ABSTRACT

Welfare Improving Tax Evasion*

We study optimal income taxation in a framework where one’s willingness to report his income truthfully is positively correlated with his type. We show that allowing low-productivity types to cheat leads to Pareto-superior outcomes as compared to deterring them, even if audits can be performed costlessly. When there is no cheating, redistribution takes place on first- and second-best frontiers and can never make low-ability types more well-off than high-ability types. Letting low-ability types cheat allows first-best redistribution up to a limit at which low-ability types are better off than high-ability types.

JEL Classification: H20, H21, H26

Keywords: optimal taxation, tax evasion, audits, welfare-improving

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1 Introduction

It is nearly five decades since Mirrlees (1971) and Allingham and Sandmo (1972) launched the literatures on the optimal general income tax and on the tax evasion. These have now grown into two of the most fertile subdisciplines in the of economics of taxation area. Notwithstanding the fact that they both are mostly concerned with efficient and fair ways to raise tax revenues for the government, they have gone their separate ways. The optimal tax literature has mainly focused on the formulation of income tax schedules; the tax evasion literature on the design of enforcement policies taking the tax schedule as given. This is no doubt due to their opposite foundational assumption. Whereas the Mirrleesian optimal taxation literature assumes incomes are publicly observable, the tax evasion literature has unobservability of incomes as its raison d’être.

Over this period of time, the attempts to bring these two literatures together have been few and far between. One, published some twenty five ago is Cremer and Gahvari (1995) who, using the Stiglitz (1982) two-group reformulation of Mirrlees (1971), allow for incomes to be misreported and observed only through costly audits. They investigate the properties of the resulting optimal policy that consists of a general income tax schedule, an audit policy that is conditioned on reported incomes, and punishment for misreporters.1

The aim of this paper is to shed light on a hitherto ignored role that tax evasion might play in the design of an optimal general income tax schedule. We ask if the widely-accepted proposition that tax evasion is “a bad thing” (welfare reducing) and has to be deterred, when not too costly, is always correct. Are there circumstances under which tax evasion can be a “good thing” which should be glossed over even if it can be deterred at a low cost—indeed costlessly? This is a question that has not been asked previously—at least to the best of our knowledge.

We consider this question within the two-group reformulation of Mirrlees (1971) optimal income tax problem by Stiglitz (1982)—a setting we shall refer to as MS (for Mirrlees/Stiglitz). We adopt its informational structure about the public unobservability of ability types and labor supplies, but not observability of incomes. Specifically, assume that the two groups differ in their willingness to reveal their true incomes. At the most general level, the two heterogeneity sources of types and truthful income-reporting can be uncorrelated or correlated either positively or negatively. Andreoni et al. (2017) have recently argued that, contrary to popular perceptions, the rich are more likely to behave socially than the poor.2 Interestingly, a positive correlation

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1Other attempts include Sandmo (1981) and Cremer and Gahvari (1994) who restrict the income tax schedule to be linear. Schroyen (1997) too allows for non-linear taxation but restricts the penalty to be proportional to the tax evaded.

2Andreoni et al. (2017) attribute this behavior to the diminishing marginal utility of income. We should point out, however, that the aim of this paper is not to take side on Andreoni et al.’s finding—only to explore its implication for devising optimal general income tax policies in the presence of tax evasion.
between the two characteristics of productivity and “honesty” have surprising implications for
devising optimal general income tax policies in the presence of tax evasion. The paper aims to
explore these implications. We shall refer to the setting where high-wage individuals reveal their
income truthfully as EL (for evasion by the low-wage type).

That the simultaneous existence of “honest” in addition to “dishonest” taxpayers matter for
equilibrium outcomes and policy design have long been recognized in the literature but not within
the Mirrleesian optimal tax paradigm. Gordon (1987) introduces a “psychic cost of evasion” into
Allingham and Sandmo (1972) and study how that changes the latter paper’s results. Erard
and Feinstein (1994) posit a game theoretic framework to study and compare the equilibrium
solutions for a model with dishonest taxpayers and a model with a mix of honest and dishonest
taxpayers. Honesty is defined in terms of truthful reporting of incomes which are exogenously
determined. The tax system is given with constant proportional tax and penalty rates. The
policy design is limited to audits based on reported incomes.

Alger and Renault (2006) study the importance of honesty in a wider context. They consider a
principle and agent framework wherein the agent has certain private information. The agent may
or may not feel compelled to reveal his private information truthfully. If he does, he is referred
to as honest; otherwise as dishonest. The authors introduce another layer of complication to
this setup by assuming that an honest agent may or may not feel compelled to reveal that he is
honest. They show that the distinction matters significantly and that this latter “conditional”
honesty drastically affects the set of implementable allocations.

In our setup, honesty refers only to truthful reporting of incomes; taxpayer’s type always
remains hidden. Incomes are endogenous and the policy design includes the tax system whose
sole restriction is incentive compatibility (with respect to type). The tax administration is able
to uncover true incomes through an auditing policy conditioned on reported incomes.

All individuals, regardless of their type, choose their labor supply and the amount of income
they want to report. Low-wage individuals’ true income does not have to be related to their
reported income, however. Their tradeoff between labor supply and consumption is not affected
by the tax schedule. This is not the case for high-wage individuals. With their reported income
being the same as their true income, their labor supply depends on the tax schedule. And if they
want to choose the same consumption-reported-income bundle as low-wage individuals (mimic
them as the terminology goes), high-wage individuals will have to work less hours than low-wage
individuals in order to actually earn what the latter types report.

We show that allocations that can be implemented in the EL setting include the set of im-
plementable allocations under MS. The inclusion is strict; there are first-best allocations that
cannot be implemented in the MS setting with full observability of incomes, but are imple-
mentable under EL when low-productivity individuals’ incomes are not observable. Interestingly too, auditing is never optimal in this setting. This is surprising a priori because one would expect less information to yield a worse outcome. The intuition for the result is that, if the income of low-productivity individuals is not observable, the tax schedule does not affect their labor supply choice which can then be set at its first-best level. Hence their reported income can be distorted down, to relax the incentive constraints of high-productivity individuals, but at no welfare loss. Put differently, one can decrease the utility of the mimicler without hurting the mimicked individual. We derive conditions under which the no-audit solution implements the first best.

