurs more sensitive to market fluctuations.

If the $\epsilon$-shock occurs when the economy is in a regime where $\sigma = \sigma_H$, the magnitude of the IFRs are the largest (the blue lines in Figure 5). Indeed, in the case of a recession (negative $\epsilon$-shock), it is more likely that entrepreneurs choose not to open a vacant job, choosing the option of delaying their hires, waiting for uncertainty to dissipate. This precautionary behavior then leads to unemployment rising sharply. Therefore, both the precautionary behaviors of the entrepreneurs and the direct impact of the uncertainty size magnify the size of the IFRs.

### 4.3 Uncertainty Shock: Are Model IRFs Close to the VAR Estimates?

The impact of uncertainty shock is measured by a rise in the variance of business-conditions, which is modeled by a shock on the variance of productivity (e.g. shock on $\sigma$). Hence, the economy that is initially at the low variance state jumps instantly to an economy of higher variance and reacts accordingly to the decision rule.

Panel (a) of Figure 6 shows that unemployment increases by 1.5% three months after an uncertainty shock. This slightly underestimates the IFR of the VAR, but the model’s IRF is in the confidence band of the VAR. Panel (b) of Figure 6 shows that uncertainty shock increases the num-
Figure 5: **Impact of a negative $\epsilon$-shock conditionally to be initially in each regime $\sigma = \sigma_L, \sigma_H$ and in an economy without uncertainty shocks.** The sizes of the shocks are $\sigma_L$ or $\sigma_H$, according the regime at the initial period. For the economy without uncertainty, the magnitude of the shock is $\sigma_L$. The red lines represent the dynamics in the regime where $\sigma = \sigma_L$. The blue lines display the dynamics in the regime where $\sigma = \sigma_H$. The green lines correspond to the dynamics of the economy without uncertainty shocks. We report only the median of the IRF distribution.
Figure 6: **Uncertainty Shock.** The red lines display the model’s IFR. The blue lines are the IRF based on the VAR presented in section 2. The dashed lines represent the confidence bands at 95% of the VAR IRF.

number of unopened vacancies. This is explained by the fact that uncertainty increases the real option value of inaction because of the costs of opening vacancies. Indeed, uncertainty makes the future economic activity barely “forecastable.” Hence, decision-makers tend to freeze their hiring decisions, and therefore, do not open new vacancies, as they do not know if the economic activity will improve or worsen. However, there is an exogenous attrition of workers. Hence, this temporary freeze generates a drop in the number of opened vacancies and increases unemployment in the economy. However, once the uncertainty shock dissipates, the hiring decisions bounce back, as firms answer to their hold up decisions of hiring new workers and posting new vacancies.

These results underline that the model’s calibration is not only able to match second order moments (unconditional moments) but also conditional moments identified using usual structural VAR estimation. This over-identifying “test” suggests that the DMP model could be a good framework to account for U.S. labor market fluctuations. Note that we obtain this result using a solution method that preserves the non-linearities of the model.\(^{23}\)

## 5 Sensitivity Analysis

To check the robustness of our results, we propose five alternative calibrations of the model.\(^{24}\) More precisely, this section studies the sensitivity of the results to calibration. Table 3 shows how the moments generated by the model change for alternative parameter values of \(\{b, \eta, \kappa_1, \sigma_H\}\), which band those retained in our calibration. Table 3 shows that simulated moments are highly sensitive to the level of opportunity cost of employment \(b\). For values of \(b\) around its reference value \(b = 0.39\)

\(^{23}\)See Petrosky-Nadeau et al. (2018) or Adjemian et al. (2019) for other examples, where accounting for non-linearities allows the DMP model to explain the U.S. labor market dynamics.

\(^{24}\)Figures that compare the IFRs of the VAR and the model’s IFR for different parameter values are in the Appendix C.
Table 3: **Robustness.** First and second order simulated moments for \(\{ur,jfr\}\) based on model calibrated with different values for parameters \(\{b,\eta,\kappa_1,\sigma_H\}\). In parenthesis: the variations with respect to the baseline in percentage.

- of -0.2 or +0.2 \((b = 0.2) \) or \(b = 0.6\), the average unemployment rate (job finding rate) can vary from -14\% to +66\% (+10.8\% to -18.3\%). For the variances, the deviations from the reference moments are even more important; with a \(b = 0.6\), the unemployment variance can increase by 1814.2\%.

- Indeed, when unemployment benefits increase, the number of grid points where it is optimal to not open vacancies largely increases, increasing the sensitivity of the economy to shocks (by reducing the profit size, a large \(b\) magnifies the sensitivity of the economy to shocks). These results are similar to Hall and Milgrom (2008) or Hagendorn and Manovskii (2008) who have already shown that the business cycle properties of the DMP model are highly sensitive to the level of unemployment benefits. Table 3 also shows that higher the bargaining power, the lower the simulated moments generated by the model. These results have been discussed by Hagendorn and Manovskii (2008). A larger fixed cost for opening vacancy \((\kappa_1)\) magnifies the impact of the shocks on the labor market aggregate. Table 3 shows that the model’s sensitivity to this parameter is not as large as in the two previous \((b, \eta)\), suggesting that this vacancy cost function à la Petrosky-Nadeau et al. (2018) is not crucial for the results.

