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ABSTRACT

Endogenous Job Destruction and Job Matching in Cities^{*}

We propose a spatial search-matching model where both job creation and job destruction are endogenous. Workers are ex ante identical but not ex post since their job can be hit by a technological shock, which decreases their productivity. They reside in a city and commuting to the job center involves both pecuniary and time costs. Thus, workers with high wages are willing to live closer to jobs to save on time commuting costs. We show that, in equilibrium, there is a one-to-one correspondence between the productivity space and the urban location space since high-productivity workers bid away low-productivity workers in order to occupy locations close to jobs. We also show that in the bargaining process, there is a spatial element in the wage setting since firms need to compensate workers for their spatial costs. Compared to the non-spatial model, the unemployment rate and the reservation productivity are lower and the job-creation rate is higher because the urban space through commuting costs and land rent create additional frictions in the labor market.

JEL Classification: D83, J41, J64, R14

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1 Introduction

It is commonly observed that unemployment varies both within and between regions and cities (Holzer, 1989; Blanchflower and Oswald, 1994; Overman and Puga, 2002; Patacchini and Zenou, 2007). There are also stark spatial differences in incomes. For example, in the United States, the median income of central city residents is 40 percent lower than that of suburban residents. There are also strong evidence that the labor market is characterized by large flows and job turnover (Davis et al., 1996).

The aim of this paper is to provide a search-matching model where both features are present, i.e. endogenous job creation and job destruction and an explicit land market.

There is by now a small literature on the theoretical aspects of spatial search models that explicitly incorporates land and labor markets (see Zenou, 2007a,b).¹ Wasmer and Zenou (2002, 2006) and Smith and Zenou (2003) introduce a land market (as in Fujita, 1989) in a standard search-matching model (as in Pissarides, 2000). They all assume that the city is monocentric, i.e. all jobs are located in one location. The interaction between land and labor markets is performed through search intensity. Indeed, in their models, distance to jobs affects search intensity because remote workers are less likely to obtain good information about job opportunities. As a result, the land market equilibrium depends on aggregate variables (such as wages and labor market tightness) since these variables affect location choices of workers. On the other hand, the labor market equilibrium crucially depends on the land market equilibrium configuration because the efficiency of aggregate matching depends on the average location of the unemployed. Sato (2004) extends this framework by introducing ex ante heterogeneity of workers who differ by specific training costs. It is assumed that ex post all workers have been trained and are thus identical in terms of productivities. The analysis is then relatively similar to what is obtained in the models above. Sato (2001) develops a similar framework but allows ex post heterogeneity in productivities. However, this heterogeneity in the productivity space, which implies different wages, does not translate into an heterogeneity in the urban space since, in equilibrium, whatever their location in the city, all workers experience the same costs of living and thus there is no direct correspondence between the heterogeneity in the productivity space and the one in the urban space. An other interesting contribution is Coulson et al. (2001) who develop a search model in a duocentric city. They assume that the fixed entry cost of firms is greater

¹It has been recognized for a long time that space affects search activities. For example, Stigler (1961) and Diamond (1982) have put geographical dispersion as one of the main determinants of price ignorance because the spatial dispersion of agents creates more frictions and thus more unemployment. See also Simpson (1992) for an early approach of the analysis of urban labor markets.

in the Central Business District (CBD) than in the Suburban Business District (SBD) and that workers are heterogeneous in their disutility of transportation (or equivalently in their search costs). These two fundamental assumptions are sufficient to generate an equilibrium in which central city residents experience a higher rate of unemployment than suburban residents and suburban firms create more jobs than central firms. The interaction between the land and labor markets is due to the fact that workers and firms are mobile and look for a “partner” to form a match in both areas.

None of these approaches have endogenized job destruction and workers’ ex post heterogeneity. The aim of the present paper is to investigate these issues by proposing a spatial version of the search-matching model developed by Mortensen and Pissarides (1994).

More precisely, following Mortensen and Pissarides (1994), we assume that each job is characterized by a fixed irreversible technology, so that each time there is a technological shock, the quality of the match and the productivity deteriorate. New filled jobs start at the highest productivity value and a job is destroyed only if the idiosyncratic component of their productivity falls below some critical number. Workers reside and work in the city and thus commute to the employment center. Both pecuniary and time costs are taken into account. Thus, workers with high productivities and wages are willing to live closer to jobs to save on time commuting costs. We first show that, in equilibrium, there is a one-to-one correspondence between the productivity space and the urban location space since high-productivity workers bid away low-productivity workers in order to occupy locations close to jobs. We then characterize the labor-market equilibrium. Wages are set according to a bilateral bargaining between a worker and a firm and we show that wages depend on labor market variables such that the unemployment benefit or the cost of maintaining a vacancy but also on spatial variables such as the commuting cost. In fact, compared to the non-spatial model, there is a new element here. Firms need to compensate employed workers for the transport cost difference between the employed and the unemployed workers in order for them to accept the job offer. We also determine the job creation and the job destruction conditions, which depend on the commuting cost. We also characterize the urban land-use equilibrium. The difficulty here is that ex post all workers are heterogeneous in terms of productivities and thus in terms of wages. Since the latter are the opportunity cost of leisure, we show that higher productivity and higher wage workers locate closer to jobs than those with lower productivities and wages. We can finally calculate the equilibrium utilities of all workers and land rent at each location in the city. The interaction between the land and labor markets is performed here through the wages because, as stated above, firms need to compensate workers for their spatial costs.

By calibrating the model, we are able to highlight the main differences between the spatial and non-spatial models. We show that the unemployment rate is higher while the vacancy rate and the reservation productivity are lower in the spatial model. The main reason for these differences is that there are more frictions in the spatial model since workers are differentiated both in the productivity and urban spaces. An interesting feature of the model is that firms keep some currently unprofitable jobs occupied because of the possibility that a job productivity might change, which enable firms to start production at the new productivity immediately after arrival, without having to pay the recruitment cost and forgo production during search. This feature is present in both models but is much more important in the spatial model. An other interesting difference is that, in the spatial model, each labor variable has an impact on both markets. Take, for example, the unemployment benefit. An increase in the latter increases the wage, which, in turn, affects the time cost of travelling. This increases the competition in the land market since the access to the job center becomes more valuable, which in turn increases the wage since firms need to compensate more for spatial costs in order to induce workers to take a job. These amplifying effects leads to higher unemployment rate and reservation productivity and lower labor market tightness in the spatial model.

The rest of the paper is organized as follows. Section 2 presents the model. In sections 3 and 4, we characterize the urban land-use and the labor equilibrium, respectively. The steady-state equilibrium, which solves simultaneously the urban land use and the labor market equilibrium, is analyzed in section 5. In section 6, we calibrate the model and presents the different numerical results. Finally, section 7 concludes. All proofs are given in the Appendix.

2 The model

There is a continuum of ex ante identical workers whose mass is N and a continuum of identical firms. Among the N workers, there are L employed and U unemployed so that $N = L+U$. The workers are uniformly distributed along a *linear, closed* and *monocentric* city. Their density at each location is taken to be 1. There is no vacant land in the city and all land is owned by absentee landlords. All firms are exogenously located in the Central Business District (CBD hereafter) and consume no space. Workers are assumed to be infinitely lived, risk neutral and decide their optimal place of residence between the CBD and the city fringe.

