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Strategic Compromise, Policy Bundling and Interest Group Power

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ABSTRACT

Strategic Compromise, Policy Bundling and Interest Group Power*

Policy reforms are often multifaceted. In the rent-seeking literature policies are usually taken as one-dimensional. This paper models policy formation using a political contest with endogenous policy proposals containing two dimensions. The two dimensions provide an opportunity to trade off one policy over another to make the lobbying opposition less aggressive. In a first stage, the Government proposes a reform over the two policies, and in a second stage engages in a contest with an Interest Group over the enactment of the proposed reform. As a result, the Government makes a compromise, under-proposing in the policy the Interest Group opposes and over-proposing in the policy the Interest Group desires. Effectively, there will be strategic bundling of desired policies with undesired ones in an attempt to increase enactment probability and overall utility.

JEL Classification: D72, D86, H4
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1 Introduction

Policy reforms are frequently multifaceted. They are composed of a collection of provisions involving more than one policy dimension. In international trade, for instance, decisions must be made about the commodities to be regulated as well as the level of tariffs. In health care policy, insurance coverage and insurance policy standards may fall under a single health reform bill. In education, a reform could contain provisions on the salary of teachers as well as on the curriculum. In social welfare, reforms can contemplate a change in the generosity of some benefit, and also changes in the group of beneficiaries. Often, these different elements of reform are used by opposing interests as tools for bargaining in the political arena. A group that prioritizes one policy dimension over another may use the other dimension as an instrument for negotiation.

During the heated 2009 debates regarding the United States’ Obamacare health bill, certain concessions were made in order to secure the support of politicians to ensure the bill’s passage, some of those policies on which compromises were made were the federal funding for abortion and the public health insurance option, which many agreed to be minor parts of the overall reform. Another example is Australia’s controversial Higher Education Bill of 2014, whose main purpose was to reform the funding system for Australian universities through deregulation of higher education fees. The bill has undergone a series of compromises since its introduction early in 2014, including the proposed budget cut to universities by 20%, in order to win over the labor party, the Greens, and resistant cross-bench politicians.¹

In this paper we explore such strategic behavior in a political contest model with two policy elements that may be traded off to augment the probability of winning. We consider a policy-oriented Government interested in reforming a bundle of two policies, one of which is opposed and the other supported by an Interest Group. In the first stage, the Government makes a reform proposal to the legislature, which then decides on its enactment in the second stage. The decision to enact the proposed reform is influenced negatively by the lobbying effort of the Interest Group and positively by the lobbying effort of the Government. We therefore model the legislative process as a contest in which effort exerted by the interest group and the Government are costs associated with gaining votes in the legislature. In this endogenous policy formation framework, we investigate the composition of the Government’s reform proposal in the presence of the Interest Group. By threatening to block the reform enactment in congress through lobbying, the Interest Group can nudge the Government’s policy proposal closer to their desired bundle. In equilibrium, the Government makes a compromise with respect to its optimal bundle by proposing less of the Interest Group’s opposed policy and more of the Interest Group’s desired policy.

This result has an interesting implication for policy bundling. If the Government were interested in reforming only one policy dimension, then, absent opposition, he would propose to keep the other policy at the status quo. However, if this policy is desired by the Interest Group, the model implies that an opposition would induce him in equilibrium to propose a level higher than the status quo to appease the lobby. In such a case, the Government strategically bundles the policies into one proposal to maximize the chances of enactment.

The setup of our model has three features of the policy-making process that deserve preliminary comment. First, the Government does not decide on reform enactment. Existing literature models the Government as the contest

maker who decides on the proposals of two opposing interest groups that expend effort to gain the favor of the Government. Here, we assume instead a Government whose reform-making power is constrained by the legislature. Thus, the contest-maker in our analysis is the legislative process. Second, although reform enactment is not determined entirely by the Government, it has agenda-setting power\[1\] The Government can propose a reform bundle, whose enactment is then decided in congress. If congress votes favorably, the proposed reform is enacted; if not, status quo is maintained. Third, since Government and an Interest Group engage in a legislative contest over the enactment of the proposed reform, both must exert effort to increase their probability of winning. We therefore assume that the effort expended by both sides are activities conducted for the purpose of gaining votes in their favor\[2\] From the viewpoint of the interest group, lobbying effort comes in the form of campaign contributions, hiring lobbyists, organizing protests and printing newspaper ads and other publicity materials. From the viewpoint of the Government, successful reform passage requires effort put into talks and negotiations with legislators, and into trying to nudge public debate in their favor, consequently influencing the constituencies of the legislators who must cast their vote\[3\] Such a setup is well suitable to represent numerous real-world applications, particularly presidential systems and coalition Governments where the ruling party does not decide on the policy, but can propose bills that must survive the legislature. As a well known example of Government lobbying, during the 111th United States Congress where the Patient Protection and Affordable Care Act (better known as Obamacare) was introduced, time and effort were spent in negotiations to break the threat of a Republican filibuster. Former U.S. president Barack Obama himself delivered a speech to a joint session of Congress to emphasize his commitment to the reform. Another example of Government lobbying is the video released by the Italian Government in the spring of 2015 about the highly-controversial and much opposed \textit{La Buona Scuola} (The Good School) education reform. The video showed Italian prime minister Matteo Renzi discussing in detail the benefits of his administration’s proposed reform. Other examples can be found in the sociological and political science literature, for instance in the fields of welfare state (Busemeyer and Neimanns, 2017; Häusermann et al., 2019), labor market (Gallego and Marx, 2017) and pension reforms (Milkman et al., 2012).

The remainder of our paper proceeds as follows. In the next section, we review the most relevant literature. In Section 3 we present the basic setup of the model, that we solve and discuss in Section 4. Finally, Section 5 concludes.

## 2 Related Literature

Our attempt to model policy determination in the face of opposition is related to the literature on policy formation under lobbying. So far, two opposite theories have been developed in the literature, the first being the “compromise” theory that says that the lobbying induces a compromise between the policy preferences of the stakeholders.