- Table 3 shows that the amplitude of the labor market fluctuations significantly increase with the size of uncertainty. These results underline that the DMP model, with its costs to open vacancies and its congestion effects, lays the foundations of a real option of waiting, which amplifies the size of the hiring flow when uncertainty dissipates.
6 Accounting for the Economic Crisis: From 2008 to 2020

The last economic crisis started in August 2007 through the subprime crisis to become a global crisis in September 2008 (bankruptcy of Lehman Brothers). The impact of this crisis on unemployment became apparent in April 2008. Which exceptional combination of shocks on marginal returns (in the model the TFP shocks) or/and on uncertainty can generate the observed unemployment dynamics over this period? Panel (a) of Figure 7 shows that if the size of the shock is five times $\sigma_L$ and the sequence of uncertainty shocks is such that its duration is 36 months (variance in the economy is high for three years), the model can reproduce the impact of 2008 crisis on the U.S. unemployment rate.25 The model shows that when uncertainty dissipates, unemployment decreases rapidly after an initial rapid increase and stagnation during one year at a level higher than 9%.

![Figure 7: Impact of Crisis on Unemployment.](image)

If the model can reproduce the impact of the 2008 crisis on the U.S. unemployment rate, we can use it to forecast different scenarios for the current coronavirus crisis. On Wednesday, March 18, 2020, the S&P500 had its worst day in more than a decade. Its drop was the worst for stocks in the United States since December 2008, when the country was still reeling from the collapse of Lehman Brothers. It puts the index close to 20% below its record high, a drop that would have ended the bull market for stocks that began exactly 11 years ago. It is too early to predict the course of the economic downturn we face because of the coronavirus. However, a recession is inevitable. If we now look at the indicators of variation of uncertainty, the daily data from the S&P100 Volatility Index shows that since February 18, 2020, this index has gone from 14.64 to

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25 We do not re-calibrate the model by increasing $\sigma_H$ and $\pi_{HH}$. We simply draw one particular story for the shocks in their calibrated distribution.
93.85 on Tuesday, March 16, 2020. This increase in uncertainty is faster and exceeds that observed during the 2008 crisis; this index was at 14.73 in July 2007 and ended its rise with its highest value on November 2009 at 87.24.\textsuperscript{26} Panel (b) of Figure 7 forecasts the implications of the forthcoming recession on unemployment by displaying two different scenarios and by comparing them to the one used to explain the 2008 crisis. As the comparative developments in stock market statistics seem to suggest, if the most likely scenario is where the size of the shock is larger than the 2008 crisis, but where the duration of the period of great uncertainty is shorter (the coronavirus crisis reaches less fundamentals of the economy than the financial crisis of 2008), the rise in unemployment may be more significant and lasting. Indeed, the scenario presented in panel (b) of Figure 7 shows that unemployment can go from its current value of 3.5% to a peak at 10.5% in less than 12 months (against a peak at 9.5% in 2010). It will reduce to a value of 5% only in 9 years after the period of shock occurrence (5 years in the case of the 2008 financial crisis).

7 Conclusion

Business cycles are not as regular as suggested by the analyses of the framework of the DSGE models. In particular, crises often accompany an increase in uncertainty. This can have a significant impact on the labor market, as hiring is a risky decision that entrepreneurs have to make in the presence of fixed costs. In this study, we propose an extension of the Diamond-Mortensen-Pissarides model, which allows taking into account the variations of uncertainty observed during the business cycle (changes in the variance of macroeconomic shocks).

In this context, we show that uncertainty shocks explain 25% of the variance in unemployment and 20% of the variance in the job finding rate. We obtain these results with a calibration of structural parameters such as unemployment benefits or bargaining power, very close to the ones reported by the OECD, and therefore, far from the extreme calibrations chosen to allow the linearized version of DMP model to match the U.S. labor market fluctuations. Therefore, these results highlight the importance of nonlinearities coupled with uncertainty shocks in explaining the dynamics of U.S. unemployment. An over-identification test, based on the ability of our model to reproduce the IRF from unemployment to an uncertainty shock estimated by a VAR model, shows that our theoretical framework is fairly close to the stylized facts of the U.S. labor market.

With these good empirical results, we show that the model can also account for the impact of major crises on unemployment, such as the 2008 recession, via a combination of traditional shocks on the marginal revenue of companies but also via an increased uncertainty. This leads us to propose a scenario for forecasting the impact of the coronavirus crisis on the labor market; U.S. unemployment could rise from 3.5% to a peak of 10.5% in less than 12 months, and could return to a value of 5% only 9 years after the beginning of the crisis.