The paper’s other results include the finding that any utility level that a low-wage individual can attain under a second-best MS solution, is available to him as a first-best EL solution. Moreover, first-best EL solutions include the Rawlsian outcome as well as outcomes wherein low-wages individuals are better off than high-wage individuals (neither of these type of solutions are available under MS).

2 The benchmark model (MS)

Consider an economy with two types of individuals, denoted by \(i = h, \ell\), who differ in their productivity \(w_h\) and \(w_\ell\) with \(w_h > w_\ell\). There are \(n_h\) persons of type \(h\) and \(n_\ell\) persons of type \(\ell\). Preferences over consumption \(x\) and labor supply \(L\) are represented by the utility function

\[
u(x, L),
\]

satisfying the standard properties. Denote pre-tax incomes by \(I_i = w_iL_i\), tax payments by \(T_i\) and assume purely redistributive taxes. The full information Pareto-frontier is obtained by maximizing a weighted sum of utilities with weights such that \(\alpha_h + \alpha_\ell = 1\) subject to the resource constraint. It is determined by solving problem \(\mathcal{P}_F\) defined as

\[
\max_{T_h, I_h, T_\ell, I_\ell} W = \alpha_h u \left( I_h - T_h, \frac{I_h}{w_h} \right) + \alpha_\ell u \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right),
\]

s.t. \(n_hT_h + n_\ell T_\ell = 0\).
The first-best (FB) allocations, denoted by \([ (T_h^*, I_h^*), (T_\ell^*, I_\ell^*)]\), satisfy the FOCs (see Stiglitz, 1982):

\[
\begin{align}
\alpha_h u_h \left( I_h^* - T_h^*, \frac{I_h^*}{w_h} \right) &= n_h \mu, \\
 u_c \left( I_h^* - T_h^*, \frac{I_h^*}{w_h} \right) + \frac{1}{w_h} u_L \left( I_h^* - T_h^*, \frac{I_h^*}{w_h} \right) &= 0, \\
\alpha_\ell u_\ell \left( I_\ell^* - T_\ell^*, \frac{I_\ell^*}{w_\ell} \right) &= n_\ell \mu, \\
u_c \left( I_\ell^* - T_\ell^*, \frac{I_\ell^*}{w_\ell} \right) + \frac{1}{w_\ell} u_L \left( I_\ell^* - T_\ell^*, \frac{I_\ell^*}{w_\ell} \right) &= 0,
\end{align}
\]

where \( \mu > 0 \) denotes the multiplier associated with the resource constraint. These conditions, along with the resource constraint determine the Pareto frontier \( PF \) represented in Figure 1, where \( u_h \) and \( u_\ell \) denote the utility of \( h \)- and \( \ell \)-type individuals.

![Figure 1: Pareto frontier and implementable allocations under MS and EL.](image)

By the first theorem of welfare economics, the competitive equilibrium is on \( PF \); it is shown by point \( a \) which is above the 45 degree line because \( w_h > w_\ell \). In what follows, we concentrate on the part of the frontier which is “to the right” of the competitive equilibrium. This implicitly assumes that the weights are such that the solution involves redistribution from the high-wage to the low-wage individuals so that \( T_h^* > T_\ell^* \). We know from Stiglitz (1982) that this includes the utilitarian FB obtained when \( \alpha_i = n_i \). We also assume \( I_\ell^* > 0 \).

The Mirrlees-Stiglitz problem assumes that incomes \( I_i = w_i L_i \) are publicly observable at no cost but \( w_i \) and \( L_i \) are not (for \( i = h, \ell \)). One then has to add an incentive compatibility constraint (IC) to problem \( P_F \), which yields problem \( P_{MS} \) defined as

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3We shall refer to \([ (T_h^*, I_h^*), (T_\ell^*, I_\ell^*)]\) as an “allocation” even though, strictly speaking, it corresponds to the allocation \([ (x_\ell^*, I_\ell^*), (x_h^*, I_h^*)] = [(I_\ell^* - T_\ell^*, I_\ell^*), (I_h^* - T_h^*, I_h^*)] \).
\[
\max_{T_h, I_h, T_\ell, I_\ell} \quad W = \alpha_h u \left( I_h - T_h, I_h \frac{I_h}{w_h} \right) + \alpha_\ell u \left( I_\ell - T_\ell, I_\ell \frac{I_\ell}{w_\ell} \right)
\]
\[
\text{s.t.} \quad u \left( I_h - T_h, I_h \frac{I_h}{w_h} \right) - u \left( I_\ell - T_\ell, I_\ell \frac{I_\ell}{w_\ell} \right) \geq 0,
\]
\[
n_h T_h + n_\ell T_\ell = 0.
\] (2)

It is the common practice to refer to the utility of the \(h\)-type evaluated at the allocation intended for the \(\ell\)-type in the incentive constraint as the utility of a fictitious mimicker—a terminology that we also follow. Stiglitz (1982) has shown that problem \(P_{MS}\) can have two types of solution. In one, the incentive compatibility constraint in (2) is non-binding and the solution is on the Pareto frontier (PF). This is depicted by that part of PF in Figure 1 which ends at point \(b\). In the other, the solution is given by the FOC of problem \(P_{MS}\) along with the binding incentive compatibility constraint in (2) and the resource constraint. This is depicted by the MS curve in Figure 1 that starts from point \(b\).

3 The model with low-productivity type evaders (EL)

Consider the MS setting with the same information structure about the public unobservability of ability types and labor supplies. Change the assumption of observability of all incomes to that of observability of high-productivity type incomes only. Specifically, assume that the two groups differ in their willingness to reveal their true incomes. The high-wage persons, who cannot be identified by the government, willingly reveal their true income \(I_h = w_h L_h\). On the other hand, the low-productivity persons, who are not identifiable either, do not feel obligated to report their income \(I_\ell = w_\ell L_\ell\) truthfully. Their income can only be observed through audits. Denoting reported income by \(\tilde{I}\), we thus have \(\tilde{I}_h = I_h\) but \(\tilde{I}_\ell\) may differ from \(I_\ell\). Assume \(0 \leq \tilde{I}_\ell \leq I_\ell\) to rule out negative- and over-reporting of income.\(^4\) Concentrate again on the case where the binding incentive constraint (if any), is from the \(h\)-type to the \(\ell\)-type. Assuming no audits are performed (we show below that is in fact the optimal policy), the policy problem is

\[
\max_{T_h, I_h, I_\ell, \tilde{I}_\ell} \quad W = \alpha_h u \left( I_h - T_h, I_h \frac{I_h}{w_h} \right) + \alpha_\ell u \left( I_\ell - T_\ell, I_\ell \frac{I_\ell}{w_\ell} \right)
\]
\[
\text{s.t.} \quad u \left( I_h - T_h, I_h \frac{I_h}{w_h} \right) - u \left( \tilde{I}_\ell - T_\ell, \tilde{I}_\ell \frac{I_\ell}{w_\ell} \right) \geq 0,
\]
\[
\tilde{I}_\ell \geq 0,
\]
\[
n_\ell T_\ell + n_h T_h = 0,
\] (3)

\(^4\)The no-overreporting constraint simplifies the expressions; one can easily show that it will not be binding.
and referred to as $\mathcal{P}_\ell$.