Let us describe the labor market, which follows Mortensen and Pissarides (1994). We assume that each job is characterized by a *fixed irreversible technology*. To be more precise,

the value of a match is given by $y + \epsilon$, where y denotes the general productivity and ϵ is a parameter that is drawn from a distribution $G(\epsilon)$, which has a finite support $[\underline{\epsilon}, \bar{\epsilon}]$ and no mass point. To be consistent with the way we model the *linear* city, we assume a uniform distribution, so that $G(\epsilon) = (\epsilon - \underline{\epsilon}) / (\bar{\epsilon} - \underline{\epsilon})$.² It is assumed that a job is created with the highest productivity value, $y + \bar{\epsilon}$, capturing the idea that firms with new filled jobs use the best available technology. Then, at a Poisson rate δ , there is an idiosyncratic productivity shock. A job is then destroyed only if the idiosyncratic component of their productivity falls below some critical number $\tilde{\epsilon} < \bar{\epsilon}$. As a result, the rate at which jobs are destroyed is given by $\delta G(\tilde{\epsilon}) = \delta (\tilde{\epsilon} - \underline{\epsilon}) / (\bar{\epsilon} - \underline{\epsilon})$. Because there will be a distribution of ex post productivities $y + \epsilon$, there will also be a distribution of wages $w_L(\epsilon)$.

A firm is a unit of production that can either be filled by a type- ϵ worker whose production is $y + \epsilon$ units of output or be unfilled and thus unproductive. In order to find a worker, a firm posts a vacancy. A vacancy can be filled according to a random Poisson process. Similarly, workers searching for a job will find one according to a random Poisson process. In aggregate, these processes imply that there is a number of contacts per unit of time between the two sides of the market that are determined by the following matching function:³

$$M(U, V)$$

where U and V the total number of unemployed and vacancies respectively. As in the standard search-matching model (see e.g. Mortensen and Pissarides, 1999, and Pissarides, 2000), we assume that $M(\cdot)$ is increasing both in its arguments, concave and homogeneous of degree 1 (or equivalently has constant return to scale). Thus, the rate at which vacancies are filled is $M(U, V)/V$. By constant return to scale, it can be rewritten as

$$M(1/\theta, 1) \equiv q(\theta)$$

where $\theta = V/U$ is the *labor market tightness* and $q(\theta)$ is a Poisson intensity. By using the properties of $M(\cdot)$, it is easily verified that $q'(\theta) \leq 0$: the higher the labor market tightness, the lower the rate at which firm fill their vacancy. Similarly, the rate at which an unemployed worker leaves unemployment is

$$\frac{M(U, V)}{U} \equiv \theta q(\theta)$$

²None of our results in the labor-market equilibrium are affected by this assumption.

³This matching function is written under the assumption that the city is monocentric, i.e. all firms are located in one fixed location.

Again, by using the properties of $M(\cdot)$, it is easily verified that $[\theta q(\theta)]' \geq 0$: the higher the labor market tightness, the higher the rate at which workers leave unemployment since there are relatively more jobs than unemployed workers.

A steady-state equilibrium requires solving *simultaneously* an urban land use equilibrium and a labor market equilibrium. It is convenient to present first the former and then the latter.

3 Urban land-use equilibrium

To generate a location pattern by productivity type, as in Brueckner et al. (2002), we introduce a key assumption that links commuting costs to the wage paid. In a more general model, this link is achieved through a labor-leisure choice, which implies that a unit of commuting time is valued at the wage rate (see, for example, Fujita, 1989, Chapter 2). However, such a model is cumbersome to analyze, and it is likely not to yield additional insights beyond those available from our simpler approach. This one is based on a particular formulation of the labor-leisure choice, which is consistent with the empirical literature that shows that the time cost of commuting increases with the wage (see, e.g. Small, 1992, and Glaeser et al., 2007).

We assume that each worker consumes one unit of land in the city in which he/she lives, while providing a fixed amount of labor time T . With land consumption fixed, the worker's utility depends solely on the quantity of a consumption good c and on the time Λ available for leisure.

In a model with labor-leisure choice, work hours are adjusted until the marginal value of leisure time equals the wage rate. While the explicit incorporation of flexible work hours would seriously complicate the model, we incorporate the spirit of this standard result by assuming that utility can be approximated by a function that values leisure time at the wage rate. In particular, we assume that workers consume a composite good z defined as follows:

$$z = c + w_L \Lambda \tag{1}$$

The marginal utility of Λ is thus equal to the worker's wage, as would be the case if work hours were adjusted optimally in a more general setting. The worker purchases the good c produced and sold at the corresponding CBD and incurs τx in monetary commuting costs

when he/she lives at distance x from the CBD. Letting $R(x)$ denote rent per unit of land at distance x , the budget constraint of a type- ϵ worker at distance x can be written as follows:

$$w_L(\epsilon)T = c + R(x) + \tau x \quad (2)$$

where T , the amount of working hours, is assumed to be the same and constant across workers. Furthermore, commuting time from distance x is equal to tx , where $t > 0$ is time spent per unit of distance. Hence, the time constraint of a type- ϵ worker at distance x is given by

$$1 = T + \Lambda + tx \quad (3)$$

in which the total amount of time is normalized to 1 without loss of generality.

Substituting (2) and (3) in (1), the *instantaneous* (indirect) utility of a type- ϵ worker at distance x , denoted by $W_L(\epsilon)$, is then given by:

$$\begin{aligned} W_L(\epsilon) &\equiv z_L = w_L(\epsilon)T - R(x) - \tau x + w_L(\epsilon)(1 - T - tx) \\ &= w_L(\epsilon)(1 - tx) - \tau x - R(x) \end{aligned}$$

Therefore, the time cost of commuting for a type- ϵ worker residing at a distance x from the CBD is $tw_L(\epsilon)x$; in accordance with empirical observation, it increases with the income $w_L(\epsilon)T$. As usual, $w_L(\epsilon)$ in (4) does not stand for the worker's actual income (which is equal to $w_L(\epsilon)T < w_L(\epsilon)$) but for the income that would accrue to an individual working all the time ($T = 1$). Rearranging (4) yields:

$$\Psi_L(\epsilon, x, W_L(\epsilon)) = w_L(\epsilon)(1 - tx) - \tau x - W_L(\epsilon) \quad (4)$$

which is the bid rent of a type- ϵ employed worker residing at a distance x from the CBD.⁴

Inspection of (4) shows that, as usual, the bid-rent function is decreasing in x , with $\partial\Psi_L/\partial x < 0$. In the present model, this reflects the combined influence of the time cost of commuting and the monetary cost. Since $w'(\epsilon) \geq 0$ (see below), further inspection shows that, at a given x , an increase in ϵ makes the bid-rent slope more negative ($\partial^2\Psi_L/\partial\epsilon\partial x < 0$). This means that high- ϵ workers have steeper bid-rent curves than low- ϵ workers. The intuitive reason is that *an extra mile of commuting reduces income more for a high- ϵ worker than for*

⁴The bid rent is a standard concept in urban economics. It indicates the maximum land rent that a worker located at a distance x from the CBD is ready to pay in order to achieve a utility level. See Fujita (1989) for a formal definition.

a *low- ϵ worker*, a consequence of the higher net wage. Therefore, the high- ϵ worker requires a larger decline in land rent than a high- ϵ worker to maintain a given utility level. In comparing the residential locations of two groups, it is well known that the group with the steeper bid-rent curve locates closer to the CBD (see, for example, Fujita, 1989, Chapter 2). In the present model, this implies that *high- ϵ workers locate closer to the CBD than low- ϵ workers*.

