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ABSTRACT

Evaluating Alternative Representations of the Choice Sets in Models of Labour Supply

During the last two decades, the discrete-choice modelling of labour supply decisions has become increasingly popular, starting with Aaberge et al. (1995) and van Soest (1995). Within the literature adopting this approach there are however two potentially important issues that are worthwhile analyzing in their implications and that so far have not been given the attention they might deserve. A first issue concerns the procedure by which the discrete alternatives are selected to enter the choice set. For example van Soest (1995) chooses (non-probabilistically) a set of fixed points identical for every individual. This is by far the most widely adopted method. By contrast, Aaberge et al. (1995) adopt a sampling procedure suggested by McFadden (1978) and also assume that the choice set may differ across the households. A second issue concerns the availability of the alternatives. Most authors assume all the values of hours-of-work within some range $[0, H]$ are equally available. At the other extreme, some authors assume only two or three alternatives (e.g. non-participation, part-time and full-time) are available for everyone. Aaberge et al. (1995) assume instead that not all the hour opportunities are equally available to everyone; they specify a probability density function of opportunities for each individual and the discrete choice set used in the estimation is built by sampling from that individual-specific density function. In this paper we explore by simulation the implications of the procedure used to build the choice set (fixed alternatives vs. sampled alternatives) and of accounting or not accounting for a different availability of alternatives. The way the choice set is represented seems to have little impact on the fitting of observed values, but a more significant and important impact on the out-of-sample prediction performance.

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1. Introduction

The idea of modeling labour supply decisions as discrete choices has become more and more popular during the last two decades. In this paper we examine through a simulation exercise an issue that has received much less attention than it might deserve: the implications of alternative methods of representing the choice set within the discrete choice approach.

The discrete choice approach has gained a prominent position as an outcome of the process aimed at solving or circumventing some theoretical and computational problems to be faced in microeconomic research when analyzing choices subject to complicated opportunity constraints. Let us consider the standard labour supply framework:

$$(1.1) \quad \begin{aligned} & \max U(h, x) \\ & s.t. \\ & x \leq wh + I \text{ and } h \geq 0 \end{aligned}$$

where U is a deterministic utility function, h represents hours of work, w is the (constant) hourly wage rate and I is the exogenous income. Using the Kuhn-Tucker conditions associated to (1.1) – and assuming for simplicity an interior solution – under appropriate conditions one can obtain the optimal labour supply h^* as a function of w and I :

$$(1.2) \quad h^* = h(w, I)$$

Then some empirical specification of $h(w, I)$ can be estimated and used for example to simulate the effects of policies implying changes in w and/or in I .

The linear budget constraint in problem (1.1), however, very rarely corresponds to reality. Considering a well-known example, taxes and transfers on income in general imply a non-linear constraint. The budget constraint would then be:

$$(1.4) \quad x \leq wh + I - \tau(wh, I)$$

where τ represents the tax-benefit rule that computes the taxes to be paid and the transfers to be received given gross incomes (wh, I) . Taking (1.4) into account, we might still be able to characterize the optimal solution as a function of w and I ,

$$(1.5) \quad h^* = h^r(w, I)$$

and estimate $h^\tau(w, I)$. However, $h^\tau(w, I)$ depends on the current tax-benefit rule τ and therefore it cannot be used to simulate policies that introduce a different tax rule, say τ' . The problem is that the behavioural function h^τ in general mixes up preferences and constraints¹. More generally, the opportunity set might be defined by complicated budget and quantity constraints that do not even allow recovering a closed form solution for h^* . What we really need is an estimate of the utility function $U(h, x)$ itself. Once preferences are estimated, in principle we are able to simulate the effect of any policy by solving $\max U(h, x)$ subject to the appropriate constraints.

A paper by J. Heckman (1974) probably for the first time took full account of the non-linearity of the budget constraint in the estimation and simulation of microeconomic models. The problem addressed is the evaluation of a child related welfare policy that introduces significant complications in the budget set. Heckman proposed a particular method of recovering preferences by using the conditions to be fulfilled by the marginal rate of substitution for h^* to be located on a particular point of the budget set. Shortly after, a series of papers by J. Hausman and various co-authors proposed a method specifically addressed to piece-wise linear budget constraints (e.g. Hausman 1979). Both Heckman (1974) and Hausman (1979) work through the implications of the Kuhn-Tucker conditions. The solution can be located in different ranges of values along the budget constraint. Corresponding to each possible range of values there is a condition involving the preference parameters. Choosing a convenient stochastic specification, we can express the probability that those various conditions alternatively hold, write down the sample likelihood and estimate the preference parameters. Useful presentations of this class of methods are provided by Moffit (1986) and Blundell and MaCurdy (2000).

Soon it emerged that the approach described above presents three main problems. First, it works well with convex budget sets (e.g. those generated by progressive taxation) and a two-good application (e.g. h and x in the individual labour supply application) but it tends to become computationally cumbersome when the agents face non-convex budget sets and when more than two goods are object to choice (e.g. when the agent is a many-person household). Second, in view of the computational problems, the above approach essentially forces the researcher to choose relatively simple specifications for the utility function or the labour supply functions. Third – in the approach proposed by Hausman and associates – computational and statistical consistency of ML estimation of the model requires imposing a priori quasi-convexity of preferences (e.g. see MaCurdy, Green and Paarsch, 1990)..