\[\text{\textsuperscript{2}}\text{See for example Knight (2005), for a study of proposal power in a } \text{Baron and Ferejohn (1989)’s bargaining model, and Aragones et al. (2015), for an article studying agenda setting power.}\]

\[\text{\textsuperscript{3}}\text{See for example Finger (2017) argues that interest groups can also have agenda setting advantages by electing allies in the legislature. We do not consider interest groups’ electoral motives here.}\]

\[\text{\textsuperscript{4}}\text{We follow in some sense Skaperdas and Vaidya (2012) who take lobbying effort as the cost of “persuasion”. This is in contrast to recent contest lobbying literature that takes effort to be favors or bribes that enter the Government’s utility function. See for example Epstein and Nitzan (2004); Münster (2006); Epstein and Nitzan (2006a,b); Epstein and Hefeker (2003) explore lobbying contests where opponents can use more than one instrument in order to win. In this study we assume all such instruments can be aggregated into a single measure of effort.}\]
Studies of this kind include Hillman and Ursprung (1988), Grossman and Helpman (1996), Epstein and Nitzan (2004), Münster (2006), and Felli and Merlo (2006). Grossman and Helpman’s (1996) Downsian model considered lobbying as a “menu auction” and found that the equilibrium policy is a compromise between the policy preferences of the lobbies and the policy preferences of the voters. Felli and Merlo (2006) also used menu-auction lobbying to develop a citizen-candidate model of political competition where the politician selects the lobbies endogenously. In this setting, the policy outcome is a compromise between the policy preferred by the candidate and the policy preferred by the chosen lobbies. Epstein and Nitzan (2004) used contests to model lobbying and relaxes the assumption commonly made in contest games that the policy proposals are exogenous. They develop a two-stage political contest between interest groups where first stage determines the choice of policy proposal and the second stage is where the contest occurs. They show that under these circumstances groups have an incentive to strategically moderate their proposals in order to reduce the effort of the opposition, thereby increasing their chances of winning. The resulting proposals are therefore less polarized than they would be without opposition. In their model, the proposals will never coincide. Münster (2006) explored the same setup for perfectly discriminating contests and found that the proposals of the two groups will not only be less polarized, but will coincide.

The opposite theory in this strand of literature provides circumstances under which lobbying could result in extreme policies. Glazer et al. (1998) developed a simple framework showing that an incumbent may choose to implement an extreme policy if he is strongly office-motivated and the costs of a challenger reversing the policy is substantial. The intuition behind this is that the median voter with moderate preferences will prefer to reelect the incumbent with an extreme policy than vote for the challenger and incur the high cost of changing the policy when the challenger comes into office. More recently, Epstein and Nitzan (2006a) studied a two-stage public policy contest in which a politician proposes a policy and interest groups compete for its approval or rejection. Building on the results of Epstein and Nitzan (2004) and Münster (2006), they find that a politician will propose an extreme policy if his marginal benefit from the lobbying expenditures exceeds his marginal losses from the disutility of the lobbies.

Inherent in these special interest and rent-seeking studies is the one-dimensionality of the policy space. The few studies involving theoretical models with two or more policy components make simplifying assumptions about how the components affect special interests. For instance, Glazer et al. (1998) assume that one of the two types of policy issues is fixed due to predetermined positions reflecting ideology. Other studies assume that interest groups have preferences over only one of the policy components (List and Sturm, 2006; Chaturvedi and Glazer, 2005). This gap in the theoretical literature has persisted despite a number of empirical work acknowledging that reforms are multi-faceted and each facet affects interest groups in varying degrees (Kang, 2015, Lake, 2015, Fabella, 2017). We attempt to fill this gap by proposing a model of policy reform in a two-dimensional policy space, the components of which both enter directly into the preferences of the interest groups. This provides insights into the decision to trade off one component over another to augment the probability of reform enactment. Indeed, we find that compared to the Government’s preferred reform bundle when there is no Interest Group, the equilibrium proposal of the Government will have more of the Interest Group’s favored policy component and less of the opposed policy component. Hence, the Government makes a strategic compromise in an attempt to make the Interest Group less aggressive.

We are aware of the significant strand of the literature which focus on the issue of multidimensional policy space in electoral competition, starting from McKelvey (1976) to the most recent contribution of McKelvey and Patty (2006), De Donder et al. (2012), Bellani and Scervini (2015), but we are not aware of any contribution that has addressed the issue in special interest and rent-seeking models.
Moreover, the implications of our framework are similar to those of Cotton and Dellis (2016) who look at information lobbying and agenda distortion. They also find that the lobbying distorts the reform agenda towards the desired points of the interest groups, though their mechanism differs from ours in that they use information lobbying by interest groups that shifts the Government’s attention from constituency-preferred policies to interest group-preferred policies.

Finally, in focusing on issue selection and on the possibility of distorting policy in some dimensions to gain the support, our paper relates naturally also to the literature on log-rolling that emerged in the 1980s (see for instance Shepsle and Weingast 1981a,b, Weingast et al., 1981). However, we differ from this setting in that we do not explicitly consider vote trading by legislative members. Instead we look at simultaneous voting, instead of sequential, on the different issues.

3 Basic Setting

In the spirit of the lobbying literature with endogenous policy proposals (Epstein and Nitzan 2004), consider a setting with two risk-neutral actors: a policy-driven Government, $G$, and an Interest Group, $I$. The Government wishes to conduct a reform in two policy dimensions. As an illustrative example, these can be broadening access $(a)$ versus improving quality $(q)$, in the sense that applies to health policy or education policy. The proposal of the Government lies in the policy space $(a,q)$ and, without loss of generality, let $s = (0,0)$ be the status quo.

In our setup, reform enactment is not determined entirely by $G$, but by a political institution such as the legislature. $G$, however, possesses agenda-setting power and can make a proposal $r = (\tilde{a}, \tilde{q})$, which is then voted on in congress.\footnote{We shall use legislature and congress interchangeably.}

We model the policy formation as the outcome of a contest in which reform enactment is a function of $G$ and $I$’s effort to gain votes in the legislature. If $G$ gets more votes than $I$, the reform proposal $(\tilde{a}, \tilde{q})$ is enacted, otherwise, status quo is maintained.

Let us call a generic proposal bundle $b = (a,q)$ and assume that the Government wants to implement its optimal bundle $g^* = (a^*_G,q^*_G)$. The point $(a^*_G,q^*_G)$ can serve a number of interpretations. It can be the reform bundle that maximizes the utility of the group in power, or the bundle that maximizes an incumbent’s political support, or even the bundle that maximizes the preferences of $G$’s constituency. We make no assumptions about the motives that determine $(a^*_G,q^*_G)$ and this point will therefore serve just as a benchmark upon which to compare the equilibrium reform levels that will emerge.

It is worthwhile noting that outside of this political contest, any other costs to $G$ of deviating from its optimal point is assumed to be captured by a loss in utility $U_G$ from deviating from the optimal bundle $(a^*_G,q^*_G)$. This allows us to focus on the reform levels that emerge from the interaction between $G$ and $I$.

**Utility function.** Let the preferences of $G$ be a function of the Euclidean distance $d(\cdot)$ between the generic bundle $b$ and the optimal bundle $g^*$, $d(b,g^*) = \sqrt{w^a_G(a - a^*_G)^2 + w^q_G(q - q^*_G)^2}$, where $w^a_G$ and $w^q_G$ are weights representing the relative importance of each dimension. For convenience, we moreover assume without loss of
generality that the utility depends on the square of the distance, so that

\[ U_G \left( d^2 \left( b, g^* \right) \right) = U_G \left( w_G^a \left( a - a_G^* \right)^2 + w_G^q \left( q - q_G^* \right)^2 \right). \] (1)

The utility is intuitively at the maximum if \( a = a_G^* \) and \( q = q_G^* \), and is decreasing with the squared Euclidean distance, \( \partial U_G / \partial d^2 < 0 \). We assume that the Government has an incentive to propose a reform only if its utility from the reform is higher than its utility in the status quo. Formally, we can define \( M_G = U_G \left( d^2 \left( r, g^* \right) \right) - U_G \left( d^2 \left( s, g^* \right) \right) > 0 \) as the difference between the utility of the Government when the reform \( (r) \) is implemented and the utility of the status quo \( (s) \). In the remainder of the paper we focus on the case of \( M_G > 0 \). Otherwise, the Government has no incentive to make a proposal in the first place.