Let us now focus on the unemployed. Their budget constraint is given by:

$$w_U = z + R(x) + \tau x \quad (5)$$

where w_U is the unemployment benefit. It is assumed that the unemployed commute as often to the CBD as the employed since they go there to search for jobs and to be interviewed.⁵ We assume the unemployed's opportunity cost of time is negligible since they do not work, and thus time costs do not enter in their utility function. As a result, and to be consistent with the utility of the employed, the *instantaneous* (indirect) utility of an unemployed worker residing at distance x is:

$$W_U \equiv z_U = w_U - R(x) - \tau x$$

and the bid rent is equal to:

$$\Psi_U(x, W_U) = w_U - \tau x - W_U \quad (6)$$

Since the wage is never equal to zero, any employed worker (whatever his/her ϵ -type) will have a steeper bid rent than the unemployed worker. To formalize this notion, we introduce the definition of residential equilibrium:

Definition 1 *A land-use urban equilibrium consists of the following mapping $\epsilon(x)$ that assigns a worker of skill type ϵ to a location x , i.e.*

$$\epsilon(x) = \bar{\epsilon} - \left(\frac{\bar{\epsilon} - \tilde{\epsilon}}{L} \right) x \quad \text{for } 0 \leq x \leq L \quad (7)$$

a set of utility levels $W_L^*(\epsilon)$ and W_U , and a land rent curve $R^*(x)$ such that:

(i) at each $x \in [0, L]$,

$$\begin{aligned} R^*(x) &= \Psi_L(\epsilon(x), x, W_L^*(\epsilon)) \\ &= \max_{\epsilon} \Psi_L(\epsilon, x, W_L^*(\epsilon)) \end{aligned} \quad (8)$$

⁵It is easy to verify that assuming less trips for the unemployed would not change any of our results.

(ii)

$$\Psi_L(L, W_L(\tilde{\epsilon})) = \Psi_U(L, W_U) \quad (9)$$

(iii)

$$\Psi_U(N) = 0 \quad (10)$$

To define the urban equilibrium, we need to calculate the assignment rule between the location space defined on $[0, L]$ and the productivity space defined on $[\tilde{\epsilon}, \bar{\epsilon}]$, since in equilibrium only workers with productivity equals or above $\tilde{\epsilon}$ will work. Because we know that the relationship is negative and because of the linearity assumption of both the distribution x of workers in the city and the distribution ϵ of productivities, this relationship has to be linear. So we have to find a linear and negative relationship between ϵ and x so that $\epsilon(0) = \bar{\epsilon}$ and $\epsilon(L) = \tilde{\epsilon}$. The only equation that satisfies these requirements is given by (7). Moreover, equation (7) formalizes the claim above that high- ϵ workers locate closer to the CBD than low- ϵ workers. Indeed, the mapping between the physical and productivity “distances” of workers involves a correlation between these distances. It should be noted that this result depends on the assumption of a fixed lot size and uniform distribution of ϵ . As is well known, variable land consumption can overturn the present inverse association between residential distance and the time cost of commuting (see, for example, Fujita, 1989, Chapter 2). With variable consumption, however, additional conditions could be imposed to guarantee that the two distances remain perfectly. The same applies to a more general c.d.f. $G(\epsilon)$.

Let us interpret the other equations. Equation (8) says that land rent $\Psi(x)$ at location x equals the maximum of the bid rents across productivity types, and that the productivity type offering the highest bid at x is occupying the location x . Furthermore, at distance $x = L$, the bid rents between the employed with the lowest productivity, i.e. $\tilde{\epsilon}$, and the unemployed worker has to be equal to ensure the continuity of the land rent. This condition is given by (9). Finally, because of our assumptions of a fixed lot size and of no vacant land, the land rent at the edge of the city (distance N) is undetermined. Since the value of this constant does not affect our results, we say in equation (10) that it equals the opportunity cost of land, which is assumed to be zero without loss of generality.

It is important to observe that richer and more productive workers live closer to the job center but this does not necessary imply that they live close to the city-center. What matters is the distance to jobs. For example, in a city like Los Angeles, which is extremely decentralized, rich workers tend to live far away from the city center but relatively close to jobs, which are more and more located in the suburbs (Glaeser et al., 2007).

We cannot solve the urban land-use equilibrium before knowing the exact value of the wage. So let us now determine the labor-market equilibrium.

4 Labor-market equilibrium

We need first to write the different lifetime expected utilities of workers and firms. The steady-state Bellman equations for unemployed and employed workers are given by:

$$rI_U = w_U - \tau x - R(x) + \theta q(\theta) [I_L(\bar{\epsilon}) - I_U] \quad (11)$$

$$rI_L(\epsilon) = w_L(\epsilon) (1 - tx) - \tau x - R(x) + \delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_L(s) dG(s) + \delta G(\tilde{\epsilon}) I_U - \delta I_L(\epsilon) \quad (12)$$

Let us interpret these equations. Equation (11) says that an unemployed worker, who enjoys an instantaneous utility $w_U - \tau x - R(x)$ today, can find a job at rate $\theta q(\theta)$ and, in that case, will start a job at the highest productivity level $y + \bar{\epsilon}$ and thus obtains an expected utility $I_L(\bar{\epsilon})$ with the highest wage level $w_L(\bar{\epsilon})$. Equation (12) has a similar interpretation. An employed worker, who is employed at a certain productivity level ϵ , obtains an instantaneous utility given by (4), i.e. $w_L(\epsilon) (1 - tx) - \tau x - R(x)$, but can then be “hit” by a productivity shock at rate δ and will continue to work in that job only if the match is still productive, i.e. the productivity is at least equal to $y + \tilde{\epsilon}$. If not, that is if the productivity of the match is less than $y + \tilde{\epsilon}$, which happens with probability $G(\tilde{\epsilon}) = \Pr[\epsilon \leq \tilde{\epsilon}]$, the worker becomes unemployed and loses $I_L(\epsilon) - I_U$. With our uniform distribution assumption, (12) can be written as:

$$rI_L(\epsilon) = w_L(\epsilon) (1 - tx) - \tau x - R(x) + \frac{\delta}{(\bar{\epsilon} - \tilde{\epsilon})} \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_L(s) ds + \delta \left(\frac{\tilde{\epsilon} - \epsilon}{\bar{\epsilon} - \epsilon} \right) I_U - \delta I_L(\epsilon) \quad (13)$$

For firms, with a filled job and a vacancy, we have respectively:

$$rI_F(\epsilon) = y + \epsilon - w_L(\epsilon) + \frac{\delta}{(\bar{\epsilon} - \tilde{\epsilon})} \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_F(s) ds + \delta \left(\frac{\tilde{\epsilon} - \epsilon}{\bar{\epsilon} - \epsilon} \right) I_V - \delta I_F(\epsilon) \quad (14)$$

$$rI_V = -c + q(\theta) [I_F(\bar{\epsilon}) - I_V] \quad (15)$$

where c is the cost of a vacancy. The interpretation of these equations are similar to that of (11) and (12).