Due to these emerging problems, applied researchers have started to make use of another innovative research effort also matured in the first half of the 70's, i.e. the discrete choice modelling approach developed by D. McFadden (1974). As far as the labour supply application is concerned, the

¹ $h^\tau(w, I)$ is called “mongrel” labour supply function by Blomquist (1988).

approach essentially consists in representing the budget set with a set of discrete 'points'. Let $[0, H]$ be the (continuous) range of possible values for hour of work h is. Let us pick K points h_1, h_2, \dots, h_s to "represent" $[0, H]$. The utility level attained at point k is $U(x_k, h_k)$, where x_k is obtained through some budget rule such as the (1.4). Now let us assume that $U(x_k, h_k)$ is a random variable that can be decomposed additively into a systematic part containing the observable $v(x_k, h_k)$ and a random component ε_k that accounts for the effect of unobservables: $U(x_k, h_k) = v(x_k, h_k) + \varepsilon_k$. The assumption that the random term ε_k is Type I Extreme Value i.i.d. leads to the well known multinomial logit expression for the probability that point j (i.e. the job with hours h_j) is chosen²:

$$(1.1) \quad P(j) = \Pr\left(U(x_j, h_j) = \max(U(x_1, h_1), \dots, U(x_s, h_s))\right) = \frac{\exp(v(x_j, h_j))}{\sum_i \exp(v(x_i, h_i))}$$

The corresponding likelihood function can then easily be computed and maximized in order to estimate the parameters of the utility function. This approach is computationally very convenient when compared to the previous one, since it does not require going through complicated Kuhn-Tucker conditions involving derivatives of the utility function and of the budget constraints. As a consequence it is not affected by how complex it is the rule that defines the budget set or by how many goods are contained in the utility function. Equally important, the deterministic part of the utility function can be specified as very flexible without worrying for the computational problems.

During the last two decades, this approach has become increasingly popular, starting with Aaberge, Dagsvik and Strøm (1995) and van Soest (1995). Within the literature adopting this approach there are however two potentially important issues that are worthwhile analyzing in their implications and that so far have not been given the attention they might deserve.

A first issue concerns the procedure by which the discrete alternatives are included in the choice set. Most authors (e.g., among others, van Soest (1995), Duncan and Weeks (1997), Blundell, Duncan, McCrae and Meghir (2000)), Kornstad and Thoresen (2004) choose (not probabilistically) a set of fixed points identical for every individual. By contrast, Aaberge, Dagsvik and Strøm (1995) and Aaberge, Colombino and Strøm (1999) adopt a sampling procedure originally proposed by McFadden (1978) and also assume that the choice set may differ across the households.

A second issue concerns the availability of the alternatives. Most authors assume all the values in $[0, H]$ - or in some discrete subset - are equally available. At the other extreme, some authors (e.g. Zabalza et al. 1980) assume only two or three alternatives (e.g. non-participation, part-time and full-time) are available for everyone. More generally, Aaberge et al. (1995, 1999, 2000, 2004) assume that the hour opportunities in $[0, H]$ are not equally available to everyone. More specifically, they assume

² A random variable ε has a (standard) Type I extreme value distribution if $\text{Prob}(\varepsilon \leq k) = \exp(-\exp(-k))$.

that there is a probability density function of opportunities for each individual. The discrete choice set used in the estimation (and subsequently in the simulations) is built by sampling from that individual-specific density function.

In what follows we explore by simulation the implications of

- the procedure used to build the choice set (fixed alternatives vs sampled alternatives)
- accounting vs not accounting for a different availability of alternatives

upon the precision of the estimates and of policy simulation results (uniform availability vs heterogeneous availability).

As to the last issue, uniform availability (as for example in van Soest (1995) and Duncan and Weeks (1997) can be interpreted as a special case of heterogeneous availability (as in Aaberge et al. (1995, 1999) where the probability density functions of opportunities are assumed to be uniform and equal for everyone. Since the approach taken by Aaberge et al. (1995, 1999) is more general, we will use their model as the “true” one in order to generate a sample, which will then be used in the simulation experiments.

2. The “true” model

The "true" model is defined along the lines adopted in Aaberge et al. (1995) as well as in several successive papers³. The individuals maximise their utility by choosing among opportunities defined by hours of work, hourly wage and non-pecuniary attributes of the job. The utility is assumed to be of the following form

$$(2.1) \quad U(f(wh, I), h, j) = v(f(wh, I), h) + \varepsilon(w, h, j)$$

where w is the wage rate, h is hours of work, I is exogenous income including the husband's labour income, f is a function that transforms gross income into income after tax, i.e. $f(wh, I)$ is disposable income (income after tax), j is a variable that capture other job characteristics and ε is a random variable that is supposed to account for unobservables affecting tastes for a given job across individuals as well as across job opportunities for a given individual⁴. Commuting time or required skill are type of characteristics captured by j . The individual is supposed to choose a "job" from a choice set B that may differ across individuals. Each job alternative in B contains a wage rate w , hours of work h and unobserved (to the analyst) job characteristics such as environmental characteristics and skill content of the job. Moreover, B contains also non-market activities, i.e. jobs with $w=0$ and $h=0$.

³ e.g. Aaberge, Colombino and Strøm, (1999, 2000, 2004) and Aaberge, Colombino, Strøm and Wennemo (2000).

⁴ In most of the papers where the model is presented, the multiplicative specification is chosen, i.e. $U = v \varepsilon$. We formulate here the model in the additive form in order to make it more easily comparable to similar models that appear in the literature.

By assuming that ε is type I extreme value distributed and that the specification (2.1) is valid, it turns out that the probability density that opportunities with hours h and wage rate w are chosen has the following expression⁵

$$(2.2) \quad \varphi(h, w) \equiv \Pr \left[U(f(wh, I), h) = \max_{(x, y) \in B} U(f(xy, I), y) \right] = \frac{\exp(v(f(wh, I), h)) p(h, w)}{\iint \exp(v(f(xy, I), y)) p(x, y) dx dy}$$

where $p(h, w, s)$ is the density of choice opportunities which can be interpreted as the relative frequency (in the choice set) of opportunities with hours h and wage rate w . Opportunities with $h = 0$ (and $w = 0$) are non-market opportunities (i.e. alternative allocations of "leisure"). Thus, the density (2.2) will form the basis of estimating the parameters of the utility function and the choice sets. In practice, the estimation adopts a discretized version of (2.2). Let $q(h, w)$ be some known joint density function (e.g. empirically fitted to the observations on h and w). Let us represent the latent choice set B with a sample S containing M points, where one is the chosen (observed) point and the other $M-1$ are sampled from $q(h, w)$. It can be shown (McFadden 1978; Ben Akiva and Lerman 1985) that consistent estimates of $v(f(wh, I), h)$ and $p(h, w)$ can still be obtained when (2.2) is replaced by

$$(2.3) \quad \Pr \left[U(f(wh, I), h) = \max_{(x, y) \in S} U(f(xy, I), y) \right] = \frac{\exp(v(f(wh, I), h)) p(h, w) / q(h, w)}{\sum_{(x, y) \in S} \exp(v(f(xy, I), y)) p(x, y) / q(x, y) dx dy}$$