Observe that, while the policy designer does not set $I_\ell$ directly, it is effectively set indirectly. Low-wage persons are induced to choose and report $\tilde{I}_\ell$ and $T_\ell$. They then, given these values, choose $I_\ell$ to maximize $u(I_\ell - T_\ell, I_\ell/w_\ell)$. Importantly, $u(I_\ell - T_\ell, I_\ell/w_\ell)$ is the only term in problem (3) that depends on $I_\ell$. Consequently, the optimal choice of $I_\ell$ by low-wage individuals is tantamount to maximization of $W$ with respect to $I_\ell$. This allows us to reformulate problem $\mathcal{P}_\ell$ by including $I_\ell$ in the list of decision variables. This is represented by problem $\mathcal{P}_\ell'$, which is the same as $\mathcal{P}_\ell$ except that there is an extra decision variable.

The Kuhn-Tucker expression for problem $\mathcal{P}_\ell'$ is

$$
\ell = \alpha_h u \left( I_h - T_h, \frac{I_h}{w_h} \right) + \alpha_\ell u \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) + \
\lambda \left[ u \left( I_h - T_h, \frac{I_h}{w_h} \right) - u \left( \tilde{I}_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) \right] + \gamma \tilde{I}_\ell + \mu \left( n_h T_h + n_\ell T_\ell \right).
$$

Observe that $\mathcal{P}_\ell'$ is similar to $\mathcal{P}_{MS}$ except that $\tilde{I}_\ell$ replaces $I_\ell$ in the utility of the mimicker; see (2) and (3). Importantly, though, problem $\mathcal{P}_\ell'$ contains an extra choice variable in comparison with $\mathcal{P}_{MS}$. This variable is $\tilde{I}_\ell$ and can always be set equal to $I_\ell$ to obtain the MS allocation. The idea we develop below is the possibility of choosing $\tilde{I}_\ell$ such that $u \left( \tilde{I}_\ell - T_\ell, \tilde{I}_\ell/w_\ell \right) < u \left( I_\ell - T_\ell, I_\ell/w_\ell \right)$. If this is possible, reducing $u \left( \tilde{I}_\ell - T_\ell, \tilde{I}_\ell/w_\ell \right)$ relaxes the otherwise binding incentive constraint and allows for increased redistribution to enhance welfare.

The first-order (Kuhn-Tucker) conditions are

$$
\frac{\partial \ell}{\partial T_h} = - (\lambda + \alpha_h) u \left( I_h - T_h, \frac{I_h}{w_h} \right) + \mu n_h = 0, \quad (4a)
$$

$$
\frac{\partial \ell}{\partial T_\ell} = (\lambda + \alpha_h) u \left( I_h - T_h, \frac{I_h}{w_h} \right) + \frac{1}{w_h} u_L \left( I_h - T_h, \frac{I_h}{w_h} \right) = 0, \quad (4b)
$$

$$
\frac{\partial \ell}{\partial I_\ell} = -\alpha_\ell u \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) + \lambda u \left( \tilde{I}_\ell - T_\ell, \frac{\tilde{I}_\ell}{w_\ell} \right) + \mu n_\ell = 0, \quad (4c)
$$

$$
\frac{\partial \ell}{\partial \tilde{I}_\ell} = -\lambda \left[ u \left( \tilde{I}_\ell - T_\ell, \frac{\tilde{I}_\ell}{w_\ell} \right) + \frac{1}{w_h} u_L \left( \tilde{I}_\ell - T_\ell, \frac{\tilde{I}_\ell}{w_\ell} \right) \right] + \gamma = 0, \quad (4d)
$$

$$
\frac{\partial \ell}{\partial I_\ell} = \alpha_\ell u \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) + \frac{1}{w_\ell} u_L \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) = 0, \quad (4e)
$$

$$
\lambda \frac{\partial \ell}{\partial \lambda} = -\lambda \left[ u \left( I_h - T_h, \frac{I_h}{w_h} \right) - u \left( \tilde{I}_\ell - T_\ell, \frac{\tilde{I}_\ell}{w_\ell} \right) \right] = 0, \quad (4f)
$$

$$
\gamma \frac{\partial \ell}{\partial \gamma} = \gamma \tilde{I}_\ell = 0. \quad (4g)
$$

It is clear that $\mu > 0$ in the above expressions because the resource constraint must be binding. However, it is possible that the other multipliers may be zero and we only have $\lambda \geq 0$ and $\gamma \geq 0$. 

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8 Unobservability of $I_\ell$ disconnects it from the incentive compatibility constraint.
Consequently, Problem $P'_\ell$ may yield different solution regimes depending on the pattern of the binding and non-binding constraints. To study this issue in the most efficient way, we organize our analyses around the results that are already known for the MS problem and examine if or how they may change.

4 EL: Solution regimes

As with the MS setting, we distinguish between first- and second-best regimes.

4.1 First best

The solutions under EL will be first best if $\lambda = \gamma = 0$ in problem $P'_\ell$. The notable point about them is that they include all first-best allocations that are implementable under MS. First, we know from Stiglitz (1982) that, because the competitive equilibrium satisfies the incentive compatibility constraint with strict inequality, the Pareto efficient allocations in the neighborhood of this equilibrium can be implemented under the MS information structure. Intuitively, the IC constraint is not violated when the amount of redistribution is “small”. Denote the first-best value of a variable by a “star” on top of it. They satisfy the IC constraint

$$u(I^*_h - T^*_h, \frac{I^*_h}{w_h}) \geq u(I^*_\ell - T^*_\ell, \frac{I^*_\ell}{w_h}),$$

and are represented diagrammatically by the segment $ab$ on the Pareto frontier in Figure 1. To see that these allocation are also implementable under EL, one can simply duplicate them by setting $[(T^*_h, I^*_h), (T^*_\ell, I^*_\ell)] = [(T^*_h, I^*_h), (T^*_\ell, I^*_\ell)]$. Although low-wage individuals can now cheat without fearing of audits, they will not do so because $I^*_\ell = I^*_\ell$ maximizes their utility regardless of their report (as long as $T^*_\ell = T^*_\ell$).