4.1 Free-entry condition and labor demand

We assume that firms post vacancies up to a point where:

$$I_V = 0 \quad (16)$$

which is a free entry condition. From (15), the value of a *new* job is equal to:

$$I_F(\bar{\epsilon}) = \frac{c}{q(\theta)} \quad (17)$$

Now using (16), (14) can be written as:

$$I_F(\epsilon) = \frac{y + \epsilon - w_L(\epsilon)}{(r + \delta)} + \frac{\delta}{(\bar{\epsilon} - \tilde{\epsilon})(r + \delta)} \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_F(s) ds \quad (18)$$

Evaluating this last equation at $\epsilon = \bar{\epsilon}$ and using (17) leads to the following decreasing relation between labor market tightness and wages:

$$\frac{c(r + \delta)}{q(\theta)} = y + \bar{\epsilon} - w_L(\bar{\epsilon}) + \frac{\delta}{(\bar{\epsilon} - \tilde{\epsilon})} \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_F(s) ds \quad (19)$$

4.2 Wage determination

At each period, the total intertemporal surplus is shared through a generalized Nash-bargaining process between the firm and the worker. At each period, the wage is thus determined by:

$$w_L = \arg \max_{w_L} [I_L(\epsilon) - I_U]^\beta [I_F(\epsilon) - I_V]^{1-\beta} \quad (20)$$

where $0 \leq \beta \leq 1$ is the bargaining power of workers. Observe that I_U , the threat point for the worker does not depend on the current location of the worker, who will relocate if there is a transition in his/her employment status. The result of the bargaining is equal to:

$$(1 - \beta) [I_L(\epsilon) - I_U] = \beta I_F(\epsilon) \quad (21)$$

We have the following result.

Lemma 1 *Assume*

$$(1 - \beta) tL < \frac{\beta(\bar{\epsilon} - \tilde{\epsilon})}{(1 - \beta) w_U + \beta(y + c\theta + \bar{\epsilon})} \quad (22)$$

Then the wage $w_L(\epsilon)$ is always strictly positive, given by

$$w_L(\epsilon) = \frac{(1 - \beta) w_U + \beta(y + \epsilon + c\theta)}{1 - (1 - \beta) tL(\bar{\epsilon} - \epsilon) / (\bar{\epsilon} - \tilde{\epsilon})} \quad (23)$$

and increasing in ϵ .

First, condition (22) guarantees that $1 - (1 - \beta)tx > 0, \forall x \in [0, L]$ so that the wage is always strictly positive in the area of the city where the employed workers live, i.e. in $[0, L]$. This condition also guarantees that $w'(\epsilon) > 0$. Condition (22) is a natural requirement because it says that commute time from the border between the employed and the unemployed, which equals tL , is less than $tL < \frac{\bar{\epsilon} - \tilde{\epsilon}}{\bar{\epsilon} + \epsilon} < 1$, and thus less than the total time available (unity). Second, the wage (23) is a generalization of the non-spatial wage obtained by Mortensen and Pissarides (1994) (see for example in Pissarides (2000) equation (2.10) page 42, which is (23) when $t = 0$). As in the standard search-matching model, we have the same positive effects of w_U, y, c and θ on the wage since they all increase the outside option of workers. The new element here is the spatial aspect of the negotiation where the employer has to take into account the location x of workers. As in Wasmer and Zenou (2002), firms must *compensate* workers for the transport cost difference between the employed and the unemployed workers in order for them to accept the job offer. However, the main difference with Wasmer and Zenou is that here workers are ex post heterogenous in terms of productivity ϵ and, because of the assignment rule (7), this translates into heterogeneity in terms of distance to jobs x . So the compensation is more complex and this leads to a non-linear relationship between w and ϵ .

Third, compared with the non-spatial model, what is interesting is that *here* the range of the distribution $\bar{\epsilon} - \tilde{\epsilon}$ does affect the wage. Indeed, in the non-spatial model, wages depend on job productivity y but not on other jobs' productivities. Here, because of the spatial compensation, the productivity of the highest and lowest productivity has an impact on the wage of every worker, whatever his/her productivity. Fourth, because the employed and unemployed workers have the same monetary commuting cost, only time cost (i.e. t) affects the wage. Not surprisingly, the higher t , the higher the wage since firms need to compensate more workers to induce to accept the job offer. Of course, this interpretation (like the ones above) is done for a given θ and a given L . In equilibrium, θ and $L = (1 - u)N$ will be affected by t and thus there will also be indirect effects of t on w via θ and L . Finally, the wage obtained in (23) is independent of the uniform distribution assumption of $G(\epsilon)$.

4.3 Endogenous job creation and job destruction

To determine the job-acceptance rule, we have to solve the following equations

$$I_F(\tilde{\epsilon}) = 0$$

$$I_L(\tilde{\epsilon}) - I_U = 0$$

These two equations are equivalent by the sharing rule (21). Let us thus solve the first one. We show in the Appendix that, by solving this equation and using (17), the job-creation condition can be written as:

$$\frac{c}{q(\theta)} = \left(\frac{1-\beta}{r+\delta} \right) \frac{(\bar{\epsilon} - \tilde{\epsilon})(1-tL) + [(1-\beta)w_U + \beta(y + \bar{\epsilon} + c\theta)]tL}{[1 - (1-\beta)tL]} \quad (24)$$

while the job-destruction condition is given by:

$$(y + \tilde{\epsilon})(1-tL) - w_U - \frac{\beta c}{1-\beta}\theta + \frac{\delta(1-tL)}{2(r+\delta)}(\bar{\epsilon} - \tilde{\epsilon}) = 0 \quad (25)$$

Equation (24) states that the firm's expected gain from a new job is equal to the expected hiring cost paid by the firm. In the non-spatial case, when $t = 0$, we obtain

$$\frac{c}{q(\theta)} = \left(\frac{1-\beta}{r+\delta} \right) (\bar{\epsilon} - \tilde{\epsilon}) \quad (26)$$

which is equation (2.14) page 43 in Pissarides (2000). First, in the non-spatial case, described by (26), the relationship between the reservation productivity $\tilde{\epsilon}$ and the labor market tightness θ is *negative* because at higher $\tilde{\epsilon}$, the expected lifetime of a job is shorter since, in any short interval of time dt , the job is destroyed with probability $\delta G(\tilde{\epsilon})dt$. As a result, firms create fewer jobs, which leads to a fall in θ and thus less job creation. This is not true anymore since by differentiating (24), one obtains:

$$\frac{\partial \theta}{\partial \tilde{\epsilon}} \geq 0 \Leftrightarrow (1-\beta)tL \geq \frac{\eta(\theta)(r+\delta)}{\theta q(\theta)\beta + \eta(\theta)(r+\delta)} \quad (27)$$

where

$$\eta(\theta) = -\frac{q'(\theta)}{q(\theta)}\theta > 0$$

is the elasticity of $q(\theta)$ in absolute value. This is because, as highlighted above, wages need to be spatially compensated. So, when $\tilde{\epsilon}$ increases, jobs are shorter but workers need to be compensated less, which increases θ . The net effect is thus ambiguous. If the spatial compensation effect is not too strong, i.e. if t is not too high, one can see from (27) that the relationship between θ and $\tilde{\epsilon}$ is negative, as in the non-spatial model. This is what we assume now. Second, the impact of the discount rate r or the mismatch rate δ is similar to that of the non-spatial model since higher r or δ decrease job creation because the future returns from new jobs are discounted at higher rates. Third, contrary the non-spatial model, the general productivity y , the cost of maintaining a vacancy c as well as the unemployment

benefit w_U enter in the equation and thus affect the relationship between $\tilde{\epsilon}$ and θ . Again this is because of the spatial aspect of wages, which implies that, contrary to (26), the firm's expected revenues and costs are not anymore proportional to y . Finally, the spatial variable t does affect the relationship between $\tilde{\epsilon}$ and θ . If t increases, then firms will create less jobs because wages increase due to higher spatial compensation. Observe that all our comments were made at a given $L = (1 - u)N$. At the general equilibrium, there will be additional effects since u is also affected by θ and $\tilde{\epsilon}$ (see below).