We select a sample of married/cohabitating females. The systematic part of their utility function (2.1) is specified as follows

$$(2.4) \quad v(h, w) = \alpha_2 \left(\frac{f(wh, I)^{\alpha_1} - 1}{\alpha_1} \right) + (\alpha_4 + \alpha_5 \log A + \alpha_6 (\log A)^2 + \alpha_7 Ch_1 + \alpha_8 Ch_8 + \alpha_9 Ch_9) \left(\frac{L^{\alpha_3} - 1}{\alpha_3} \right)$$

where L is leisure, defined as $L = 1 - (h/8736)$, A is age and Ch_1 , Ch_2 and Ch_3 are number of children below 3, between 3 and 6 and between 7 and 14 years old. In the specification of the probability density of opportunities $p(h, w)$ we will assume that offered hours and offered wages are independently distributed. The justification for this is that offered hours, in particular normal working hours, are typically set in rather infrequent negotiations between employers and employees associations, while wage negotiations are far more frequent in which the hourly wage tend to be set

⁵ For the derivation of the choice density (2.2) see Aaberge, Colombino and Strøm (1999). Note that (2.2) can be considered as a special case of the more general multinomial type of framework introduced by Ben-Akiva and Watanatada (1981) and Dagsvik (1994).

independent of working hours. Thus, we specify the density of opportunities requiring h hours of work and paying hourly wage w as follows

$$(2.5) \quad p(h, w) = \begin{cases} p_0 g_1(h) g_2(w) & \text{if } h > 0 \\ 1 - p_0 & \text{if } h = 0 \end{cases}$$

where p_0 is the proportion of market opportunities in the opportunity set, and g_1 and g_2 are respectively the densities of hours and wages, conditional upon the opportunity being a market job.

In view of the empirical specification it is convenient to divide both numerator and denominator by $1 - p_0$ and define $g_0 = \frac{p_0}{1 - p_0}$. We can then rewrite the choice density (2.2) as follows:

$$(2.6) \quad \varphi(h, w) = \frac{\exp(v(h, w)) g_0 g_1(h) g_2(w)}{\exp(v(0, 0)) + \int_{x>0} \int_{y>0} \exp(v(x, y)) g_0 g_1(h) g_2(w) dx dy}$$

for $\{h, w\} > 0$, and

$$(2.7) \quad \varphi(0, 0) = \frac{\exp(v(0, 0))}{\exp(v(0, 0)) + \int_{x>0} \int_{y>0} \exp(v(x, y)) g_0 g_1(h) g_2(w) dx dy}$$

for $\{h, w\} = 0$

Offered hours are assumed to be uniformly distributed except for possible peaks corresponding to part time (pt), 18-20 weekly hours, and to full time (ft), 37-40 weekly hours. Thus, g_1 is given by

$$(2.8) \quad g_1(h) = \begin{cases} \gamma & \text{if } h \in [1, 17] \\ \gamma \exp(\pi_1) & \text{if } h \in [18, 20] \\ \gamma & \text{if } h \in [21, 36] \\ \gamma \exp(\pi_2) & \text{if } h \in [37, 40] \\ \gamma & \text{if } h \in [41, H] \end{cases}$$

where H is the maximum observed value of h . Thus, this opportunity density for offered hours

implies that it is far more likely to find jobs with hours that accord with full-time and standard part time positions than jobs with other working loads.

Since the density values must add up to 1, we can also compute γ according to:

$$(2.9) \quad \gamma = \left((17-1) + (20-18)\exp(\pi_1) + (36-21) + (40-37)\exp(\pi_2) + (H-41) \right)^{-1}$$

Moreover we write

$$(2.10) \quad g_0 = \exp(\theta_0)$$

In Table 2.1 we refer to π and θ as the parameters of the *job opportunity density*. The density of offered wages is assumed to be lognormal with mean that depends on length of schooling (Ed) and on past potential working experience (Exp), where experience is defined to be equal to age minus length of schooling minus five, i.e.

$$(2.11) \quad \log w = \beta_0 + \beta_1 Exp + \beta_2 Exp^2 + \beta_3 Ed + \sigma \eta,$$

where η is standard normally distributed.

Using (2.8) and (2.10) we can write the choice density as follows:

$$(2.12) \quad \varphi(w, h) = \frac{\exp(v(f(wh, I), h) + (\theta_0 + \ln g_2(w))d_0(h) + \pi_1 d_1(h) + \pi_2 d_2(h))}{\int \int_{x y} \exp(v(f(xy, I), y) + (\theta_0 + \ln g_2(x))d_0(h) + \pi_1 d_1(y) + \pi_2 d_2(y)) dx dy}$$

where

$$(2.13) \quad \begin{aligned} d_0(h) &= 1 \text{ if } h > 0; 0 \text{ otherwise} \\ d_1(h) &= 1 \text{ if } h \in [18, 20]; 0 \text{ otherwise} \\ d_2(h) &= 1 \text{ if } h \in [37, 40]; 0 \text{ otherwise} \end{aligned}$$

In what follows we will refer to $d_0(h)$ as the "job" dummy, since it captures the relative frequency of market opportunities to non-market opportunities; we will refer to $d_1(h)$ and $d_2(h)$ as the "peaks" dummies, since they are meant to capture the "peaks" in the density of hours corresponding to part-time and full-time jobs.