The preferences of the Interest Group are modeled in a similar manner. Let its optimal bundle be denoted by \( i^* = (a_I^*, q_I^*) \) and its preferences be described by \( U_I (a, q) \), as follows

\[ U_I \left( d^2 \left( b, i^* \right) \right) = U_I \left( w_I^a \left( a - a_I^* \right)^2 + w_I^q \left( q - q_I^* \right)^2 \right). \] (2)

\( I \)'s utility decreases with the distance between a generic bundle \( b \) and \( i^* \), \( \partial U_I / \partial d < 0 \). Also in this case, we make no explicit assumption on the determinants of \( i^* \): the Interest Group’s optimal bundle could have been chosen to maximize political prestige, the utility of its supporters, or funds raised. The interpretation of the utility function and its maximization conditions are the same as eq. (1). Figure 1 represents an example of Government and Interest Group utility functions.

Figure 1: Illustrative example of utility functions in case the Government (red) wants to improve only the quality \( (g^* \) is its optimal policy bundle) with respect to the status quo \( s \), while the Interest Group (blue) wants to improve only the access. Indifference curves farther from \( g^* \) \((i^*) \) represent lower levels of utility for the Government (Interest Group).

\(^8\)See appendix A.1 for the discussion of the first order condition.
**Agents’ Incentives.** The attitude of the Interest Group depends on the relative distances between the *status quo* $s$, the proposal, $r = (\tilde{a}, \tilde{q})$, and its own preferred bundle $i^*$, that in turn affects the difference in utilities. If we define $M_I$ analogously to $M_G$ as the difference between utility under the reform and utility under the *status quo* $s$, we can state that:

i. if $d(r, i^*) > d(s, i^*) \Rightarrow M_I < 0$, then $I$ prefers the *status quo*,

ii. if $d(r, i^*) < d(s, i^*) \Rightarrow M_I > 0$ then $I$ supports the reform,

iii. if $d(r, i^*) = d(s, i^*) \Rightarrow M_I = 0$ then $I$ is indifferent between the two options.

The only relevant case to study is the one in which the Interest Group has an incentive to oppose the reform. Otherwise, the Government and the Interest Group agree in support of the reform and the Interest Group has no incentive to oppose the reform. For this reason, the remainder of the paper focuses on the first case in which $M_I < 0$.

Figure 2 shows the possible cases. If the Government has no incentive to propose a reform, then the *status quo* is maintained irrespective of the preferences of the Interest Group. Otherwise, the reform is implemented if it is proposed by the Government ($M_G > 0$) and not opposed by the Interest Group ($M_I \geq 0$), while some kind of compromise should be reached if the Interest Group exerts some effort against the reform ($M_I < 0$).

![Decision Tree](image)

**Figure 2:** Representation of the decision tree.

The no-proposal case can be seen as a trivial case where the *status quo* $(\tilde{a}, \tilde{q}) = (0, 0)$ is proposed. The no-opposition case, on the other hand, occurs when both the Government and the Interest Group prefer the reform over *status quo*. In both these cases, the Government’s utility is maximized. Furthermore, both cases have corner solutions in effort$^{10}$ No efforts are exerted and the proposal is passed with full certainty.

From this point forward, let us consider only the case with an interior solution in efforts, i.e. $M_G > 0$ and $M_I < 0$ (highlighted red in Figure 2).

$^9 M_I = U_I (d^2 (r, i^*)) - U_I (d^2 (s, i^*))$.

$^{10}$These corner solutions can be formalized as $e_G = e_I = 0$ and $p (\tilde{a}, \tilde{q}) = 1$, that will be formally defined in the next paragraph.
The game. The timing of the game is as follows. In stage 1, $G$ decides on his reform proposal $(\tilde{a}, \tilde{q})$. In stage 2, $G$ and $I$ engage in a contest over the enactment of $(\tilde{a}, \tilde{q})$ in the legislature. If $G$ wins, his proposed reform $(\tilde{a}, \tilde{q})$ is enacted, if $I$ wins, status quo is maintained.

In the second-stage contest, let $e_G$ and $e_I$ be the effort levels exerted by $G$ and $I$, respectively, to gain votes in the legislature. Denote the probability that the proposal is enacted by the Tullock success function $p(e_G, e_I) \in [0, 1]$, 

$$p(e_G, e_I) = \frac{\alpha_G e_G}{\alpha_G e_G + \alpha_I e_I}, \quad (3)$$

where $\alpha_j$ denotes the ‘productivity’ of each contestant’s effort, with $\alpha_j > 0$, $j \in \{G, I\}$. As in the standard contest defined by Tullock (1980), whatever the outcome of the contest, the invested effort of both players are lost. This success function is increasing and concave in the lobbying effort of $G$, while it is decreasing and convex in the effort of $I$. Such assumptions ensure a positive but diminishing marginal effect of each player’s effort on their own probability of winning the contest; moreover, they ensure that an increase in each player’s effort harms the other, making it strategically desirable for each player to induce a lower effort from the other.

The expected payoffs of $G$ and $I$ from the two-stage game are given by:

$$EU_G = p U_G \left( d^2 \left( r, g^* \right) \right) + (1 - p) U_G \left( d^2 \left( s, g^* \right) \right) - e_G, \quad (4)$$

$$EU_I = p U_G \left( d^2 \left( r, i^* \right) \right) + (1 - p) U_G \left( d^2 \left( s, i^* \right) \right) - e_I. \quad (5)$$

We consider subgame-perfect Nash equilibria consisting of reform proposals $(\tilde{a}, \tilde{q})$ and effort levels $e_G^*$ and $e_I^*$ such that at every stage each player takes an action that maximizes his expected payoff given the other’s behavior.

4 Equilibria

In this section we characterize the optimal behavior of the two agents using backward induction. Anticipating the level of effort of Government and Interest Group in the second stage, the Government chooses in the first stage the policy bundle that maximizes its expected utility, that is the balance between the benefit from the policy implemented and the cost from the effort necessary to counteract the opposition of the Interest Group. We will show that it is rational to choose a bundle closer to the Interest Group’s preferences in the first stage if this is overcompensated by the gain in terms of reduced-effort in the second stage.

4.1 Second stage equilibrium

Our interest lies in the comparison between $G$’s resulting reform proposals in the presence of an Interest Group and his proposal without an Interest Group. In the absence of $I$, the second stage legislative contest becomes trivial, and $G$ will win the contest with no effort. In the first stage, $G$ can therefore propose his optimal reform $(a_G^*, q_G^*)$.