Let us now comment the job-destruction equation (25) and compared it with the non-spatial case, which is given by:

$$y + \tilde{\epsilon} - w_U - \frac{\beta c}{1 - \beta} \theta + \frac{\delta}{2(r + \delta)} \frac{(\bar{\epsilon} - \tilde{\epsilon})^2}{(\bar{\epsilon} - \underline{\epsilon})} = 0 \quad (28)$$

and corresponds to equation (2.15) page 44 (for the uniform distribution) in Pissarides (2000). Contrary to the job-creation equations, the two equations (25) and (28) are quite similar. This is mainly because job creation is directly affected by wages while job destruction is not. First, as in the non-spatial model, there is a positive relationship between θ and $\tilde{\epsilon}$ because at higher θ , the worker's outside opportunities are better, wages are thus higher, and so more marginal jobs are destroyed, which increase $\tilde{\epsilon}$. Second, all the variables except t enter in a similar way and have the same effects on the relationship between θ and $\tilde{\epsilon}$ as in the non-spatial model. Finally, the new element is t the commuting cost per unit of distance. At given θ and L , it is easy to verify that an increase in t increases $\tilde{\epsilon}$, which implies that more marginal jobs are destroyed. Indeed, when t is higher, net wages increase and thus the value of employment at each ϵ , i.e. $I_L(\epsilon)$, increases. As a result, since the value of unemployment I_U is not affected, $\tilde{\epsilon}$ has to increase to satisfy the condition $I_L(\tilde{\epsilon}) = I_U$.

4.4 Unemployment rate

The number of workers who enter unemployment is $\delta G(\tilde{\epsilon})(1 - u)N$ and the number who leave unemployment is $\theta q(\theta)uN$. The evolution of unemployment is thus given by the difference between these two flows,

$$\dot{u} = \delta G(\tilde{\epsilon})(1 - u) - \theta q(\theta)u \quad (29)$$

where \dot{u} is the variation of unemployment with respect to time. In steady state, the rate of unemployment is constant and therefore these two flows are equal (flows out of unemployment

equal flows into unemployment). We have:

$$u^* = \frac{\delta G(\tilde{\epsilon})}{\delta G(\tilde{\epsilon}) + \theta q(\theta)} = \frac{\delta(\tilde{\epsilon} - \underline{\epsilon})}{\delta(\tilde{\epsilon} - \underline{\epsilon}) + (\tilde{\epsilon} - \underline{\epsilon})\theta q(\theta)} \quad (30)$$

This equation is exactly the same as in the non-spatial model. All spatial variables will affect the unemployment rate only indirectly through both job creation θ (see (24)) and job destruction $\tilde{\epsilon}$ (see (25)).

5 Steady-state equilibrium

The steady-state equilibrium is a 7-tuple $(\theta^*, \tilde{\epsilon}^*, u^*, R_L^*(x), W_L^*(\epsilon), W_U^*, L^*)$ such that equations (24), (25), (30), (8), (9), (10), and

$$L^* = (1 - u^*)N \quad (31)$$

are satisfied. We can in fact solve separately the urban and the labor equilibrium. We have a first result:

Proposition 1 *Assume*

$$\frac{tL}{\bar{\epsilon} - \tilde{\epsilon}} < \min \left\{ \frac{\beta / (1 - \beta)}{(1 - \beta)w_U + \beta(y + c\theta + \bar{\epsilon})}, \frac{1}{\bar{\epsilon} - \epsilon} \right\} \quad (32)$$

Then, there exists a unique residential equilibrium characterized the assignment rule (7).

The equilibrium instantaneous utilities of employed workers of type ϵ are given by:

$$\begin{aligned} W_L^*(\epsilon) &= \frac{\beta^2(\bar{\epsilon} - \tilde{\epsilon})}{(1 - \beta)^2 tL} \log \left[\frac{\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta)tL(\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta)tL(\bar{\epsilon} - \tilde{\epsilon})} \right] - \tau N \\ &\quad + \frac{\beta(\epsilon - \tilde{\epsilon})}{(1 - \beta)} + \frac{[(1 - \beta)w_U + \beta(y + \epsilon + c\theta)](1 - tL)}{1 - (1 - \beta)tL} \end{aligned} \quad (33)$$

which are increasing and convex in ϵ , while the unemployed's utility is:

$$W_U^* = w_U - s\tau N \quad (34)$$

The equilibrium land rent in the employment zone $R_L^(x)$, i.e. for all $x \in [0, L]$, is equal to*

$$\begin{aligned} R_L^*(x) &= \frac{[(1 - \beta)w_U + \beta(y + \bar{\epsilon}(1 - 1/L) + x\tilde{\epsilon}/L + c\theta)](1 - tx)}{1 - (1 - \beta)tx} \\ &\quad - \frac{[(1 - \beta)w_U + \beta(y + \tilde{\epsilon} + c\theta)](1 - tL)}{1 - (1 - \beta)tL} + \tau(N - x) \\ &\quad + \frac{\beta(\bar{\epsilon} - \tilde{\epsilon})}{(1 - \beta)L} \left[\frac{\beta}{(1 - \beta)t} \log \left[\frac{1 - (1 - \beta)tx}{1 - (1 - \beta)tL} \right] - 1 + x \right] \end{aligned} \quad (35)$$

and is decreasing and convex in x . In the unemployment zone, i.e. for all $x \in [L, N]$, the equilibrium land rent $R_U^*(x)$ is given by:

$$R_U^*(x) = \tau (N - x) \quad (36)$$

This result establishes that there is a unique urban land-use equilibrium and gives the exact values of all the endogenous variables as a function of the exogenous parameters and the endogenous variables in the labor market, i.e. $\theta^*, \tilde{\epsilon}^*, u^*$. Observe that the way these labor variables affect the spatial variables $W_L^*(\epsilon)$, W_U^* and $R^*(x)$ are through the wage setting. Indeed, as we have seen above, the wage (23) is affected by $\theta^*, \tilde{\epsilon}^*, u^*$, but the wage also affects the land market through the bid rent. Indeed, workers with higher wages want to reduce their commuting time, and thus outbid workers with lower wages at the outskirts of the city. Concerning the labor market, we have three unknowns $\theta^*, \tilde{\epsilon}^*, u^*$, and three equations (24), (25) and (30). By imposing the restriction in the parameters as given by (27), we have a decreasing relationship between θ and $\tilde{\epsilon}$ in (24). Then, using similar arguments as in Mortensen and Pissarides (1994), one can show that the steady-state equilibrium exists and is unique. What is interesting now is to analyze the interaction between the land and the labor market. Because the model is quite cumbersome, we now turn to numerical simulations.