The more interesting question is whether there are first-best allocations that can be implemented under EL but not under MS. Consider a Pareto efficient allocation $[(T^*_h, I^*_h), (T^*_\ell, I^*_\ell)]$ for which (5) does not hold and thus is not implementable under MS. Then formulate an EL policy that consists of the tax function

$$T(\tilde{I}) = I^*_\ell - c^*_\ell = T^*_\ell \quad \text{if} \quad \tilde{I} = 0,$$

$$= I^*_h - c^*_h = T^*_h \quad \text{if} \quad \tilde{I} > 0,$$

and no audits. The $\ell$-types’ best option is to report $\tilde{I} = 0$ and pay $T^*_\ell$ which then leads them to earn $I^*_\ell$. To report $\tilde{I} > 0$ and pay $T^*_h > T^*_\ell$ will only reduce their utility. As to the $h$-types, recall that they cannot misreport their income. Consequently, if they were to report $\tilde{I} = 0$ in order to receive $-T^*_\ell$, they must also earn $I^*_h = 0$. Their options are thus either (i) pay $T^*_h$ and earn $I^*_h$ or (ii) receive and consume $-T^*_\ell$ and earn $I^*_h = 0$. 

8
The first-best solution will be implementable under EL if and only if

\[ u\left(I_h^* - T_h^*, \frac{I_h^*}{w_h}\right) \geq u\left(-T_h^*, 0\right). \] (8)

Now, as long as \( I_t^* > 0 \), the right-hand side of the incentive compatibility constraint (5) is larger than that of (8):

\[ u\left(I_t^* - T_t^*, \frac{I_t^*}{w_t}\right) > u\left(0 - T_t^*, 0\right). \]

Condition (8) is thus strictly weaker than (5). Consequently, there must exist FB allocations that can be implemented under EL even though they are not attainable under MS. These are the allocations that satisfy (8) but not (5). They are represented by segment be on the Pareto frontier in Figure 1.

Observe also that while (8) must hold for some Pareto efficient allocations, it will not hold for all. As long as \( u\left(-T_t^*, 0\right) > 0 \), there will be a non-empty segment below and to the right of point b which is implementable under EL. However, when the first-best utility of high-wage individuals gets sufficiently close to zero, the direction of this inequality will unavoidably be reversed. Consequently, there must exist a non-empty subset of the Pareto frontier that cannot be implemented through EL. This is illustrated in the numerical example given in Section 5 below.

Finally, we have thus far assumed that no audits are performed. This is in fact the optimal policy because audits can only do harm. With a positive probability of audits, low-wage individuals might find it optimal to report \( \tilde{I}_t > 0 \). Which will then mean that they have to pay \( T_h^* \) rather than \( T_t^* \) making the implementation of \( [(T_h^*, I_h^*), (T_t^*, I_t^*)] \) no longer feasible. Moreover, when \( \tilde{I}_t > 0 \), incentive constraint (8) would have to be amended thus making the mimicking option more attractive for the \( h \)-type. This in turn will reduce the set of FB allocations that can be implemented.

The results derived thus far are summarized in the following proposition.

**Proposition 1** Consider the EL and MS settings as defined in the paper. (i) The set of first-best allocations that can be implemented under EL includes the set of allocations that is implementable in the MS setting. (ii) As long as \( I_t^* > 0 \), the inclusion is strict so that there exist FB allocations that can be implemented under EL but not under MS. (iii) Not all FB allocations are implementable under EL. (iv) Auditing is never desirable.

That the unobservability of \( I_t \) leads to the implementability of first-best allocations, unattainable when \( I_t \) is observable at no cost, is a rather striking result. Having less information is expected to bring about a worse outcome, not a better one. To garner intuition for this, remember
that in the MS setting, distorting the low-wage individuals’ labor supply downwards is needed to make mimicking more costly to the high-wage individuals. The distortion is no longer needed for this purpose, nor is it possible to induce it, under EL. The incentive constraint is manipulated through $\tilde{I}_f$ in a way that makes mimicking even more costly than under MS. Low-wage individuals report $\tilde{I}_f$ and choose their most desired level of labor supply which would be FB (regardless of the marginal tax rate on $\tilde{I}_f$).

This also explains why evasion by high-wage individuals is not interesting or relevant for our purposes. The MS allocation entails no distortion of the $h$-types to be corrected. Moreover, their evasion cannot weaken the IC constraint (5); nor is it necessary to achieve it. Effectively, a setting with evasion by high-wage individuals becomes equivalent to MS.\(^6\)

**4.2 Second best**

We now turn to the case where FB allocations violate condition (8) so that they cannot be implemented. These are the allocations that lie to the right of and below point $c$ on the Pareto frontier in Figure 1. Kuhn-Tucker conditions of problem $P'_f$ then yield second-best solutions at which condition (8) is binding and $\lambda > 0$. Of course, with the resource constraint always binding, $\mu$ must also be positive. Lemma 1 proves that $\gamma > 0$ as well because the optimal policy continues to imply $\tilde{I}_1 = 0$, though this time as a corner solution.

**Lemma 1** The Lagrange multiplier $\gamma$ in Problem $P'_f$ is positive.

**Proof.** To begin with, observe that $\tilde{I}_1$ has to be set to minimize the utility of the mimicker $u^{ht} (\tilde{I}_f - T_f, \tilde{I}_h/w_h)$ which is the only term in problem $P'_f$ that it affects. Specifically, given our assumptions (in particular that of no overreporting), we must minimize $u^{ht}$ over $\tilde{I}_f \in [0, I_f]$.