\textsuperscript{26}See these daily data on https://fred.stlouisfed.org/series/VXOCLS.
References


A Uncertainty shocks

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<th>Event</th>
<th>Max Volatility</th>
<th>First Volatility</th>
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<td>Cuban missile crisis</td>
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<td>October 1962</td>
<td>Terror</td>
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<tr>
<td>Assassination of JFK</td>
<td>November 1963</td>
<td>November 1963</td>
<td>Terror</td>
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<td>Vietnam buildup</td>
<td>August 1966</td>
<td>August 1966</td>
<td>War</td>
</tr>
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<td>Cambodia and Kent State</td>
<td>May 1970</td>
<td>May 1970</td>
<td>War</td>
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<td>October 1974</td>
<td>September 1974</td>
<td>Economic</td>
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<td>OPEC II</td>
<td>November 1978</td>
<td>November 1978</td>
<td>Oil</td>
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<td>October 1990</td>
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<td>War</td>
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<tr>
<td>Chinese stock market crash</td>
<td>September 2015</td>
<td>January 2015</td>
<td>Economic</td>
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Table 4: Summary of the events, nature, and dates of major stock-market volatility shocks

B VAR estimation

To ensure stationarity and to focus on the cyclical components, we use the HP filter to de-trend all variables in log, using smoothing parameter equals to $\lambda = 2.5 \times 10^5$. Bloom (2009) use the value $\lambda = 129,600$ for the HP filter; we have checked that this does not change the results. We have chosen the value $\lambda = 2.5 \times 10^5$ because it is retained in the literature on unemployment fluctuations (see, e.g., Shimer (2005), Robin (2011) or Lise and Robin (2017)). To remove the seasonal trend, we differentiate the series using 12 lags, as the data is monthly.

The estimated VAR model is

$$y_t = A_1 y_{t-1} + \ldots + A_k y_{t-k} + e_t$$

with

$$
\begin{align*}
y_t &= \begin{bmatrix} \log(S&P500), I_{S&P500}, \log(u_t), \log(v_t) \end{bmatrix}' \\
E(e_t'e_t) &= \Sigma
\end{align*}
$$

where $y_t$ is the vector of the endogenous variables: log($S&P500$), the stock-market volatility indicator ($I_{S&P500}$), log(unemployment) and log(vacancies). $A_i$ are the coefficient associated with the VAR, $k$ the number of lags (in our case $k = 12$), $e_t$ are the error terms and $\Sigma$ is the covariance matrix of the errors. The ordering of the variable in the VAR is based on the assumption that shocks in-
stantaneously influence the stock market (levels and volatility) first then quantities (unemployment and vacancies). Moreover, to ensure that the impact of volatility shocks is already controlled for, we include the stock-market levels as the first variable in the VAR. To allow for contemporaneous relationships among its variables, we can rewrite the previous VAR to

\[ Ay_t = C_1 y_{t1} + \ldots + C_k y_{tk} + \epsilon_t \]

where the \( A \) matrix represents the contemporaneous relationships among the variables in the VAR. As we only study the effect of uncertainty shock using the impulse response function, we must hold all other shocks constant. To decompose the error terms into orthogonal shocks, we have to rewrite the error terms as a linear combination of the structural shocks, as follows: \( \epsilon_t = Bu_t \), while imposing \( E(u_t'u_t') = I \). Henceforth, we have to identify \( A, C_i, \) and \( B \). To do so, we start by estimating the reduced-form matrices \( A_i \) and \( \Sigma \) from the first VAR equation. We then rewrite \( A_i = A^{-1}C_i \) and \( A^{-1}BB'A^{-1'} = \Sigma \). At this point, we need to impose some restrictions on \( A \) and \( B \) to obtain their estimation from \( \Sigma \). Hence, using the Cholesky identification, we impose \( A = I \) and \( B \) is set to be a lower-triangular matrix because of our imposed ordering condition. For example, shocks to unemployment rate equation contemporaneously affect vacancies but only affect stock market levels and volatility equations with a lag. Finally, we can recover \( B \) from a Cholesky decomposition of \( \Sigma \): \( BB' = \Sigma \).

C Robustness Analysis

![Figure 8: Robustness. Comparison between VAR’s and model’s IFR after an uncertainty shock, \( b = 0.85 \). The red lines display the model’s IFR. The blue lines are the IRF based on the VAR presented in section 2. The dashed lines represent the confidence bands at 95% of the VAR IRF.](image)
Figure 9: **Robustness.** Comparison between VAR’s and model’s IFR after an uncertainty shock, $\eta = 0.5$. The red lines display the model’s IFR. The blue lines are the IRF based on the VAR presented in section 2. The dashed lines represent the confidence bands at 95% of the VAR IRF.

Figure 10: **Robustness.** Comparison between VAR’s and model’s IFR after an uncertainty shock, $\kappa_1 = 0$. The red lines display the model’s IFR. The blue lines are the IRF based on the VAR presented in section 2. The dashed lines represent the confidence bands at 95% of the VAR IRF.
Figure 11: Robustness. Comparison between VAR’s and model’s IFR after an uncertainty shock, $\sigma_H = 3 \times \sigma_L$. The red lines display the model’s IFR. The blue lines are the IRF based on the VAR presented in section 2. The dashed lines represent the confidence bands at 95% of the VAR IRF.