Without Interest Group, the optimal behavior of the Government is $(\tilde{a}, \tilde{q}) = (a_G^*, q_G^*)$, that is implemented with $p = 1$ with $e_G = 0$. This is a corner solution of the Government maximization problem at the maximum level of utility.

11 Formally, $\partial p/\partial e_G > 0$, $\partial^2 p/\partial e_G^2 < 0$ and $\partial p/\partial e_I < 0$, $\partial^2 p/\partial e_I^2 > 0$.
In the presence of $I$, $G$ and $I$ maximize their expected payoffs in equations (4) and (5) with respect to their respective lobbying efforts. An interior equilibrium is then characterized by the following conditions.\(^{12}\)

**Lemma 1.** The equilibrium lobbying efforts of $G$ and $I$ are given by:

\[
\begin{align*}
e_G^* &= -\delta M_G^2 M_I \\
e_I^* &= \delta M_G M_I^2
\end{align*}
\]

with $\delta = \frac{\alpha_G M_G - \alpha_I M_I}{(\alpha_G M_G - \alpha_I M_I)^2}$.

Lemma 1 states that the effort of both the Government and the interest groups depends positively on their own incentive to propose/oppose the reform and on their opponent’s incentive to oppose/propose it (recall that $M_G > 0$ and $M_I < 0$). Moreover, the equilibrium condition in Lemma 1 implies that $\frac{\partial e_G}{\partial q} = -\frac{M_I}{M_G}$, meaning that the relative effort exerted by the two groups equals the relative utility gain between the reform and status quo.\(^{13}\)

Since equilibrium efforts $e_G^*$ and $e_I^*$ are functions of $G$’s policy proposal $(\tilde{a}, \tilde{q})$, comparative static properties of the second stage contest can be characterized.

**Lemma 2.** Equilibrium effort of the Interest Group, $e_I^*$, varies with the reform levels $\tilde{a}$ and $\tilde{q}$ such that

\[
\frac{\partial e_I^*}{\partial \tilde{x}} = \delta M_I \left[ -M_I \left( \frac{\alpha_G M_G + \alpha_I M_I}{\alpha_G M_G - \alpha_I M_I} \right) \frac{\partial M_G}{\partial \tilde{x}} + M_G \left( \frac{2\alpha_G M_G}{\alpha_G M_G - \alpha_I M_I} \right) \frac{\partial M_I}{\partial \tilde{x}} \right]
\]

where $x \in \{a, q\}$ and

\[
\frac{\partial e_I^*}{\partial \tilde{x}} = -\delta M_G \left[ -M_I \left( \frac{2\alpha_I M_I}{\alpha_G M_G - \alpha_I M_I} \right) \frac{\partial M_G}{\partial \tilde{x}} + M_G \left( \frac{\alpha_G M_G + \alpha_I M_I}{\alpha_G M_G - \alpha_I M_I} \right) \frac{\partial M_I}{\partial \tilde{x}} \right]
\]

In order to study the sign of eq. (8), let us recall that $M_G > 0$ and $M_I < 0$, that all the parameters $(\alpha_G, \alpha_I, w_G, w_I)$ are positive, and that the two derivatives have opposite signs.\(^{14}\) Just as an illustrative example, let us assume that the change of $\tilde{x}$ increases the utility of the Interest Group $\frac{\partial M_G}{\partial \tilde{x}} > 0$ and decreases the utility of the Government $\frac{\partial M_I}{\partial \tilde{x}} < 0$. In this case, eq. (8) is unambiguously negative if $(\alpha_G M_G + \alpha_I M_I)$ is below zero, that is, if $|\alpha_G M_G| < |\alpha_I M_I|$. In other words, if the utility gain of $I$ from the reform is larger than the utility loss of $G$, then $I$’s effort decreases with $\tilde{x}$. The same condition can be stated in terms of elasticity as follows:

\[
\frac{\partial M_I}{\partial \tilde{x}} - \frac{\partial M_G}{\partial \tilde{x}} > \frac{\alpha_G M_G + \alpha_I M_I}{2\alpha_G M_G}
\]

Intuitively, the sign of eq. (8) is surely positive if we assume that the proposal goes toward the preferences of the Government instead of the Interest Group.\(^{15}\) With respect to the effort put by the Government (eq. 9), it is surely negative if the opposite condition holds, that is if $(\alpha_G M_G + \alpha_I M_I)$ is positive, that can be stated also as $|\alpha_G M_G| > |\alpha_I M_I|$ or $\frac{\partial M_G / \partial \tilde{x}}{\partial M_I / \partial \tilde{x}} < \frac{\alpha_G M_G + \alpha_I M_I}{2\alpha_G M_G}$.

Since the sign of eq. (8) - (9) depends on the term $(\alpha_G M_G + \alpha_I M_I)$, Figure 3 represents the conditions that make the two equations positive or negative under the assumptions made above. Interestingly, if $\alpha_G M_G + \alpha_I M_I$ is

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12 All proofs can be found in the Appendix A.
13 This also justifies the assumption made above about the incentives to exert effort. If $M_G < 0$ and/or $M_I > 0$, both efforts would be negative, which is a meaningless result. If we bound the efforts to be non-negative, then they would be both zero in this case.
14 Indeed, this is a condition for the Interest Group to oppose the proposal of the Government. If a reform increased the utility of both the Government and the Interest Group (the two derivatives have the same sign), then there would be no bargaining.
15 $\frac{\partial M_G}{\partial \tilde{x}} < 0$ and $\frac{\partial M_I}{\partial \tilde{x}} > 0$. 
sufficiently close to zero, that is if \( |\alpha_G M_G| \) and \( |\alpha_I M_I| \) are not too different, shifting the bundle towards the preferences of the Interest Group decreases the effort of both the Interest Group and the Government.

\[
\frac{\partial e_G}{\partial \tilde{x}} > 0
\]
\[
\frac{\partial e_I}{\partial \tilde{x}} < 0
\]

Figure 3: Signs of the derivatives of the G and I’s effort with respect to the policy proposal \( \tilde{x} \), \( x \in \{a, q\} \).

4.2 First stage equilibrium

In this subsection we first explore the case with a single-policy reform, as a reference, and then we analyze the policy bundle model with two policies that can be traded off.

Let us start by assuming that the G can propose each time a reform on a single policy only. Without loss of generality, we assume that G can only propose a reform on \( q \), leaving a possible reform on \( a \) for a successive bill. Then, irrespective of the actual preferences of G and I on \( a \), \( \tilde{a} = 0 \). In order to determine its best proposal, \( \tilde{q} \), the Government maximizes with respect to \( q \) its expected utility in eq.(4), where the probability of implementing the policy is described in eq.(3) and depends on the efforts from the second stage maximization problem as in Lemma 1. The problem faced by the Government in then the following:

\[
\max_q U_G = \frac{\alpha^2 G M_G^3}{(\alpha_G M_G - \alpha_I M_I)} + U_G \left( d^2 (s, g^*) \right) \tag{11}
\]

whose solution is in the following proposition 1.