Let $MRS$ denote the marginal rate of substitution between consumption and income. Given the standard assumptions on $u$, $u^{ht}$ is concave with at an interior maximum where $MRS^{ht} (\tilde{I}_f - T_f, \tilde{I}_h/w_h) = w_h$. Now, from the first-order condition (4e), we have $MRS^{ht} (I_f - T_f, I_h/w_h) = w_f$. With $w_h > w_f$, $MRS^{ht} (\tilde{I}_f - T_f, \tilde{I}_h/w_h) > MRS^{ht} (I_f - T_f, I_h/w_f)$. Moreover, from single-crossing property $MRS^{ht} (I_f - T_f, I_h/w_h) < MRS^{ht} (I_f - T_f, I_h/w_f)$. These two inequalities imply

$$MRS^{ht} (\tilde{I}_f - T_f, \tilde{I}_h/w_h) > MRS^{ht} (I_f - T_f, I_h/w_h)$$

(9)

It follows from inequality (9) and the concavity of $u^{ht}$ that $\tilde{I}_f > I_f$. Consequently, $u^{ht}$ is increasing over $[0, I_f]$ which in turn implies that it is minimized at $\tilde{I}_f = 0$. This also implies that the bracketed expression in equation (4d) is positive resulting in a positive solution for $\gamma$. $\blacksquare$

\(^6\)The story would be different, of course, if one was concerned with an upward-binding IC constraint.
Set \( \tilde{I}_\ell = 0 \) in the Kuhn-Tucker conditions (4a)-(4g) to simplify equations (4b), (4e)-(4f) into

\[
\begin{align*}
&u_c \left( I_h - T_h, \frac{I_h}{w_h} \right) + \frac{1}{w_h} u_L \left( I_h - T_h, \frac{I_h}{w_h} \right) = 0, \\
u \left( I_h - T_h, \frac{I_h}{w_h} \right) - u (-T_\ell, 0) = 0, \\
u_c \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) + \frac{1}{w_\ell} u_L \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) = 0.
\end{align*}
\]

These equations, along with the resource constraint \( n_\ell T_\ell + n_h T_h = 0 \), determine a unique set of values for \( I_h, T_h, I_\ell, T_\ell \). In particular, these values are independent of the weights assigned to \( u^h \) and \( u^\ell \). Moreover, these equations are precisely the same equations that determine the first-best allocation \( ([T^*_h, I^*_h], [T^*_\ell, I^*_\ell]) \) at point \( c \) in Figure 1. Recall that \( c \) is the boundary point on the Pareto frontier satisfying the IC constraint as an equality (beyond it the IC constraint will be binding).

That increasing \( \alpha_\ell \) above its value at point \( c \) does not change the values of \( ([T^*_\ell, I^*_\ell], [T^*_h, I^*_h]) \) is interesting, if at first surprising. However, one has to bear in mind that IC constraints limit the extent of redistribution in second-best environments by making the bundle “intended” for the \( \ell \)-types less appealing to the \( h \)-types; that is, by reducing \( u^{h\ell} \). Often, there is a floor to how much \( u^{h\ell} \) can be pushed down. In the MS setting, what usually limits the extent of redistribution as \( \alpha_\ell \to 1 \), is when \( I_\ell \) hits zero. In the EL setting, \( u^{h\ell} \) depends on \( \tilde{I}_\ell \) and not \( I_\ell \). The floor to redistribution is thus dictated by \( \tilde{I}_\ell = 0 \) with \( \tilde{I}_\ell \leq I_\ell \). When redistribution is limited by a positive value of \( I_\ell \), it will no longer be possible to make the \( \ell \)-types any better-off by decreasing \( I_\ell \) notwithstanding the fact that higher values of \( \alpha_\ell \) call for it. The numerical example of Section 5 below illustrates this point.

Given that EL does not allow for redistribution beyond point \( c \), one may wonder if the utility of the \( \ell \)-types can be pushed beyond this point under MS. The answer is no. Take any MS solution \( ([T^*_\ell, I^*_{\ell}]^M, [T^*_h, I^*_h]^M) \); then set \( T_h = T_h^M, I_h = I_h^M, T_\ell = T_\ell^M \) and \( \tilde{I}_\ell = I_\ell^M \) under EL. This leaves the \( h \)-type’s utility and the incentive and resource constraints unaffected. This allows the \( \ell \)-type individuals to choose a level of income that differs from their reported level thus increasing their utility over its level under MS. The suggested reallocation is not necessarily the best policy. Which means the optimal policy can only do better.

The EL policy can in fact increase the utility of the \( \ell \)-types beyond what is feasible under MS. There, one can never make the \( \ell \)-types more well-off than the \( h \)-types.\(^7\) In contrast, point \( c \)

---

\(^7\)To see this, observe that with \( w_h > w_\ell \), and the fact that utility changes negatively with labor supply, we have

\[
u \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) > u \left( I_\ell - T_\ell, \frac{I_\ell}{w_h} \right).
\]

The IC constraint is

\[
u \left( I_h - T_h, \frac{I_h}{w_h} \right) > u \left( I_\ell - T_\ell, \frac{I_\ell}{w_h} \right).
\]
is necessarily below the 45 degree line. At point $c$, where the incentive constraint starts to bind, we have

$$u_h = u \left( I^*_h - T^*_h, \frac{I^*_h}{w_h} \right) = u \left( -T^*_\ell, 0 \right) < u_\ell.$$ 

The inequality follows because $I_\ell = 0$ makes the consumption bundle $(-T^*_\ell, 0)$ available to the $l$-types. The inequality is strict as long as $I^*_\ell > 0$. Compared to the competitive equilibrium, the ranking of utilities is reversed. This property also implies that EL can implement the Rawlsian FB solution (where the 45 degree line intersects the Pareto frontier). It follows that the solutions that cannot be implemented are rather “extreme” and go beyond the usual notion of income redistribution in the sense that inequalities are reversed.

Finally, observe that auditing can only increase the utility of the mimiccker. Consequently, the optimal policy involves no audits. The results derived so far are summarized in the following proposition.

**Proposition 2** In the second best:

(i) The solution under EL, $([T^*_h, I^*_h], [T^*_\ell, I^*_\ell])$, is unique. It is represented by point $c$ in Figure 1 regardless of $h$- and $l$-types’ weights in the social welfare function.

(ii) Point $c$ must be below and to the right of where the 45 degree line intersects the Pareto frontier so that $u_\ell > u_h$.

(iii) The MS frontier must lie everywhere to the left of point $c$ in Figure 1.

5 A numerical example

This section illustrates our results through a numerical example. Details of the derivations are presented in the Appendix. Assume preferences are represented by the following quasilinear utility function:

$$u = 2 \left[ c + \beta \ln(1 - L) \right]^{0.5},$$

where $0 < \beta < w_\ell$. The two ability-types are of equal size with the population size being normalized at one. Hence $n_\ell = n_h = 1/2$. Additionally, set the parameter values to $\beta = 10$, $w_\ell = 14$ and $w_h = 20$. Given these values, the Pareto frontier is represented by $u_h^2 + u_\ell^2 = 14.82$ as depicted in Figure 2. The two types utility levels then range from 0 to 3.85.