6 Calibration

We calibrate the model to obtain reasonable values of the unemployment rate and the job creation rate. We will in particular focus our discussion on the differences between the spatial and the now spatial model. As it is usual, we use the following Cobb-Douglas function for the matching function:

$$M(uN, V) = (uN)^{0.5} (vN)^{0.5}$$

where v is the vacancy rate. This implies that $q(\theta) = \theta^{-0.5}$, $\theta q(\theta) = \theta^{0.5}$ and, the elasticity of the matching rate (defined as $\eta(\theta) = -q'(\theta)\theta/q(\theta)$) is equal to 0.5. The values of the parameters (in yearly terms) are the following: the output y is normalized to unity, as is the scale parameter of the matching function. The relative bargaining power of workers is equal to $\eta(\theta)$, i.e. $\beta = \eta(\theta) = 0.5$. Unemployment benefits have a value of 2 and the costs of maintaining a vacancy c are equal to 2 per unit of time while the general productivity is 4. The uniform distribution of ϵ is defined on the support $[0, 2]$, which implies that new jobs have a productivity equal to $y + \bar{\epsilon} = 6$ while for the worse jobs it is $y + \underline{\epsilon} = 4$. Pecuniary commuting costs τ are equal to 0.1 whereas time cost are equal to 0.07. The discount rate

is $r = 0.05$, whereas the mismatch rate is $\delta = 0.15$, which means that, on average, there is a technological shock every six and half years. Finally, the total population is normalized to 1. Table 1 summarizes these different values.

Table 1. Parameter values

$y=4$ General productivity	$r=0.01$ Pure discount rate
$\delta=0.15$ Job-specific shock arrival rate	$\beta=0.5$ Workers' share of surplus
$\underline{\epsilon}=0, \bar{\epsilon}=2$ Parameters of the distribution	$\eta(\theta)=0.5$ Search elasticity of matching
$N=1$ Total population	$t=0.07$ Time commuting cost
$w_U=2$ Unemployment benefit	$\tau=0.1$ Pecuniary commuting cost
$c=2$ Cost of a vacant job	

6.1 Steady-state equilibrium

Let us calculate the steady-state equilibrium for the non-spatial ($t = 0$) and the spatial models ($t > 0, \tau > 0$) using the parameter values given in Table 1. The numerical results of these two equilibria are displayed in Table 2.

Table 2. Steady-state equilibrium

	<i>Non-spatial model ($t = 0$)</i>	<i>Spatial model ($t > 0, \tau > 0$)</i>
u^* (%)	6.07	8.11
θ^*	1.77	1.69
v^* (%)	10.77	13.71
$\tilde{\epsilon}^*$	1.15	1.53
$\epsilon^*(x)$	-	$2 - 0.51x$
$w_L^*(\epsilon)$	$4.69 + 0.5\epsilon$	$(4.69 + 0.5\epsilon) / (0.86 - 0.068\epsilon)$
$I_F(\epsilon)$	$-3.59 + 3.13\epsilon$	$-19.96 + 6.25\epsilon + 146.83 / (12.60 + \epsilon)$
$W_L^*(\epsilon)$	$4.69 + 0.5\epsilon$	$2.95 + 1.48\epsilon + 7.30 \log[0.89 + 0.07\epsilon]$
W_U^*	2	1.95

The unemployment rate u^* and the reservation productivity $\tilde{\epsilon}^*$ are lower and the job-creation rate θ^* is higher in the non-spatial model. Indeed, the urban space through commuting costs and land rent creates additional frictions in the labor market that lead to these differences. Let us understand more closely these differences.

Figure 1 displays the assignment rule $\epsilon^*(x) = 2 - 0.51x$, which establishes a perfect negative correlation between the urban and the productivity spaces. This provides evidence of the double differentiation of workers in both the productivity and urban spaces.

[Insert Figure 1 here]

To better understand the differences between the two models, let us examine the wage setting. In the non-spatial model, the wage, given by $4.69 + 0.5\epsilon$ varies between $w_L^*(\tilde{\epsilon}^*) = 5.27$ and $w_L^*(\bar{\epsilon}) = 5.69$ while the total productivity varies between $y + \tilde{\epsilon}^* = 5.15$ and $y + \bar{\epsilon} = 6$. This means that firms keep some currently unprofitable jobs occupied. In fact, it is easy to verify that all jobs with a productivity $\epsilon \leq 1.38$ are not currently profitable because $y + \epsilon \leq w_L^*(\epsilon)$ while job with a $\epsilon \in]1.38, 2]$ are profitable. As observed by Pissarides (2000), this means that some firms keep some currently unprofitable jobs occupied because of the possibility that a job productivity might change, which enable firms to start production at the new productivity immediately after arrival, without having to pay the recruitment cost (which is here quite high since $c = 2$) and forgo production during search. This is confirmed by looking at $I_F(\epsilon)$, the lifetime expected utility of a firm with a filled job, which is calculating using (42) and given by $-3.59 + 3.13\epsilon$. It is easy to check that it is always strictly positive for all $\epsilon \in [\tilde{\epsilon}^*, \bar{\epsilon}]$. So even if jobs with an $\epsilon \in [1.15, 1.38[$ are not currently profitable, firms keep them because their lifetime expected utility $I_F(\epsilon)$ are strictly positive in this interval. This is even more true in the spatial model. Indeed, because firms need to compensate for spatial costs, wages are always higher but productivity is the same, and it is easy to verify that, in our example, all jobs are currently unprofitable since $w_L^*(\epsilon) > y + \epsilon, \forall \epsilon \in [\tilde{\epsilon}^*, \bar{\epsilon}]$. However, in the same interval, it is also easy to check that the lifetime expected utility $I_F(\epsilon)$ is strictly positive. Figure 2a displays the wage distribution in the urban space, showing that workers living further away have a lower wage because they also have a lower productivity. Figure 2b confirms what we said before. The spatial wage $w_L^*(\epsilon)$ (solid curve) is always higher than the non-spatial one $w_L^{NS}(\epsilon)$ (dotted curve) because of the need for firms to spatially compensate workers for commuting and land costs. Figure 2c shows that, in the spatial model, $I_F(\tilde{\epsilon}^*) = 0$ and then is always strictly positive for any $\epsilon > \tilde{\epsilon}^*$. Finally, Figure 2d compares the two expected lifetime utilities $I_F(\epsilon)$ (solid and dotted curves for the spatial and non-spatial, respectively) and shows that it is always higher in the spatial model for $\epsilon \in [\tilde{\epsilon}^*, \bar{\epsilon}]$ because wages are much lower. To summarize, when comparing the spatial and the non-spatial model, we find that, because spatial wages are higher and there are more frictions since workers are differentiated both in the productivity and urban spaces, the

unemployment rate as well as the reservation productivity are higher and job creation lower.

[Insert Figures 2a, 2b, 2c, 2d here]

Let us now focus on the land market. Figure 3a shows that the instantaneous utility of employed workers is decreasing with the distance to jobs because workers who live further away have a lower productivity and thus earn a lower wage. Figure 3b compares the equilibrium instantaneous utilities in the spatial model (solid curve), given by $W_L^*(\epsilon)$, and in the non-spatial model (dotted curve), given by $w_L^*(\epsilon)$. It is not clear which utility is higher because, on the one hand, spatial wages are higher but workers incur commuting and land costs. In Figure 3b, one sees that for low values of ϵ , the non-spatial utility is higher while we have the reverse for high productivity values. This is quite intuitive because when ϵ is low, wage differences are not too high but spatial costs exist, and thus the non-spatial utility has a higher value. For high ϵ , we have the reverse because the wage difference is sufficiently high to compensate the spatial cost difference. This is because the spatial wage (23) is non-linear in ϵ . Finally, in Figure 4, we have plotted the land rent for type- $\tilde{\epsilon}$ workers. It shows that the land rent is decreasing with distance to jobs.