Proposition 1. In the presence of an opposition, the proposed reform \( \tilde{q} \) will be such that \( |q_I^*| < |\tilde{q}| < |q_G^*| \).

Proposition 1 shows that – in a single-policy framework – the optimal behavior of the Government is to propose a policy (\( \tilde{q} \)) which is closer to the preferences of the Interest Group with respect to its unconstrained optimal policy (\( q_G^* \)) in order to decrease the amount of effort needed to win the opposition of the Interest Group. Indeed, a more moderate policy would induce the Interest Group to put less effort in opposing the reform and, in turn, this requires less effort from the Government. This result coincides with the strategic restraint model of Epstein and Nitzan (2004). Essentially, the level of \( \tilde{q} \) proposed by the Government serves two functions: a policy reform that contributes to the utility of the Government, and a “bargaining tool” that affects the incentive of the opposition to engage in lobbying efforts against the reform.

Now let us assume instead that the Government is allowed to propose a reform in both dimensions, \( (a, q) \). In this case, the maximization problem is as follows:

\[
\max_{a,q} U_G = \frac{\alpha^2 G M_G^2}{(\alpha_G M_G - \alpha_I M_I)} + U_G \left( d^2 (s, g^*) \right) , \tag{12}
\]

which only differs from eq.(11) in that the Government maximizes its utility with respect to two policy dimensions, \( a \) and \( q \). In this case, the optimal behavior of the Government is described in the following Proposition 2.

16The figure is qualitatively analogous if we switch the signs of the partial derivatives and of the two loss functions \( M_G \) and \( M_I \).
**Proposition 2.** In the presence of an opposition, the proposed reform $\tilde{b} = (\tilde{a}, \tilde{q})$ lies in between the two preferred policy bundles $b_I^* = (a_I^*, q_I^*)$ and $b_G^* = (a_G^*, q_G^*)$.

Case 1: $q_I^* < q_G^*$ and $a_G^* < a_I^*$
Case 2: $q_I^* < q_G^*$ and $a_I^* < a_G^*$

Case 3: $q_G^* < q_I^*$ and $a_I^* < a_G^*$
Case 4: $q_G^* < q_I^*$ and $a_G^* < a_I^*$

Figure 4: Illustrative example of political compromise: case 1 and the symmetric case 3 describe the behavior of the Government when it over-proposes in one policy and under-proposes in the other to get closer to the bundle preferred by the Interest Group. Case 2 shows a situation in which the Government maximizes its utility by under-proposing in both policies. Case 4 is characterized by no opposition, since the Interest Group will always prefer the Government proposal with respect to the status quo. A similar picture could be drawn for negative values of any of the preferred policies.

In seeking to maximize expected payoffs, $G$ decides to propose a policy bundle that is closer to the preferred bundle of the opposition (when it exists). The rationale is quite straightforward: the relative loss due to a policy different from its preferred bundle is compensated by the decreased effort put in supporting the bill against the opposition.

Let us define the preference of the Government and the Interest Group over the policies as:

**Definition 1.** **Conflicting Preferences.** If the preferred bundle of the Interest Group is such that $x_I^* > x_G^*$ and $y_I^* < y_G^*$ with $x = a, q$ and $y = q, a$.

One interesting case among the results of Proposition 2 is the following:

**Corollary 1.** If the Interest Group and the Government exhibit conflicting preferences, the Government makes a strategic compromise by over-proposing in the policy component that the lobby favors and under-proposing in the component that the lobby opposes.
The intuition behind the results is simple. In the face of opposition, $G$ recognizes that it is dealing with a group whose lobbying efforts are affected by the extent of its proposed reforms. As a result, we can have a case in which although proposing an $\tilde{a} > a^*_G$ and a $\tilde{q} < q^*_G$ reduces his payoffs, doing so also reduces the stake of $I$, thereby reducing their incentive to exert as much lobbying effort. In effect, it gets a higher probability of reform enactment and a lower cost from effort, increasing its expected utility in the first stage of the game.

The rationale behind this result is that any change in the policy bundle proposed by the Government triggers a reaction by the Interest group, as described in equation (8) of Lemma 2. If the Government proposed a policy reform only in one dimension (e.g. an increase of $q$ from the status quo $s$ toward $g^*$ in Figure 1), this would make the effort of the Interest Group stronger and would therefore imply a larger effort from $G$ too, increasing its cost and decreasing its (expected) utility. The only way to balance costs and benefits is to trade some utility from the policy $a$ for a lower cost from efforts. Corollary 1 ensures that the in order to make the costs smaller than the benefit, the proposed bundle must lay in a specific region of the $a - q$ plane, that is with $\tilde{a} > a^*_G$ and $\tilde{q} < q^*_G$.

Based on these results, it is possible to go a step further and stating that:

**Corollary 2.** In the presence of an opposition, the Government will strategically bundle different policies in the same reform.

Referring to the situation represented in Figure 1 as an illustrative example, Corollary 2 ensures that – starting from $s$ – the Government has to bundle the desired increase of $q$ with an increase of $a$, so that $\tilde{q} > 0$ and $\tilde{a} > 0$. This implies that, in this case, an increase of $q$ causes always a cost in terms of larger effort that is larger than the increase of utility. To balance the two, the Government has to increase also $a$, so that the effort of the interest groups declines, and so the Government effort. This result is clear because, in this example, the status quo coincides with the preferences on one dimension for both agents. Things are slightly more complicated if the status quo and the preferences are different in both dimensions. What is important to notice is that the Government trades-off the two policy dimensions with respect to its preferred policy bundle and not with respect to the status quo.

5 Conclusion

In this paper we set up a game-theoretic framework to explain policy formation under multiple reform dimensions, such as health (e.g. Obamacare health bill) and education reforms (e.g. Australia’s Higher Education Bill and Italy’s La Buona Scuola (The Good School) bill). In our setting a policy-oriented Government can endogenously propose a bundle of two policies which is then voted on in the legislature. We model this legislative process as a contest between the Government and an Interest Group and find that the Government compromises on his reform proposal to appease the opposition. In effect, he uses the two policy dimensions as tools of bargaining. He proposes more of what the Interest Group supports and less of what it opposes. We also find that strategic bundling of policies may occur when the Government prefers the status quo for one policy but proposes a positive value in equilibrium, to make the opposition less aggressive. The resulting bundle of reforms contains more of the policy desired by the Interest Group, and less of the policy disfavored by the Interest Group.

This result is a first attempt to rationalize the frequent behavior of governments and policy makers that bargain over two policy dimensions. Consistently, the model is able to predict the legislative process whenever a bargaining over different policies is possible. Finally, it is also relevant from a policy perspective: by identifying the policy
dimension valued the most by the interest groups, policy makers can reduce their opposition by over-proposing in that dimension. Indeed, the 'size' of over- and under-proposing depends on the relative weights attached to the policy dimensions by the Government and the Interest Group.