The laissez-faire allocation is found to be $I_\ell = c_\ell = 4$ and $I_h = c_h = 10$ with the corresponding utility levels of $u_\ell = 1.59$ and $u_h = 3.50$. This is the FB allocation if $\alpha_\ell = 1/3$ and is shown by

\[\text{It follows from these two inequalities that} \]

$$u \left( I_h - T_h, \frac{I_h}{w_h} \right) \geq u \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) > u \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right).$$

In this regard, Figure 2.1 in Stiglitz (1987, p. 2.1) which shows the FB and SB frontiers is misleading.
point $a$ on the Pareto frontier in Figure 2. Redistribution towards the $\ell$-types from $a$ becomes desirable when $\alpha_\ell$ exceeds $1/3$. This limits the considered FB allocations to segment $af$ on the Pareto frontier. Segment $ea$ on the PF represents the FB allocations that entail redistribution to the $h$-types and are desired when $\alpha_\ell < 1/3$.

Under MS, the IC constraint (5) is satisfied as a strict inequality if and only if $\alpha_\ell < 0.42$. Under EL, the IC constraint (8) holds as a strict inequality if and only if $\alpha_\ell < 0.54$. Consequently, for all $\alpha_\ell \in [1/3, 0.42]$, the first-best is implementable under both MS and EL settings. These allocations correspond to the points on $ab$ segment of the Pareto frontier. When $\alpha_\ell$ exceeds $0.42$, the first-best allocations can no longer be implemented under MS. However, as long as $\alpha_\ell \in [0.42, 0.54]$, the first-best is implementable under EL. These allocations are shown in Figure 2 as segment $bc$ on the PF. When $\alpha_\ell$ exceeds $0.54$, the EL setting too cannot implement the corresponding FB allocations. These are shown by segment $cf$ on the PF in Figure 2.

Turning to allocations that are second-best, they will be attained under MS for $\alpha_\ell > 0.42$ and shown in Figure 2 by the $bb'$ curve that lies everywhere below the PF. It approaches the 45 degree line as $\alpha_\ell \to 1$. The limiting $\ell$-types’ allocation is $I_\ell = 0, c_\ell = -T_\ell = 1.53$ resulting in $u_\ell = 2.48$. The corresponding values for the $h$-types are $I_h = 10, c_h = 8.48$, and $u_h = 2.47731$. Under EL, on the other hand, there is no second-best allocation that extends beyond the boundary point $c$ — however high one raises $\alpha_\ell$ above 0.54. This is the case because at $\alpha_\ell = 0.54$, $\tilde{I}_\ell = 0$. Interestingly though, whereas $u_\ell = 2.4773$ for $\alpha_\ell = 1$ under MS, $u_\ell = 2.95 > 2.4773$ for $\alpha_\ell \geq 0.54$.

Table 1 illustrates the laissez-faire allocations as well as the first best, MS, and EL allocations for different welfare weights. When low-productivity individuals have a welfare weight equal to $\alpha_\ell = 0.35 < 0.42$, the first best is implementable under both MS and EL. When $\alpha_\ell = 0.5 > 0.42$,
Utilities have the functional form $u = 2(c + \gamma \ln(1 - L))^{0.5}$, with $w_\ell = 14$, $w_h = 20$, and $\gamma = 10$.

The first-best allocation continues to be implementable under EL but not MS. This is the Rawlsian FB solution with $u_\ell = u_h = 2.72$. The second-best MS solution for $\alpha_\ell = 0.5$ entails a lower utility level for the $\ell$-types as compared to what they can attain under EL (2.34 versus 2.72). But the $h$-types enjoy a higher utility level (3.05 versus 2.72.) At $\alpha_\ell = 0.54$, the EL solution remains first-best with $u_\ell = 2.95$ and $u_h = 2.48$. This corresponds to point $c$ on PF and caps the utility level the $\ell$-types can attain under EL. Observe also that $u_\ell > u_h$ at this point.

Raising $\alpha_\ell$ further does not change the optimal allocations under EL—not even as a second-best solution. Table 1 illustrates this point by finding the solution for $\alpha_\ell = 0.65$ for which we continue to have $u_\ell = 2.95$ and $u_h = 2.48$ under EL. However, while we have the same allocation for $\alpha_\ell = 0.65$ as for $\alpha_\ell = 0.54$, this allocation is not FB for $\alpha_\ell = 0.65$. At the FB allocation for this value of $\alpha_\ell$, the $\ell$-types attain a higher utility level equal to $u_\ell = 3.39$ (and the $h$-types a lower utility level equal to $u_h = 1.82$). The second-best MS allocation at $\alpha_\ell = 0.65$ results in $u_\ell = 2.42$ (and $u_h = 2.93$) which is worse for the $\ell$-types as compared to EL.