The estimation of the model is based on data for 1842 married/cohabitating females from the 1995 Norwegian Survey of Level of Living. We have restricted the ages of the females to be between

20 and 62 years in order to minimize the inclusion in the sample of individuals who in principle are eligible for retirement, since analysis of retirement decisions is beyond the scope of this study. Although the model adopted was originally developed for analysing simultaneous household partners' behaviour, we focus here on women's behaviour in order to simplify the execution and the interpretation of the simulation exercise. Husband's income as well as the couple's non-labour income are treated as exogenous and included in disposable income $f(wh, I)$.

The parameters appearing in expressions (2.3) – (2.10) are estimated by the method of maximum likelihood using the sampling procedure illustrated in Section 2.3. Each of the choice sets are represented by a set S that includes the observed choice plus 999 independent draws (h, w) from densities $q(w, h)$ previously fitted to the observed values of w and h . If (w_s, h_s) are the observed values for a particular individual, the corresponding contribution to the likelihood function is:

$$(2.14) P(w_s, h_s | S) = \frac{v(f(w_s h_s, I), h_s) \exp((\theta_0 + \ln g_2(w_s))d_0(h_s) + \pi_1 d_1(h_s) + \pi_2 d_2(h_s) - \ln q(w_s, h_s))}{\sum_{i \in S} v(f(w_i h_i, I), h_i) \exp((\theta_0 + \ln g_2(w_i))d_0(h_i) + \pi_1 d_1(h_i) + \pi_2 d_2(h_i) - \ln q(w_i, h_i))}$$

The estimates of the parameters of the opportunity density parameters and the parameters of the utility function are reported in Tables 2.1 and 2.2.

Based on the empirical distribution of the exogenous variable and on the estimates of Tables 2.1 and 2.2 (to simulate the endogenous variables and choices) we generate a sample of $6 \times 1842 = 11052$ observations, which is then used in the simulation exercise described in what follows. This mean that we simulate six independent optimums point for each of the females in the original data set.

Table 2.1. Hours and Wage densities, Norway 1994

	Parameter	Estimate	Std. Dev.
<i>Job opportunity</i>			
	θ_0	-0.60	(0.10)
<i>Hours</i>			
<i>Part-time</i>	π_1	0.46	(0.10)
<i>Full-time</i>	π_2	1.57	(0.07)
<i>Wage</i>			
	β_0	0.24	(0.01)
	β_1	3.62	(0.05)
	β_2	2.41	(0.26)
	β_3	-3.67	(0.58)
	σ	4.10	(0.35)

Table 2.2. Estimates of the parameters of the utility functions for married/cohabitating females. Norway 1994

Variable	Parameter	Estimate	Std. Dev.
<i>Consumption</i>			
	α_1	0.39	(0.11)
	α_2	4.42	(0.44)
<i>Leisure</i>			
	α_3	-4.57	(0.53)
	α_4	168.88	(27.47)
Log age	α_5	-94.29	(15.32)
Log age squared	α_6	13.35	(2.16)
Number of children below 3 years old	α_7	0.44	(0.23)
Number of children 3-6 years old	α_8	1.23	(0.24)
Number of children 7-14 years old	α_9	1.05	(0.19)

3. Alternative representations of the choice sets

3.1 Selection of alternatives

As we have already mentioned in the Introduction, the first issue in choice set representation concerns the procedure used to select the alternatives. In many applications, including labour supply modelling, the choice set contains a very large (or even infinite) number of alternatives. For instance, if we model couples labour supply and the decision period is the year, considering 1 hour intervals and 16 hours available during the day, there are $(16 \times 365)^2 = 34,105,600$ alternatives. This would imply a very heavy computational burden, since for each alternative we must compute the couple's budget by applying a possibly complicated tax rule. Thus it is convenient to work with a smaller choice set somehow representative of the true one. Ben-Akiva and Lerman (1985) present a detailed treatment of the procedures that might be used when the number of alternatives contained in the choice set is very large (or even infinite) so that a complete enumeration is computationally too costly:

- Aggregation of alternatives
- Sampling of alternatives

The procedure consisting in selecting a fixed number of hours' values can be interpreted as an aggregation procedure. Instead of using all the possible values between 0 and T, the (0,T) range is divided into sub-intervals and then the mid (or maybe the average) value of h in each interval is chosen to 'represent' all the values of that interval. The authors adopting this procedure realize that it introduces measurement errors, but tend to assume they are of minor importance. For example van Soest (1995) reports that some experiments with a different number of points did not show significant differences in parameter estimates, however a systematic investigation of the implication of that procedure has never been done either theoretically or empirically. However, if one interprets the approximation as an aggregation procedure, the analysis provided by Ben-Akiva and Lerman (1985) can be applied to clarify the issue.

We will assume the average of h in each sub-interval is chosen as representative (instead of the more common procedure of choosing the mid point: of course the two are very close and in fact coincide if the values of h are continuous or if each interval contains a an uneven number of values). Let us define (we drop the subscript of the household to simplify the notation):

$$(3.1) \quad v_j \equiv v(f(wh_j, I), h_j).$$

Furthermore, let $\bar{v}^L \equiv \frac{1}{N^L} \sum_{j \in L} v_j$ = average systematic utility in sub-interval L, where N^L = number of elements in L and \bar{h}^L = average value of h in sub-interval L.

Ben-Akiva and Lerman show that the expected maximum utility attained on subinterval ℓ is

$$(3.2) \quad \hat{v}^\ell = \bar{v}^\ell + \ln(N^\ell) + \ln(D^\ell)$$

where $D^\ell \equiv \sum_j \exp(v_j^\ell - \bar{v}^\ell) \frac{1}{N^\ell}$.

This last term is a measure of dispersion of v in sub-interval ℓ .