[Insert Figures 3a, 3b, 4 here]

6.2 Comparative statics

We would like to pursue our analysis of the interaction between land and labor markets by analyzing the impact of the key spatial variable, commuting cost t , on the equilibrium labor market variables, u^* , $\tilde{\epsilon}^*$ and θ^* . The effects are complex since t directly affects the land market through the land rent and the instantaneous utility $W_L(\epsilon)$ and W_U but also indirectly affects the labor market through the wage. Figures 5a, 5b and 5c display the comparative static results of the impact of an increase in t on u^* , $\tilde{\epsilon}^*$ and θ^* . The relationships between u^* and t and between $\tilde{\epsilon}^*$ and t are clearly increasing while the one between θ^* and t is decreasing. This should not be a surprise since higher commuting costs lead to an increase in wages (firms need to compensate more workers for their spatial costs), which makes travelling to the job center even more costly. As a result, land rents increase and firms must compensate even more workers. Therefore, firms enter less in the labor market, thus creating less jobs, which decreases θ^* . Also since wages have increased, $I_L(\epsilon)$ the lifetime expected value of employment increases for each job ϵ , which implies that $\tilde{\epsilon}^*$, the reservation productivity below which jobs are destroyed, must increase for the condition $I_L(\tilde{\epsilon}^*) = I_U$

to be satisfied. Because more marginal jobs are destroyed and less jobs are created, the equilibrium unemployment u^* is reduced following an increase in t .

[Insert Figures 5a, 5b, 5c here]

One can also notice that the equilibrium values are quite sensitive to a variation in the commuting cost t . Indeed, looking at Figures 5a, 5b, and 5c, an increase in the commuting cost from $t = 0$ to $t = 0.15$ leads to an increase in the unemployment rate from 6.07% to 10.59%, an increase in the reservation productivity from 1.15 to 1.995, and a decrease in the job-creation rate from 1.77 to 1.6 (which corresponds to an increase in the vacancy rate from 10.77% to 16.91%). These are important effects due to the fact that the interaction between land and labor markets tends to amplify the effect of one variable on the other. To illustrate this point, let us consider an increase in the unemployment benefit w_U on u^* , $\tilde{\epsilon}^*$ and θ^* both in the spatial and non-spatial models. In both models, we obtain without surprise that an increase in w_U leads to an increase in u^* and $\tilde{\epsilon}^*$ and a decrease in θ^* . What is interesting is the difference in the values of the endogenous variables. Table 3 reports these values for $w_U = 0, 1, 2, 3, 4$.

Table 3. Variation of the unemployment benefit w_U

	<i>Non-spatial model ($t = 0$)</i>				<i>Spatial model ($t > 0, \tau > 0$)</i>			
	$u^*(\%)$	θ^*	$v^*(\%)$	$\tilde{\epsilon}^*$	$u^*(\%)$	θ^*	$v^*(\%)$	$\tilde{\epsilon}^*$
$w_U = 0$	4.12	2.72	11.20	0.94	5.58	2.63	15.17	1.32
$w_U = 1$	4.95	2.25	11.12	1.04	6.76	2.16	14.61	1.42
$w_U = 2$	6.07	1.77	10.77	1.15	8.11	1.69	13.71	1.53
$w_U = 3$	7.69	1.31	10.04	1.27	10.07	1.22	12.33	1.65
$w_U = 4$	10.34	0.84	8.72	1.41	13.32	0.77	10.21	1.79

It is easy to see that the values of the endogenous variables are much higher in the spatial model than in the non-spatial one because of amplifying effects due to the interaction between the land and labor markets. Indeed, when $w_U = 0$, the unemployment rate is 4.12% and the vacancy rate is 11.20% in the non-spatial model while, in the spatial model, these figures are 5.58% and 15.17%, respectively. When $w_U = 4$, the differences are even larger, with for example 3 percent more unemployment in the spatial model. This is because, in the spatial model, each labor variable, like for example w_U , has an impact on both markets.

Indeed, the unemployment benefit directly affects the wage since, at given θ and L , higher w_U implies higher wages, which in turn affects the time cost of travelling. This increases the competition in the land market since the access to the job center becomes more valuable, which in turn increases the wage since firms need to compensate more for spatial costs in order to induce workers to take a job. These amplifying effects lead to higher unemployment rate and reservation productivity and lower labor market tightness in the spatial model.

7 Conclusion

Most people live and work in cities, particularly in developed countries. Since the urban economy is almost the microcosm of the national economy, understanding its problems requires the application of many economic subdisciplines. In the present paper, we focus on one of them, namely the labor market. We embed a search-matching model a la Mortensen-Pissarides into a land market a la Alonso-Fujita. To the best of our knowledge, this is the first paper that brings together a search matching model with endogenous job destruction and a land market in a unified framework. We characterize the general equilibrium of this economy. In the urban space, workers with high productivities and wages reside close to jobs, have low commuting costs but pay high land rents. In the productivity space, ex ante identical workers are ex ante heterogenous and we obtain a wage distribution. In this respect, workers are heterogenous in both the urban and productivity spaces. We show that, in equilibrium, there is a perfect negative correlation between these two spaces since high-productivity workers are also those who occupy locations close to jobs. We also show that in the bargaining process, there is a spatial element in the wage setting since firms need to compensate workers for their spatial costs. Compared to the non-spatial model, the unemployment rate and the reservation productivity are lower and the job-creation rate is higher in the non-spatial model because the urban space through commuting costs and land rent create additional frictions in the labor market.

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APPENDIX

Proof of Lemma 1.

We start the demonstration of this Lemma by using a general c.d.f $G(\epsilon)$.

The sharing rule (21) implies in particular that:

$$I_L(\bar{\epsilon}) - I_U = \frac{\beta}{1-\beta} I_F(\bar{\epsilon}) \quad (37)$$

Note that the sharing rule (21) can also be written as

$$(1-\beta) I_L(\epsilon) - \beta I_F(\epsilon) = (1-\beta) I_U \quad (38)$$

Now, using (37) and (17), we have:

$$\begin{aligned} rI_U &= w_U - \tau x - R(x) + \theta q(\theta) [I_L(\bar{\epsilon}) - I_U] \\ &= w_U - \tau x - R(x) + \theta q(\theta) \frac{\beta}{1-\beta} I_F(\bar{\epsilon}) \\ &= w_U - \tau x - R(x) + \frac{\beta}{1-\beta} c\theta \end{aligned} \quad (39)$$

Now multiplying the asset equation $I_F(\epsilon)$ defined in (14) by β , we obtain:

$$r\beta I_F(\epsilon) = \beta [y + \epsilon - w_L(\epsilon)] + \beta\delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s) - \delta\beta I_F(\epsilon)$$

Also, multiplying the asset equation $I_L(\epsilon)$ defined in (13) by $(1-\beta)$, we get:

$$\begin{aligned} r(1-\beta) I_L(\epsilon) &= (1-\beta) [w_L(\epsilon) (1-tx) - \tau x - R(x)] \\ &\quad + (1-\beta) \delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_L(s) dG(s) + (1-\beta) \delta G(\tilde{\epsilon}) I_U \\ &\quad - \delta (1-\beta) I_L(\epsilon) \end{aligned}$$