### Table 1: Incomes, taxes, and utility levels in first-best, EL, MS, and laissez-faire allocations.

<table>
<thead>
<tr>
<th>$\alpha_\ell$</th>
<th>FB</th>
<th>EL</th>
<th>MS</th>
<th>LF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>$I_\ell = 4, I_h = 10$</td>
<td>$I_\ell = 4, I_h = 10$</td>
<td>$I_\ell = 3.52, I_h = 10$</td>
<td>$I_\ell = 4, I_h = 10$</td>
</tr>
<tr>
<td></td>
<td>$T = -1.22$</td>
<td>$T = -1.53$</td>
<td>$T = -0.74$</td>
<td>$T = -0.92$</td>
</tr>
<tr>
<td></td>
<td>$u_\ell = 2.72, u_h = 2.72$</td>
<td>$u_\ell = 2.95, u_h = 2.48$</td>
<td>$u_\ell = 2.34, u_h = 3.05$</td>
<td>$u_\ell = 2.37, u_h = 3.02$</td>
</tr>
<tr>
<td>0.50</td>
<td>$I_\ell = 4, I_h = 10$</td>
<td>$I_\ell = 3.27, I_h = 10$</td>
<td>$I_\ell = 2.67, I_h = 10$</td>
<td>$I_\ell = 4, I_h = 10$</td>
</tr>
<tr>
<td></td>
<td>$T = -1.53$</td>
<td>$T = -0.80$</td>
<td>$T = -0.92$</td>
<td>$T = -0.80$</td>
</tr>
<tr>
<td></td>
<td>$u_\ell = 2.95, u_h = 2.48$</td>
<td>$u_\ell = 2.95, u_h = 2.48$</td>
<td>$u_\ell = 2.42, u_h = 2.93$</td>
<td>$u_\ell = 2.42, u_h = 2.93$</td>
</tr>
<tr>
<td>0.54</td>
<td>$I_\ell = 4, I_h = 10$</td>
<td>$I_\ell = 3.27, I_h = 10$</td>
<td>$I_\ell = 2.67, I_h = 10$</td>
<td>$I_\ell = 4, I_h = 10$</td>
</tr>
<tr>
<td></td>
<td>$T = -1.53$</td>
<td>$T = -0.80$</td>
<td>$T = -0.92$</td>
<td>$T = -0.80$</td>
</tr>
<tr>
<td></td>
<td>$u_\ell = 2.95, u_h = 2.48$</td>
<td>$u_\ell = 2.95, u_h = 2.48$</td>
<td>$u_\ell = 2.42, u_h = 2.93$</td>
<td>$u_\ell = 2.42, u_h = 2.93$</td>
</tr>
<tr>
<td>0.65</td>
<td>$I_\ell = 4, I_h = 10$</td>
<td>$I_\ell = 3.27, I_h = 10$</td>
<td>$I_\ell = 2.67, I_h = 10$</td>
<td>$I_\ell = 4, I_h = 10$</td>
</tr>
<tr>
<td></td>
<td>$T = -2.24$</td>
<td>$T = -1.53$</td>
<td>$T = -0.92$</td>
<td>$T = -0.80$</td>
</tr>
<tr>
<td></td>
<td>$u_\ell = 3.39, u_h = 1.82$</td>
<td>$u_\ell = 2.95, u_h = 2.48$</td>
<td>$u_\ell = 2.42, u_h = 2.93$</td>
<td>$u_\ell = 2.42, u_h = 2.93$</td>
</tr>
</tbody>
</table>

Raising $\alpha_\ell$ further does not change the optimal allocations under EL—not even as a second-best solution. Table 1 illustrates this point by finding the solution for $\alpha_\ell = 0.65$ for which we continue to have $u_\ell = 2.95$ and $u_h = 2.48$ under EL. However, while we have the same allocation for $\alpha_\ell = 0.65$ as for $\alpha_\ell = 0.54$, this allocation is not FB for $\alpha_\ell = 0.65$. At the FB allocation for this value of $\alpha_\ell$, the $\ell$-types attain a higher utility level equal to $u_\ell = 3.39$ (and the $h$-types a lower utility level equal to $u_h = 1.82$). The second-best MS allocation at $\alpha_\ell = 0.65$ results in $u_\ell = 2.42$ (and $u_h = 2.93$) which is worse for the $\ell$-types as compared to EL.

### 6 Concluding remarks

Three decades ago Slemrod (1990, p. 157) wrote “...in its current state, optimal tax theory is incomplete as a guide to action concerning the questions that began this paper and for other issues in tax policy. It is incomplete because it has not yet come to terms with taxation as a system of coercively collecting revenues from individuals who will tend to resist”. Clearly, there is still a long way ahead of us in this regard. In this paper, we have tried to take a short step in this direction by assuming that one’s willingness to misreport his income depends on his productivity type. Interestingly, and rather surprisingly, we have found that all available allocations with
truthful reporting of incomes have a corresponding weakly Pareto-superior allocation in a setting where low-productivity types misreport their income but high-productivity do not. Moreover, auditing is never desirable even if it can be done at no cost.

Specifically, we have shown that (i) every FB allocation that can be implemented under MS can also be implemented under EL; (ii) every utility level that low-wage individuals can have under a second-best MS solution is available to them as a first-best EL solution; (iii) the first-best EL solutions include the Rawlsian outcome as well as outcomes wherein low-wages individuals are better off than high-wage individuals (neither of these type of solutions are available under MS).

There are many directions in which this work can be extended to integrate tax evasion into the Mirrleesian optimal income tax framework. The first obvious direction is to relax the assumption of perfect correlation between ability type and unwillingness to misreport income. Either one of the two individual types, or both, may include honest and dishonest income reporters. One can introduce a psychic evasion cost into the model which can be different for the types. Another avenue is to explore the implications of introducing some kind of conditional honesty along the lines of Alger and Renault (2006). The message of Slemrod (1990) remains as relevant today as it was three decades ago.
Appendix

A Details of derivations for the example

In the laissez-faire, with \( c_i = w_iL_i \), each individual maximizes

\[
2[w_iL_i + \beta \ln(1 - L_i)]^{0.5},
\]

with respect to \( L_i \). The first-order condition reduces to \( w_i - \beta / (1 - L_i) = 0 \). Assuming \( w_i - \beta > 0 \) for \( i = h, \ell \), this yields interior solutions:

\[
\begin{align*}
L_i^{LF} &= (w_i - \beta) / w_i, \\
c_i^{LF} &= w_i - \beta, \\
u_i^{LF} &= 2 [w_i - \beta + \beta (\ln \beta - \ln w_i)]^{0.5}
\end{align*}
\]

First-best solution The Lagrangian expression for the maximization of \( \alpha_h u_h + \alpha_\ell u_\ell \) subject to the resource constraint is

\[
\mathcal{L} = 2\alpha_h [c_h + \beta \ln(1 - L_h)]^{0.5} + 2\alpha_\ell [c_\ell + \beta \ln(1 - L_\ell)]^{0.5} + \mu (w_hL_h + w_\ell L_\ell - c_h - c_\ell)
\]

The first-order conditions for this problem are, for \( i = 1, 2, \)

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial c_i} &= \alpha_i [c_i + \beta \ln(1 - L_i)]^{-0.5} - \mu = 0, \\
\frac{\partial \mathcal{L}}{\partial L_i} &= -\alpha_i \frac{\beta}{1 - L_i} [c_i + \beta \ln(1 - L_i)]^{-0.5} + \mu w_i = 0.
\end{align*}
\]

Substitute for \( \mu \) from (A1) into (A2) and solve for \( L_i \) to get

\[
L_i^* = (w_i - \beta) / w_i, \quad i = h, \ell.
\]