Accordingly, the probability that a value of h belonging to sub-interval L is chosen is

$$(3.3) \quad P(\ell) = \frac{\exp(\bar{v}^L + \ln(N^L) + \ln(D^L))}{\sum_\ell \exp(\bar{v}^\ell + \ln(N^\ell) + \ln(D^\ell))}$$

To compare this with the expressions used in the fixed-alternatives approach it is useful to Taylor-expand v_j up to 2-order terms to get

$$(3.4) \quad P(L) \cong \frac{\exp(v(f(w\bar{h}^L, I), \bar{h}^L) + 0.5\sigma_{hh}^L v_{hh}^L + \ln(N^L) + \ln(D^L))}{\sum_\ell \exp(v(f(w\bar{h}^\ell, I), \bar{h}^\ell) + 0.5\sigma_{hh}^\ell v_{hh}^\ell + \ln(N^\ell) + \ln(D^\ell))}$$

where σ_{hh}^ℓ is the variance of the values of h in sub-interval ℓ and v_{hh}^L is the second (total) derivative of $v(f(w\bar{h}^L, I), \bar{h}^L)$ with respect to \bar{h}^L .

It would be pointless to use (3.4) for estimation since it require the very same computations that one wishes to avoid by aggregating alternatives. However (3.4) is useful in order to understand the type and the extent of the errors we incur by using various approximations. The expression typically used in the literature is:

$$(3.5) \quad \hat{P}(L) = \frac{\exp(v(f(w\bar{h}^L, I), \bar{h}^L))}{\sum_\ell \exp(v(f(w\bar{h}^\ell, I), \bar{h}^\ell))}$$

Clearly, in expression (3.5) all the terms $0.5\sigma_{hh}^\ell v_{hh}^\ell + \ln(N^\ell) + \ln(D^\ell)$ are dropped. If these terms were equal across all the sub-intervals they would cancel out from (3.4) and (3.5) would be exact. In general however they will not be equal, and dropping them will lead to biased estimates. Nonetheless there are

ways by which we could improve upon (3.5) when adopting aggregation as an approximation strategy, which however have never been considered in the literature on labour supply modelling:

- The dimension of N^ℓ of the sub-intervals - when not equal for all of them - is typically known and can be explicitly accounted for;
- σ_{hh}^ℓ can also be computed;
- Depending on the functional form used for the utility function, the term v_{hh}^ℓ might be explicitly evaluated and accounted for;
- The terms $\ln(D^\ell)$ in general will vary both across sub-intervals and across individuals; however we might capture at least some of their effect by introducing a set of dummies (as many as the number of sub-intervals - 1).

Summing up, the aggregation of alternatives implies biased estimates. The bias could be moderated by using various possible corrections suggested by expression (3.4) itself. Up to now, however, it must be said that the literature on labour supply has treated this issue in a rather superficial way (when compared, for instance to the literature on transportation or location choices).

Sampling of alternatives, on the other hand, offers the possibility of working with a relatively small choice set and at the same time preserving the consistency of the estimates. The basic results were established by McFadden (1978). Ben-Akiva and Lerman (1985) also provide a very useful and more practically oriented survey, together with some additional theoretical results.

Let us represent the true choice set B with a sample S containing M points, where one is the chosen (observed) point and the other M-1 are sampled from. Let q_i be the probability of sampling point h_i . It can be shown (McFadden 1978; Ben Akiva and Lerman 1985) that consistent estimates of $v(f(wh, I), h)$ and $p(h, w)$ can still be obtained when the true choice set B is replaced by S and the probability of observing choice j is evaluated as follows:

$$(3.6) \quad P(j|S) = \frac{\exp(v(f(wh_j, I), h_j) - \ln(q_j))}{\sum_{h_i \in S} \exp(v(f(wh_i, I), h_i) - \ln(q_i))}$$

If a simple random sampling is adopted, all the q 's are equal and cancel out. Typically more sophisticated sampling procedures are used since they are expected to be more efficient. For instance, a common procedure consists in using as sampling probabilities the observed relative frequencies of choice possibly differentiated according to personal characteristics of the decision units. Besides Ben-Akiva and Lerman (1985), also Train et al. (1987) present a very detailed application of this procedure.

3.2 Availability of alternatives

A second and possibly even more substantial issue is whether or not account is taken of the different availability of job-types on the market. Some authors have made the extreme choice of assuming the choice set contains only two or three alternatives (e.g. non-participation, part-time and full-time). More common, however, is the approach of choosing a few equally spaced points in the interval $[0,H]$, without taking into account the possibility that some type of opportunities maybe more easily available than others. Other authors (Aaberge et al. 1995, 1999, 2000, 2004) do account for this possibility as well as for the relative density of jobs as a function of personal characteristics (see section 2). In practice, their specification boils down to “augmenting” the term v with a set of appropriately defined dummy variables. Also van Soest (1995) introduces similar dummies, although he gives them a different interpretation in terms of utility costs or *premia* attached to some range of hour values⁶.

3.3 The simulation exercise

In what follows we use the sample generated according to the true model to estimate various versions of models generated according to the various possible representation of the choice set as discussed above. In these models the wage rate is kept fixed for each individual, i.e. it does not vary across alternatives as it happens in the “true” model; moreover it is simultaneously estimated as in (2.12), instead we use a wage equation to predict the wage for non-participants⁷. This simplification is introduced in order to make the simulation results more easily interpretable.

The more general versions of the models are

$$(3.7) \quad P(w_s, h_s | S) = \frac{\exp(v(f(w_s, h_s, I), h_s) + \theta_0 d_0(h_s) + \pi_1 d_1(h_s) + \pi_2 d_2(h_s) - \ln q(w_s, h_s))}{\sum_{i \in S} \exp(v(f(w_i, h_i, I), h_i) + \theta_0 d_0(h_i) + \pi_1 d_1(h_i) + \pi_2 d_2(h_i) - \ln q(w_i, h_i))}$$

when sampled alternatives are used, and

$$(3.8) \quad P(w_s, h_s | R) = \frac{\exp(v(f(w_s, h_s, I), h_s) + \theta_0 d_0(h_s) + \pi_1 d_1(h_s) + \pi_2 d_2(h_s))}{\sum_{i \in R} \exp(v(f(w_i, h_i, I), h_i) + \theta_0 d_0(h_i) + \pi_1 d_1(h_i) + \pi_2 d_2(h_i))}$$

⁶ There is still another approach, the so called Dogit model (Gaudry and Dagenais, 1979), to represent a non-uniform availability of alternatives. It is a generalization of the logit model, where the decision-maker may – with a given probability – be “captive” to one of the alternatives or otherwise choose freely from the whole choice set. The Dogit model has been recently used by Harris and Duncan (2002) in a labour supply application. We do not consider the Dogit model in the simulation exercise presented here.