References


A  Mathematical Appendix

A.1  Utility maximization (Equations 1-2).

The utility function in 1 is

\[ U_G \left( d^2 (b, g^*) \right) = U_G \left( w_G^2 (a - a_G^*)^2 + w_G^q (q - q_G^*)^2 \right). \]

It follows that the first order conditions are the following:

\[ \frac{\partial U_G}{\partial x} = \frac{\partial U_G}{\partial d^2} \cdot \frac{\partial d^2}{\partial x} = \frac{\partial U_G}{\partial d^2} \cdot 2w_G^2 (x - x_G^*) = 0, \text{ for } x \in \{a, q\} \]

(13)

The first order conditions are satisfied only if both policies are set at the optimal level \( x = x_G^* \). Otherwise, since utility decreases with the squared distance, \( \frac{\partial U_G}{\partial d^2} < 0 \), the derivative is negative if \( x > x_G^* \) and positive if \( x < x_G^* \). An increase in the policy \( x \) increases (decreases) utility if \( x \) is below (above) the optimal level.

Analogously, these are the first order conditions for the utility of the Interest Group (eq. 2):

\[ \frac{\partial U_I}{\partial x} = \frac{\partial U_I}{\partial d^2} \cdot \frac{\partial d^2}{\partial x} = \frac{\partial U_I}{\partial d^2} \cdot 2w_I^2 (x - x_I^*) = 0, \text{ for } x \in \{a, q\}. \]

(14)

For simplicity, we denote \( \frac{\partial U_G}{\partial d^2} \) by \( U_G^d \) and \( \frac{\partial U_I}{\partial d^2} \) by \( U_I^d \) in the rest of the proofs.

A.2  Proof of Lemma 1

The expected payoffs of \( G \) and \( I \) from the two-stage game are given by:

\[ EU_G = pU_G \left( d^2 (r, g^*) \right) + (1 - p) U_G \left( d^2 (s, g^*) \right) - e_G, \]

\[ EU_I = pU_G \left( d^2 (r, i^*) \right) + (1 - p) U_G \left( d^2 (s, i^*) \right) - e_I. \]

where \( p = \frac{\alpha_G e_G}{\alpha_G e_G + \alpha_I e_I} \).

Notice that the probability of implementation depends on the efforts as follows

\[ \frac{\partial p}{\partial e_G} = \frac{\alpha_G \alpha_I}{(\alpha_G e_G + \alpha_I e_I)^2} e_I = \gamma e_I \]

\[ \frac{\partial p}{\partial e_I} = \frac{-\alpha_G \alpha_I}{(\alpha_G e_G + \alpha_I e_I)^2} e_G = -\gamma e_G \]

while utility functions do not depend on efforts:

\[ \frac{\partial U_G}{\partial e_G} = 0, \quad \frac{\partial U_G}{\partial e_I} = 0, \quad \frac{\partial U_I}{\partial e_G} = 0, \quad \frac{\partial U_I}{\partial e_I} = 0 \]
so that the first order conditions are:

\[
\begin{align*}
\frac{\partial E U_G}{\partial e_g} &= 0 \\
\frac{\partial E U_i}{\partial e_i} &= 0 \\
\Rightarrow \left\{ \begin{array}{l}
\gamma e_I U_G \left( d^2 (r, g^*) \right) - \gamma e_I U_G \left( d^2 \left( s, g^* \right) \right) = 1 \\
-\gamma e_G U_I \left( d^2 \left( r, i^* \right) \right) + \gamma e_G U_I \left( d^2 \left( s, i^* \right) \right) = 1 \\
\end{array} \right. \\
\Rightarrow \left\{ \begin{array}{l}
\gamma e_I M_G = 1 \\
-\gamma e_G M_I = 1 \\
\end{array} \right. \\
\Rightarrow \left\{ \begin{array}{l}
e_I = \frac{\alpha_G G}{\alpha_G M_G - \alpha_I M_I} M_G M_I^2 \\
\gamma e_G = -\frac{\alpha_G G}{\alpha_G M_G - \alpha_I M_I} M_G^2 M_I \\
\end{array} \right. \\
\Rightarrow \left\{ \begin{array}{l}
e^*_I = \delta M_G M_I^2 \\
\gamma e^*_G = -\delta M_G^2 M_I \\
\end{array} \right.
\]

\[\text{(15)}\]

\[\text{(16)}\]

\[\text{(17)}\]

\[\text{(18)}\]

\section*{A.3 Proof of Lemma 2}

From Lemma 1, we can take the derivative of \( e_I^* \) with respect to any reform component \( \tilde{x} \), where \( x \in \{ a, q \} \)

Recall that \( e_I^* = \delta M_G M_I^2 \) where \( \delta = \frac{\alpha_G G}{\alpha_G M_G - \alpha_I M_I} \).

The first derivative is

\[
\frac{\partial e_I}{\partial \tilde{x}} = \frac{\partial \delta}{\partial \tilde{x}} M_G M_I^2 + \delta \frac{\partial M_G}{\partial \tilde{x}} M_I^2 + 2\delta M_G \frac{\partial M_I}{\partial \tilde{x}} M_I
\]

where

\[
\frac{\partial \delta}{\partial \tilde{x}} = -\frac{2\delta}{\alpha_G M_G - \alpha_I M_I} \left( \frac{\partial \alpha_G}{\partial \tilde{x}} M_G \frac{\partial M_I}{\partial \tilde{x}} - \frac{\partial \alpha_I}{\partial \tilde{x}} M_I \right)
\]

so that

\[
\frac{\partial e_I}{\partial \tilde{x}} = -\frac{2\delta}{\alpha_G M_G - \alpha_I M_I} \left( \frac{\partial \alpha_G}{\partial \tilde{x}} M_G \frac{\partial M_I}{\partial \tilde{x}} - \frac{\partial \alpha_I}{\partial \tilde{x}} M_I \right) M_G M_I^2 + \delta \frac{\partial M_G}{\partial \tilde{x}} M_I^2 + 2\delta M_G \frac{\partial M_I}{\partial \tilde{x}} M_I =
\]

\[
\left[ \delta M_I \left( \frac{\partial \alpha_G}{\partial \tilde{x}} M_G + \frac{\partial \alpha_I}{\partial \tilde{x}} M_I \right) \frac{\partial M_G}{\partial \tilde{x}} + M_G \left( \frac{2\alpha_G M_G}{\alpha_G M_G - \alpha_I M_I} \frac{\partial M_I}{\partial \tilde{x}} \right) \right]
\]

The analogous derivative for \( e_G \) is

\[
\frac{\partial e_G}{\partial \tilde{x}} = -\delta M_G \left[ -M_I \left( \frac{2\alpha_I M_I}{\alpha_G M_G - \alpha_I M_I} \frac{\partial M_G}{\partial \tilde{x}} \right) + M_G \left( \frac{\alpha_G M_G + \alpha_I M_I}{\alpha_G M_G - \alpha_I M_I} \frac{\partial M_I}{\partial \tilde{x}} \right) \right]
\]

\[\text{(18)}\]

\section*{A.4 Proof of Proposition 1}

The first stage expected utility of the Government in case of a second stage interior equilibrium can be written as follows:

\[
EU_G = \frac{\alpha_G^2 M_G^3}{(\alpha_G M_G - \alpha_I M_I)} + U_G \left( d^2 \left( s, g^* \right) \right)
\]

Without loss of generality let us normalize \( U_G \left( d^2 \left( s, g^* \right) \right) \) to zero.