We also have, from (A1),

\[
(\alpha_h)^2 [c_\ell + \beta \ln(1 - L_\ell)] = (\alpha_\ell)^2 [c_h + \beta \ln(1 - L_h)].
\]

Solving equations (A3)–(A4) and the resource constraint for \( c_h, c_\ell, \) and \( T_\ell \) yields

\[
\begin{align*}
T_\ell^* &= \alpha_h^2 [w_\ell - \beta + \beta \ln(\beta/w_\ell)] - \alpha_\ell^2 [w_h - \beta + \beta \ln(\beta/w_h)] / (\alpha_h^2 + \alpha_\ell^2), \\
c_h^* &= w_h - \beta + T_\ell^*, \\
c_\ell^* &= w_\ell - \beta - T_\ell^*.
\end{align*}
\]

Given the specification for the utility function (10), and using the first-best values of \( c_h, c_\ell, L_h, \) and \( L_\ell \) from above equations, we have

\[
u_h^2 + u_\ell^2 = 4 [w_h + w_\ell - 2\beta + \beta \ln(\beta/w_h) + \beta \ln(\beta/w_\ell)]
\]
The Pareto frontier is found from this equation to be
\[ u(\ell) = 2 \left\{ w_h + w_\ell - 2\beta + \beta \ln(\beta/w_h) + \ln(\beta/w_\ell) \right\}^{5/4}. \]

Using the above values for \( c^*_h, L^*_h, c^*_\ell, L^*_\ell \) in the utility function (10), we have:
\[ u(c^*_h, \frac{I^*_h}{w_h}) = 2\alpha_h \left\{ \frac{(w_h + w_\ell - 2\beta + \beta \ln(\beta/w_\ell) + \ln(\beta/w_h))}{\alpha_h^2 + \alpha_h^2} \right\}^{5/4}, \quad (A5) \]
\[ u(c^*_\ell, \frac{I^*_\ell}{w_h}) = 2 \left\{ \beta \ln \frac{w_h - w_\ell + \beta}{w_h} + \frac{\alpha_h^2 [w_h - w_\ell - 2\beta + \beta \ln(\beta/w_\ell)] - \beta \alpha_h^2 \ln(\beta/w_\ell)}{\alpha_h^2 + \alpha_h^2} \right\}^{0.5}, \quad (A6) \]
\[ u(-T^*_\ell, 0) = 2 \left\{ \frac{\alpha_h^2 [w_h - \beta + \beta \ln(\beta/w_\ell)] - \alpha_h^2 [w_\ell - \beta + \beta \ln(\beta/w_\ell)]}{\alpha_h^2 + \alpha_h^2} \right\}^{0.5}. \quad (A7) \]

Comparing (A6) with (A7), one finds that \( u(-T^*_\ell, 0) < u(c^*_\ell, \frac{I^*_\ell}{w_h}) \) if and only if
\[ w_\ell - \beta + \beta \ln \left(1 - \frac{w_\ell - \beta}{w_h}\right) > 0, \quad (A8) \]
which is true for all \( w_h > w_\ell > \beta > 0 \) (as required for \( I^*_\ell > 0 \)).

**Second best under MS** Under MS, the first-best allocation is implementable as long as (A8) is satisfied. If this is not the case, the problem of the social planner is
\[
\max_{I_h, I_\ell, T} \quad 2\alpha_h \left[ I_h + T + \beta \ln \left(1 - \frac{I_h}{w_h}\right) \right]^{0.5} + 2\alpha_\ell \left[ I_\ell - T + \beta \ln \left(1 - \frac{I_\ell}{w_\ell}\right) \right]^{0.5} \quad (A9) \\
\text{s.t.} \quad I_h + T + \beta \ln \left(1 - \frac{I_h}{w_h}\right) \geq I_\ell - T + \beta \ln \left(1 - \frac{I_\ell}{w_\ell}\right).
\]

It is straightforward to show that the second-best allocation is characterized by no distortion at the top, i.e., \( I^*_{h, MS} = w_h - \beta \). Using this condition, the incentive compatibility constraint can be rewritten as
\[ w_h - \beta + T + \beta \ln \left(\frac{\beta}{w_h}\right) \geq I_\ell - T + \beta \ln \left(1 - \frac{I_\ell}{w_\ell}\right), \]
which, when binding, implies that
\[ T = \frac{I_\ell + \beta - w_h - \beta \ln(\beta) + \beta \ln(w_h - I_\ell)}{2}. \]
Substituting this expression for \( T \) and \( I^*_{h, MS} = w_h - \beta \) in (A9), the problem of the social planner can be rewritten as
\[
\max_{I_\ell} \quad 2\alpha_h \left[ \frac{w_h - \beta + \beta \ln(\beta)}{2} - \beta \ln(w_h) + \frac{I_\ell + \beta \ln(w_h - I_\ell)}{2} \right]^{0.5} \\
+ 2\alpha_\ell \left[ \frac{w_\ell - \beta + \beta \ln(\beta)}{2} + \frac{I_\ell - \beta \ln(w_h - I_\ell)}{2} + \beta \ln \left(1 - \frac{I_\ell}{w_\ell}\right) \right]^{0.5}. \]

\[ ^8 \text{With } w_h > w_\ell \text{ and } w_\ell > \beta, \quad w_\ell - \beta + \beta \ln \left(1 - \frac{w_\ell - \beta}{w_h}\right) > w_\ell - \beta + \beta \ln \left(\frac{\beta}{w_\ell}\right) > 0. \]
The first-order condition, for an interior solution, is

\[
\left[ \frac{1}{2} + \frac{\beta}{2(w_h - I_{MS}^h)} \right] \frac{\alpha_h}{u_h} + \left[ \frac{1}{2} + \frac{\beta}{2(w_h - I_{MS}^h)} + \frac{\beta}{w_{\ell} - I_{MS}^\ell} \right] \frac{\alpha_{\ell}}{u_{\ell}} = 0.
\]

**Second best under EL** The second-best outcome occurs when \( \tilde{I}_{\ell} = 0 \) and we have a unique solutions with \( I_{h}^{EL} \) and \( I_{\ell}^{EL} \) set at their first-best levels and \( T \) being determined from (4f) as

\[
T^{EL} = -\frac{I_{h}^* + \beta \ln(1 - I_{h}^*/w_h)}{2} \\
= -\frac{w_h - \beta + \beta \ln(\beta/w_h)}{2}.
\]
References