⁷ The estimates of the wage equation are available upon request from the authors.

when fixed alternatives are used. The dummies $d_0(h_i)$ and $(d_1(h_i), d_2(h_i))$ are defined as in (2.13).

Dropping the job dummy $d_0(h_i)$ and/or the peak dummies $(d_1(h_i), d_2(h_i))$ generates a more restrictive version of the model. The choice sets S and R contain alternatively 6 or 24 points. Altogether with have 16 models resulting from the combinations of the following possibilities:

1. *alternative generation*: fixed or sampled
2. *number of alternatives*: 6 or 24
3. *job dummy*: included or dropped
4. *peaks dummies*: included or dropped

In the following sections they are named as in Table 3.1. The parameter estimates of the 16 models are reported in the Appendix.

Table 3.1 Types of models

Model	Fixed alternatives				Sampled alternatives			
	Model Ia	Model Ib	Model Ic	Model Id	Model IIa	Model IIb	Model IIc	Model IId
<i>Job dummy</i>	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Yes</i>
<i>Peaks dummies</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Yes</i>
<i>Number of alternatives</i>	<i>6</i>	<i>6</i>	<i>6</i>	<i>6</i>	<i>24</i>	<i>24</i>	<i>24</i>	<i>24</i>

The parameter estimates of the 16 models are reported in Tables A.1 and A.2 of the Appendix.

4. Evaluation of the different modelling approaches

In order to evaluate the impact of alternative representations of the choice set on the performance of the models we proceed in the following way. First, for each of the 16 models we predict participation rates, hours of work and disposable income. The predictions are obtained individual by individual, evaluating the utility function – including the stochastic component drawn from the Type I extreme value distribution – at each alternative and identifying the selected alternative as the one the highest utility level. The individual predictions are then aggregated into the 10 means of the 10 income deciles. Next, we introduce the following summary measure of prediction performance z_i for model i ,

$$(4.1) \quad z_k = \sqrt{\sum_{j=1}^{10} \left(\frac{(\tilde{y}_{kj} - y_j)}{y_j} \right)^2}, \quad k=1, 2, \dots, 16,$$

where y_j and \tilde{y}_{kj} denote the outcomes in decile j of the true model and alternative model k , respectively. The outcomes are alternatively defined to be the job participation rate, hours of work and disposable income after tax.

Next, we carry out a regression analysis where z is treated as a response variable and the following variables are treated as co-variables,

$x_1 = 1$ if the choice alternatives are sampled (= 0 if the choice alternatives are fixed),

$x_2 = 1$ if the number of choice alternatives is equal to 24 (= 0 if the number of alternatives is equal to 6),

$x_3 = 1$ when it is accounted for job entry (= 0 when it is *not* accounted for job entry),

$x_4 = 1$ when it is accounted for part-time and full-time peaks (= 0 when it is *not* accounted for part-time and full-time peaks).

The following regression equation forms the basis of the evaluation of alternative modelling approaches,

$$(4.2) \quad z = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_{34} (x_3 * x_4)$$

where the coefficients a will measure the relevance of the different ways of specifying the choice set. Since the most important application of labour supply models is the evaluation of tax and welfare policy reforms, we focus on the prediction performance under alternative tax regime. Namely, the steps above are repeated twice:

- Prediction of the outcomes under the current tax regime
- Prediction of the outcomes after the introduction of a Flat Tax (keeping total tax revenue constant).

4.1. Outcomes under the current tax regime

Tables 4.1 – 4.3 illustrate the results of the exercise under the current tax regime. Tables 4.1 and 4.2 refer to the eight models with fixed alternatives. In order to simplify the illustration we limit ourselves to the models without job and peaks dummies and to the models with both types of dummies. For each of the models and each of the 10 income deciles, we report the predictions of participation rates and hours of work in Tab. 4.1 and of after tax disposable income in Tables 4.2. We do not report here the analogous results from the models with sampled alternatives, since they are very close to those with fixed alternatives. Even a causal inspection of the tables suggests that the prediction performance is pretty good whatever the model considered. Possibly the only entries where there seems to be some substantial error depending on the model used are the predictions of outcomes for the first decile. In any case, in order to systematically assess the impact of the characteristics of all the 16 models we run the regression (4.2) and report the results in Table 4.3.

Table 4.1 Prediction of participation rates and hours of work under the 1994 tax system. Fixed-alternatives models

Deciles	True model		Model Ia		Model Id		Model IIa		Model IId	
	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work
1	58	568	55	627	43	514	87	733	55	568
2	65	715	73	818	61	730	93	837	67	708
3	79	937	81	1000	71	890	95	989	79	941
4	86	1157	87	1179	80	1130	97	1125	85	1153
5	91	1389	92	1375	87	1397	96	1276	90	1352
6	93	1527	94	1494	91	1541	98	1429	93	1528
7	93	1606	95	1638	91	1650	99	1598	94	1631
8	94	1695	94	1701	92	1735	98	1667	93	1672
9	94	1757	95	1812	93	1838	99	1746	96	1771
10	88	1523	89	1631	83	1566	97	1676	87	1567
Mean	84	1287	86	1327	79	1299	96	1308	84	1289

Table 4.2 Prediction of disposable income under the 1994 tax system. Fixed-alternatives models