This function always exist whenever we have an interior solution for the second stage efforts, i.e. \( M_G > 0 \) and
Given our single-policy assumption that \( \tilde{a} = a_s = 0 \), \( M_G > 0 \) if \( w_G^a \tilde{q} (2q_G^* - \tilde{q}) > 0 \) and \( M_I < 0 \) if \( w_I^a \tilde{q} (2q_I^* - \tilde{q}) < 0 \). These conditions imply that \( \tilde{q} \in (\max(0, 2q_I^*), 2q_G^*) \) or \( \tilde{q} \in (2q_G^*, \min(0, 2q_I^*)) \). Also note that by transitivity, \( |q_I^*| < |q_G^*| \) is implied.

We can rewrite eq. (11), expanding \( M_G \) and \( M_I \) using eq. (4) as follows:

\[
EU_G = \alpha_G^2 \left( U_G \left( w_G^a (-a_G^*)^2 + w_G^I (\tilde{q} - q_G^*)^2 \right) \right)^3
\]

\[
\left( \alpha_G \left( U_G \left( w_G^a (-a_G^*)^2 + w_G^I (\tilde{q} - q_G^*)^2 \right) \right) - \alpha_I \left( U_I \left( w_I^a (-a_I^*)^2 + w_I^I (\tilde{q} - q_I^*)^2 \right) - U_I \left( w_I^a (-a_I^*)^2 + w_I^I (\tilde{q} - q_I^*)^2 \right) \right) \right)
\]

and

\[
\frac{\partial EU_G}{\partial \tilde{q}} = \frac{2\alpha_G^2 M_G^2 \left( U_G \left( \alpha_G \left( w_G^a (-a_G^*)^2 + w_G^I (\tilde{q} - q_G^*)^2 \right) \right) - \alpha_I \left( U_I \left( w_I^a (-a_I^*)^2 + w_I^I (\tilde{q} - q_I^*)^2 \right) - U_I \left( w_I^a (-a_I^*)^2 + w_I^I (\tilde{q} - q_I^*)^2 \right) \right) \right)}{(\alpha_G M_G - \alpha_I M_I)^2}
\]

(21)

(a) if \( q_G^* > 0 \) and \( \tilde{q} \in (\max(0, 2q_I^*), 2q_G^*) \),

1. \( \frac{\partial EU_G}{\partial \tilde{q}} < 0 \) for every \( \tilde{q} \geq q_G^* \)

2. Since \( \frac{\partial EU_G}{\partial \tilde{q}} \) is continuous and \( \frac{\partial EU_G}{\partial \tilde{q}} \bigg|_{\tilde{q}=q_G^*} = \frac{2\alpha_G^2 M_G^2 U_G \alpha_G \left( w_G^a (-a_G^*)^2 + w_G^I (\tilde{q} - q_G^*)^2 \right)}{(\alpha_G M_G - \alpha_I M_I)^2} < 0 \), there exists \( q_0 < q_G^* \) such that \( \frac{\partial EU_G}{\partial \tilde{q}} < 0 \) for every \( \tilde{q} \geq q_0 \).

3. Since \( EU_G = 0 \) if \( \tilde{q} = 0 \), and \( EU_G > 0 \) for all \( \tilde{q} \in (\max(0, 2q_I^*), 2q_G^*) \),

   - if \( q_I^* \leq 0 \), the derivative \( \frac{\partial EU_G}{\partial \tilde{q}} \) must be positive for all \( \tilde{q} \) in a neighborhood of 0, i.e., for all \( \tilde{q} \in [0, q_1] \) with some small \( q_1 > 0 \). Hence, we know that the function must have at least one local maximum between \( q_1 \) and \( q_0 \) with one of these local maxima being the global maximum.

   - if \( q_I^* > 0 \), we know that the derivative \( \frac{\partial EU_G}{\partial \tilde{q}} > 0 \) for all \( \tilde{q} > 2q_I^* \) and \( EU_G \) is decreasing for every \( \tilde{q} \geq q_0 \). Hence, we know that the function must have at least one local maximum between \( 2q_I^* \) and \( q_0 \) with one of these local maxima being the global maximum.

4. Let’s call this global maximum \( \tilde{q}^* \), we have shown that \( q_I^* < \tilde{q}^* < q_G^* \).

(b) if \( q_G^* < 0 \), and \( \tilde{q} \in (2q_G^*, \min(0, 2q_I^*)) \), can be proved symmetrically that there exist a global maximum \( \tilde{q}^* \) such that \( q_I^* > \tilde{q}^* > q_G^* \).

A.5 Proof of Proposition 2

We can rewrite eq. (11) as follows:

\[
EU_G = \alpha_G^2 \left( U_G \left( w_G^a (\tilde{a} - a_G^*)^2 + w_G^I (\tilde{q} - q_G^*)^2 \right) \right)^3
\]

\[
\left( \alpha_G \left( U_G \left( w_G^a (\tilde{a} - a_G^*)^2 + w_G^I (\tilde{q} - q_G^*)^2 \right) \right) - \alpha_I \left( U_I \left( w_I^a (\tilde{a} - a_I^*)^2 + w_I^I (\tilde{q} - q_I^*)^2 \right) - U_I \left( w_I^a (\tilde{a} - a_I^*)^2 + w_I^I (\tilde{q} - q_I^*)^2 \right) \right) \right)
\]

From here we can analyze the relevant cases in which an opposition to the reform is present, as also depicted in case 1.2 and 3 of Figure 4.
EU\_G = 0 \text{ iff } w^0_G (\tilde{a}^2 - 2\tilde{a}a^*_G) = w^0_G (2\tilde{q}q^*_G - \tilde{q}^2), \text{ and } M_G > 0 \text{ if } w^0_G \tilde{a}^2 + w^0_G \tilde{q}^2 - 2 (w^0_G \tilde{a}a^*_G + w^0_G \tilde{q}q^*_G) < 0 \text{ while } M_I < 0 \text{ if } w^I_G \tilde{a}^2 + w^I_G \tilde{q}^2 - 2 (w^I_G \tilde{q}q^*_I + w^I_G \tilde{a}a^*_I) > 0.