Deciles	True model	Model Ia	Model Id	Model IIa	Model IId
1	168915	170648	169098	171945	168690
2	216080	217801	215357	219415	216333
3	244914	245504	243740	245176	243672
4	268880	268308	267340	267880	267659
5	290441	290083	290556	288798	289893
6	312088	312113	313719	310410	312446
7	336247	335829	337305	334374	336148
8	363833	364607	365453	362513	363739
9	403513	405063	405654	403401	404046
10	600841	605283	602163	608705	604516
Mean	320575	321524	321038	321262	320714

Table 4.3. Estimates of equation (4.2) outcomes under the current tax regime

Outcome variable	α_0	α_1	α_2	α_3	α_4	α_{34}	R ²
Probability of participation	.406 (.123)	-.075 (.100)	.126 (.100)	-.266 (.142)	.086 (.142)	-.076 (.201)	.54
Hours of work	.266 (.057)	-.023 (.046)	.070 (.046)	-.178 (.007)	-.004 (.007)	.031 (.092)	.60
Income after tax	.021 (.008)	-.002 (.004)	.007 (.004)	-.009 (.006)	-.002 (.006)	-.003 (.008)	.50

*Standard deviation in parentheses

The results of Table 4.3 confirm the message conveyed by Tables 4.1 – 4.2. Very few coefficients are significant at standard levels (the significant ones are in bold italics). Overall one can conclude that there is little evidence of an important impact of alternative modes of representing the choice set as long as the replication of current values is concerned.

4.2 Outcomes under a Flat Tax reform.

In this second part of the simulation exercise, the models are run as after a hypothetical tax reform. Namely, a fixed proportional tax (Flat Tax) replaces the current tax system. The flat tax is determined running recursively the true model until the total tax revenue is the same as under the current system. Tables 4.4 and 4.5 are analogous to Tables 4.1 and 4.2. Tables 4.6 and 4.7 replicate Tables 4.4 and 4.5 but with sampled-alternatives models. When it comes to reform simulations rather than current values replication, the differences in outcomes are somewhat more marked, and this is confirmed by Table 4.8 where eq. (4.2) is estimated, analogous to Table 4.3 but with reference here to post-Flat-Tax outcomes. There is a rather clear pattern of the effects of different modelling strategies in particular on the prediction of disposable income. For example, using 24 alternatives instead of 6 reduces the average percentage error by 0.8%. Using sampled alternatives instead of fixed alternatives reduces it by 1.9%, introducing job and peaks dummies reduces it by 3.2%. Moreover, the detailed information provided by Tables 4.4 - 4.7 demonstrates that the less satisfactory out-of-sample prediction performance arises from discrepancies between the lower parts of the predicted and "observed" flat-tax distributions of hours of work and disposable income.

Table 4.4 Prediction of participation rates and hours of work under a flat tax reform. Fixed-alternatives models

Deciles	True model		Model Ia		Model Id		Model Iia		Model Iid	
	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work
1	69	987	62	835	55	826	89	946	63	890
2	75	1022	77	943	68	966	95	1041	74	943
3	84	1160	83	1100	76	1117	96	1145	83	1134
4	89	1315	89	1260	83	1279	97	1271	87	1291
5	93	1491	93	1432	89	1488	97	1392	91	1459
6	94	1609	94	1542	92	1626	98	1543	93	1609
7	94	1659	95	1677	92	1717	99	1685	94	1670
8	95	1742	94	1735	92	1786	98	1727	93	1720
9	95	1794	96	1843	94	1898	99	1811	96	1821
10	88	1549	89	1647	84	1619	97	1721	88	1606
Mean	88	1487	87	1401	82	1432	96	1428	86	1414

Table 4.5 Prediction of disposable income under a flat tax reform. Fixed-alternatives models

Deciles	True model	Model Ia	Model Id	Model Iia	Model Iid
1	194076	171081	177612	173092	177934
2	234263	214268	220564	222704	220524
3	259189	242704	250457	247374	248492
4	279624	266384	272361	271441	271579
5	301124	289038	294062	293453	294681
6	323777	314124	320755	319278	319492
7	350809	342509	349310	346358	344397
8	383958	375740	379893	378941	377972
9	431297	426513	431747	430622	428668
10	651815	649764	651885	657771	652667
Mean	340993	329213	334865	334103	333641

Table 4.6 Prediction of participation rates and hours of work under a flat tax reform. Sampled-alternatives models

Deciles	True model		Model Ia		Model Id		Model Iia		Model Iid	
	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work	Participation rates (per cent)	Annual hours of work
1	69	987	76	915	65	883	76	921	65	880
2	75	1022	83	982	74	993	84	985	75	992
3	84	1160	90	1159	83	1131	90	1151	83	1133
4	89	1315	92	1288	88	1330	93	1307	89	1338
5	93	1491	94	1449	91	1493	94	1460	91	1485
6	94	1609	95	1580	94	1650	95	1579	94	1646
7	94	1659	95	1671	93	1691	96	1675	93	1695
8	95	1742	97	1759	96	1775	97	1771	96	1774
9	95	1794	98	1806	96	1811	98	1807	96	1814
10	88	1549	92	1606	88	1587	92	1617	88	1586
Mean	88	1487	91	1422	87	1434	91	1427	87	1434

Table 4.7 Prediction of disposable income under a flat tax reform. Sampled- alternatives models

Deciles	True model	Model Ia	Model Id	Model Iia	Model IId
1	194076	175360	178959	175829	178558
2	234263	221008	223384	220745	222943
3	259189	248332	249373	247584	249304
4	279624	272276	275414	273516	275739
5	301124	293241	296123	293368	295567
6	323777	318317	321883	318698	321400
7	350809	346147	348328	346124	348868
8	383958	377469	379296	378295	378984
9	431297	430380	430587	429954	431015
10	651815	651514	650805	652383	650766
Mean	340993	333404	335415	333650	335314

Table 4.8 Contributions to the prediction performance: outcomes under a flat tax reform