Therefore to have an interior solution in the second stage contest game, the proposal must lie inside the following domain:

if \( q^*_G < q^*_I \)

\[
\tilde{q} \in \left( q^*_G - \frac{w^0_G}{w^*_G} \tilde{a}(2a^*_G - \tilde{a}), q^*_I - \frac{w^0_G}{w^*_I} \tilde{a}(2a^*_I - \tilde{a}) \right)
\]

or if \( q^*_G > q^*_I \)

\[
\tilde{q} \in \left( q^*_I + \frac{w^0_G}{w^*_I} \tilde{a}(2a^*_I - \tilde{a}), q^*_G + \frac{w^0_G}{w^*_G} \tilde{a}(2a^*_G - \tilde{a}) \right)
\]

and if \( a^*_G < a^*_I \)

\[
\tilde{a} \in \left( a^*_G - \frac{w^0_G}{w^*_G} \tilde{q}(2q^*_G - \tilde{q}), a^*_I - \frac{w^0_G}{w^*_I} \tilde{q}(2q^*_I - \tilde{q}) \right)
\]

or if \( a^*_G > a^*_I \)

\[
\tilde{a} \in \left( a^*_I + \frac{w^0_G}{w^*_I} \tilde{q}(2q^*_I - \tilde{q}), a^*_G + \frac{w^0_G}{w^*_G} \tilde{q}(2q^*_G - \tilde{q}) \right)
\]

In that domain \( EU\_G > 0 \) and

\[
\frac{\partial EU\_G}{\partial \tilde{a}} = \frac{2\alpha^2_G M^2_G (U^d_G w^0_G (\tilde{a} - a^*_G)(2\alpha_G M_G - 3\alpha_I M_I) + U^d_I \alpha_I M_G w^q_G (\tilde{a} - a^*_I))}{(\alpha_G M_G - \alpha_I M_I)^2} \tag{23}
\]

\[
\frac{\partial EU\_G}{\partial \tilde{q}} = \frac{2\alpha^2_G M^2_G (U^d_G w^q_G (\tilde{q} - q^*_G)(2\alpha_G M_G - 3\alpha_I M_I) + U^d_I \alpha_I M_G w^q_G (\tilde{q} - q^*_I))}{(\alpha_G M_G - \alpha_I M_I)^2} \tag{24}
\]

We can show that:

(1) If \( a^*_G < a^*_I \) and \( q^*_G < q^*_I \):

\[
\cdot \frac{\partial EU\_G}{\partial \tilde{a}} > 0 \text{ for every } \tilde{a} \leq a^*_G \text{ and } \frac{\partial EU\_G}{\partial \tilde{a}} < 0 \text{ for every } \tilde{a} \geq a^*_I
\]

\[
\cdot \text{Since } \frac{\partial EU\_G}{\partial \tilde{a}} \text{ is continuous and } \frac{\partial EU\_G}{\partial \tilde{a}} \bigg|_{\tilde{a}=a^*_G} > 0, \text{ there exists } a_0 > a^*_G \text{ such that } \frac{\partial EU\_G}{\partial \tilde{a}} > 0 \text{ for every } \tilde{a} \leq a_0.
\]

\[
\cdot \text{Let’s call } a_1 \text{ and } a_2 \text{ the } \tilde{a} \text{ for which } EU\_G = 0. \text{ Where, } a_1 = a^*_G - \sqrt{a^2_G - \frac{w^q_G}{w^*_G} \tilde{q} \tilde{q} - 2q^*_G} \text{ and } a_2 = a^*_G + \sqrt{a^2_G - \frac{w^q_G}{w^*_G} \tilde{q} \tilde{q} - 2q^*_G}
\]

\[
\cdot a_1 = \overline{a} \text{ and we know that } EU\_G \text{ is increasing in the right neighborhood of } a_1, \text{ being } a_1 < a^*_G.
\]

\[
\cdot a_2 \text{ is outside the relevant domain, i.e } a_2 > \overline{a} = a^*_I - \sqrt{a^2_I + \frac{w^q_G}{w^*_I} \tilde{q} \tilde{q} - 2q^*_I}, \text{ therefore the maximum must be lower than } \overline{a}.
\]

\[
\cdot \text{ Hence, we know that the function must have at least one local maximum between } a_0 \text{ and } \overline{a}, \text{ where } \frac{\partial EU\_G}{\partial \tilde{a}} = 0, \text{ with one of these local maxima being the global maximum.}
\]

\[
\cdot \text{ Let’s call this global maximum } \tilde{a}^*, \text{ we have shown that } a^*_G > \tilde{a}^* > a^*_I.
\]
• \( \frac{\partial EU_G}{\partial q} < 0 \) for every \( \tilde{q} \geq q_G^* \) and \( \frac{\partial EU_G}{\partial q} > 0 \) for every \( \tilde{q} \leq q_I^* \), for \( q_G^* > q_I^* \).

• Since \( \frac{\partial EU_G}{\partial q} \) is continuous and \( \frac{\partial EU_G}{\partial \tilde{q}} \big|_{\tilde{q}=q_G^*} < 0 \), there exists \( q_1 \) in the neighborhood of \( q_G^* \) such that \( \frac{\partial EU_G}{\partial \tilde{q}} < 0 \) for every \( \tilde{q} \geq q_{\tilde{q}} = q_G^* \).

• Let’s call \( q_1 \) and \( q_2 \) the \( \tilde{q} \) for which \( EU_G = 0 \). Where, \( q_1 = q_G^* - \sqrt{q_G^* - \frac{w_G}{\alpha_G} \tilde{a} - 2a_G^*} \) and \( q_2 = q_G^* + \sqrt{q_G^* - \frac{w_G}{\alpha_G} \tilde{a} - 2a_G^*} \)

  - \( q_2 = \tilde{q} \) and therefore is outside our domain.

  - \( q_1 \) is also outside the relevant domain, i.e. \( q_1 < q = q_I^* + \sqrt{q_I^* + \frac{w_I}{\alpha_I} \tilde{a} (2a_I^* - \tilde{a})} \).

• Hence, we know that the function must have at least one local maximum \( \in (q, q_G^*) \) with one of these local maxima being the global maximum.

• Let’s call this global maximum \( \tilde{q}^* \), we have shown that \( q_I^* < \tilde{q}^* < q_G^* \).

(2) If \( a_G^* > a_I^* \) and \( q_I^* > q_G^* \): can be proved symmetrically that there exist a maximum in \( (\tilde{a}^*, \tilde{q}^*) \) such that \( a_I^* < \tilde{a}^* < a_G^* \) and \( q_I^* > \tilde{q}^* > q_G^* \).

(3) If \( |a_I^*| > |a_G^*| \) and \( |q_I^*| > |q_G^*| \): following the same rational behind point (1) and (2) it can be proved that \( |a_I^*| > |\tilde{a}^*| > |a_G^*| \) and \( |q_I^*| > |\tilde{q}^*| > |q_G^*| \).

A.5.1 Proof of Corollary 2

It follows directly from the proof of Proposition 2 that, once the Government is allowed to propose policies in more dimensions, it will propose \( \tilde{a} = a_s = 0 \) only if \( \frac{w_I}{w_G} = \phi \) with \( \phi = -\frac{\alpha_s M_G}{\alpha_G (2\alpha_G M_G - 3\alpha_I M_I)} \).