Outcome variable	α_0	α_1	α_2	α_3	α_4	α_{34}	R ²
Probability of participation	.215 (.008)	-.090 (.064)	.060 (.062)	-.110 (.091)	.077 (.091)	-.073 (.129)	.47
Hours of work	.093 (.013)	-.061 (.010)	-.023 (.010)	-.003 (.014)	-.001 (.014)	.012 (.020)	.82
Income after tax	.128 (.006)	-.019 (.005)	-.008 (.005)	-.029 (.007)	-.039 (.007)	.036 (.01)	.84

*Standard deviation in parentheses

4.4 Computational costs

The different representations of the choice set imply different computational burdens, particularly with regards to the number of alternatives and to the procedure used to generate the alternatives. Depending on the availability of computing resources and time, the advantages of the various approaches to represent the choice set should be balanced against the computational costs. Table 4.9 reports the relative elapsed time (= 1 for the simplest model⁸) of a typical estimation run with four different type of models: fixed vs sampled alternatives and 6 vs 24 alternatives (accounting or not accounting for job and peaks dummies does not make any significant difference in terms of computing time).

Table 4.9 Relative computation time (estimation) for different models

	6 alternatives	24 alternatives
Fixed alternatives	1	4.62
Sampled alternatives	6.70	8.46

5. Conclusions

We have performed a series of simulation exercises aimed at exploring the performance of different versions of a labour supply model, where different approaches to represent choice sets are used. The various models are estimated using a large sample generated by a “true” model, to which they can then be compared. In evaluating the models, we focus upon their ability replicate the “true” outcomes under different tax regimes. It turns out that as far as the replication of the current-tax-regime outcomes are concerned, there is little evidence for important effects of alternative choice-set-representation procedures. Not even the number of alternatives contained in the choice set seems to matter. All the models predict very well, although there are some indications favouring the sampled-alternatives procedure. However, when it comes to predicting outcomes under a flat-tax reform, the indications are more clear-cut: using sampled alternatives and accounting for heterogeneity of opportunities seems to significantly reduce the prediction errors (at least for the prediction of incomes). Clearly the sampled-alternative procedure is more costly computationally, so the benefits should eventually be balanced against the increased computational costs. The fact that the prediction performance of current values is not able to discriminate between different models and instead the prediction performance of post-reform is, conveys the important message that the ability of a model to replicate observed outcomes is not very informative. Ultimately, the models should be judge in their ability to do the job they are mainly built for, i.e. predicting the outcomes of policy changes.

⁸ The absolute computing time for estimating the simplest model was 2.42 seconds on a Alpha ES45, 1 Gb Mhz, 8 Gb workspace memory

Appendix

Here we report the parameter estimates of the true model and of the 16 alternative models.

Table A.1. Fixed-alternatives models Ix and IIx (x = a, b, c)

Variable	Parameter	True model	Model Ia	Model IIa	Model Ib	Model IIb	Model Ic	Model IIc	Model Id	Model IId
Consumption										
	α_1	0.39	0.35	0.54	0.43	0.46	0.43	0.50	0.43	0.44
	α_2	4.42	2.46	3.70	3.97	4.55	4.05	4.64	4.17	4.38
Leisure										
	α_3	-4.57	-7.53	-3.18	-7.31	-6.72	-2.07	-0.14	-3.99	-4.15
	α_4	168.88	54.20	184.85	64.76	92.39	232.99	351.30	156.91	171.12
Log age	α_5	-94.29	-30.46	-102.83	-36.27	-51.64	-128.78	-193.30	-87.38	-95.45
Log age squared	α_6	13.35	4.32	14.62	5.15	7.33	18.27	27.48	12.40	13.54
Number of children below 3 years old	α_7	0.44	0.13	0.51	0.13	0.19	0.61	0.95	0.38	0.40
Number of children 3-6 years old	α_8	1.23	0.48	1.68	0.53	0.76	1.86	2.99	1.25	1.40
Number of children 7-14 years old	α_9	1.05	0.40	1.37	0.44	0.62	1.53	2.47	1.04	1.14
Job dummy	θ_0	-0.60	-	-	-1.08	-2.33	-	-	-0.78	-2.10
Part-time dummy	π_1	0.46	-	-	-	-	-0.23	0.14	0.15	0.28
Full-time dummy	π_2	1.57	-	-	-	-	0.99	1.53	0.78	1.19

Table A.1. Sampled-alternatives models Ix and IIx (x = a, b, c)

	Parameter	True model	Model Ia	Model IIa	Model Ib	Model IIb	Model Ic	Model IIc	Model Id	Model IId
Consumption										
	α_1	0.39	0.54	0.55	0.53	0.54	0.55	0.55	0.52	0.53
	α_2	4.42	3.96	3.93	4.72	4.64	4.56	4.51	4.70	4.62
Leisure										
	α_3	-4.57	-5.15	-5.27	-5.94	-6.10	-2.40	-2.49	-3.52	-3.60
	α_4	168.88	125.90	121.50	112.19	106.31	234.88	231.26	195.26	190.72
Log age	α_5	-94.29	-70.17	-67.75	-62.54	-59.28	-129.94	-128.03	-108.43	-105.95
Log age squared	α_6	13.35	9.96	9.62	8.88	8.42	18.46	18.19	15.39	15.04
Number of children below 3 years old	α_7	0.44	0.33	0.30	0.25	0.23	0.66	0.58	0.50	0.44
Number of children 3-6 years old	α_8	1.23	1.07	1.05	0.91	0.87	1.94	1.95	1.56	1.57
Number of children 7-14 years old	α_9	1.05	0.88	0.88	0.75	0.73	1.61	1.65	1.29	1.33
Job dummy	θ_0	-0.60	-	-	-0.88	-0.86	-	-	-0.63	-0.60
Part-time dummy	π_1	0.46	-	-	-	-	0.44	0.44	0.53	0.52
Full-time dummy	π_2	1.57	-	-	-	-	1.66	1.63	1.56	1.54

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