Subtracting the second equation from the first, we obtain:

$$\begin{aligned} &r [(1-\beta) I_L(\epsilon) - \beta I_F(\epsilon)] \\ &= (1-\beta) [w_L(\epsilon) (1-tx) - \tau x - R(x)] - \beta [y + \epsilon - w_L(\epsilon)] \\ &\quad + (1-\beta) \delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_L(s) dG(s) - \beta\delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s) - \delta [(1-\beta) I_L(\epsilon) - \beta I_F(\epsilon)] \\ &\quad + (1-\beta) \delta G(\tilde{\epsilon}) I_U \end{aligned}$$

Observing that

$$(1 - \beta) \delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_L(s) dG(s) - \beta \delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s) = \delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} [(1 - \beta) I_L(s) - \beta I_F(s)] dG(s)$$

then using the sharing rule (38), we obtain:

$$\begin{aligned} & r(1 - \beta) I_U \\ &= (1 - \beta) [w_L(\epsilon) (1 - tx) - \tau x - R(x)] - \beta [y + \epsilon - w_L(\epsilon)] \\ & \quad + \delta (1 - \beta) I_U \int_{\tilde{\epsilon}}^{\bar{\epsilon}} G(s) - \delta (1 - \beta) I_U + (1 - \beta) \delta G(\tilde{\epsilon}) I_U \end{aligned}$$

Observing that $\int_{\tilde{\epsilon}}^{\bar{\epsilon}} G(s) = 1 - G(\tilde{\epsilon})$, we get:

$$\begin{aligned} & r(1 - \beta) I_U \\ &= (1 - \beta) [w_L(\epsilon) (1 - tx) - \tau x - R(x)] - \beta [y + \epsilon - w_L(\epsilon)] \\ & \quad + \delta (1 - \beta) I_U [1 - G(\tilde{\epsilon}) - 1 + G(\tilde{\epsilon})] \end{aligned}$$

which is equivalent to:

$$r(1 - \beta) I_U = (1 - \beta) [w_L(\epsilon) (1 - tx) - \tau x - R(x)] - \beta [y + \epsilon - w_L(\epsilon)]$$

Finally, using the value of rI_U in (39), we obtain

$$(1 - \beta) [w_U - \tau x - R(x)] + \beta c \theta = (1 - \beta) [w_L(\epsilon) (1 - tx) - \tau x - R(x)] - \beta [y + \epsilon - w_L(\epsilon)]$$

which after some manipulations leads to:

$$w_L(\epsilon) = \frac{(1 - \beta) w_U + \beta (y + \epsilon + c \theta)}{1 - (1 - \beta) tx} \quad (40)$$

Now we need to use the assignment rule given by (7), which assumes a uniform distribution in $G(\epsilon)$. The assignment rule (7) implies

$$x(\epsilon) = \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}} \right) L \quad \text{for } 0 \leq x \leq L$$

then the wage is given by:

$$w_L(\epsilon) = \frac{(1 - \beta) w_U + \beta (y + \epsilon + c \theta)}{1 - (1 - \beta) tL (\bar{\epsilon} - \epsilon) / (\bar{\epsilon} - \tilde{\epsilon})}$$

which is (23). For this wage to be strictly positive, it has to be that:

$$(1 - \beta) tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \tilde{\epsilon})} < 1$$

This has to be true for all $\epsilon \in [\tilde{\epsilon}, \bar{\epsilon}]$, which means that this condition must be:

$$(1 - \beta) tL < 1$$

This condition is captured by (22) since

$$\frac{\beta (\bar{\epsilon} - \tilde{\epsilon})}{(1 - \beta) w_U + \beta (y + c\theta + \bar{\epsilon})} < 1$$

Furthermore, we have:

$$w'(\epsilon) = \frac{\beta \left[1 - (1 - \beta) tL \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}} \right) \right] - [(1 - \beta) w_U + \beta (y + \epsilon + c\theta)] tL \left(\frac{1 - \beta}{\bar{\epsilon} - \tilde{\epsilon}} \right)}{\left[1 - (1 - \beta) t \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}} \right) L \right]^2}$$

For $w'(\epsilon) > 0$, it must be that

$$\beta \left[1 - (1 - \beta) tL \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}} \right) \right] > [(1 - \beta) w_U + \beta (y + \epsilon + c\theta)] (1 - \beta) \left(\frac{tL}{\bar{\epsilon} - \tilde{\epsilon}} \right)$$

which is equivalent to

$$(1 - \beta) tL < \frac{\beta (\bar{\epsilon} - \tilde{\epsilon})}{(1 - \beta) w_U + \beta (y + c\theta + \bar{\epsilon})}$$

which is captured by condition (22). ■

Determination of the job-creation and job-destruction conditions

We derive the results for a general c.d.f. $G(\epsilon)$ and then determine them for the uniform distribution. By plugging the wage $w_L(\epsilon)$ given by (23) in (14) and using the fact that $I_V = 0$, we obtain:

$$(r + \delta) I_F(\epsilon) = \frac{(1 - \beta) (y + \epsilon) \left[1 - tL \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}} \right) \right] - (1 - \beta) w_U - \beta c\theta}{\left[1 - (1 - \beta) tL \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}} \right) \right]} + \delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s) \quad (41)$$

First, evaluating (41) at $\epsilon = \tilde{\epsilon}$, we obtain:

$$(r + \delta) I_F(\tilde{\epsilon}) = \frac{(1 - \beta) (y + \tilde{\epsilon}) (1 - tL) - (1 - \beta) w_U - \beta c\theta}{[1 - (1 - \beta) tL]} + \delta \int_{\tilde{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s)$$

Now, subtracting this equation from (41) and noting that $I_F(\tilde{\epsilon}) = 0$, we get:

$$(r + \delta) I_F(\epsilon) = \frac{(1 - \beta) (y + \epsilon) \left[1 - tL \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}} \right) \right] - (1 - \beta) w_U - \beta c\theta}{\left[1 - (1 - \beta) tL \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}} \right) \right]} - \frac{(1 - \beta) (y + \tilde{\epsilon}) (1 - tL) - (1 - \beta) w_U - \beta c\theta}{[1 - (1 - \beta) tL]}$$

Replacing this value into the $I_F(s)$ in the integral in (41) yields:

$$\begin{aligned}
(r + \delta) I_F(\epsilon) &= \frac{(1 - \beta)(y + \epsilon) \left[1 - tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \tilde{\epsilon})} \right] - (1 - \beta)w_U - \beta c \theta}{\left[1 - (1 - \beta)tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \tilde{\epsilon})} \right]} \\
&+ \frac{\delta}{(r + \delta)} \int_{\tilde{\epsilon}}^{\bar{\epsilon}} \frac{(1 - \beta)(y + \epsilon) \left[1 - tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \tilde{\epsilon})} \right] - (1 - \beta)w_U - \beta c \theta}{\left[1 - (1 - \beta)tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \tilde{\epsilon})} \right]} dG(\epsilon) \\
&- \frac{\delta}{(r + \delta)} \int_{\tilde{\epsilon}}^{\bar{\epsilon}} \frac{(1 - \beta)(y + \tilde{\epsilon})(1 - tL) - (1 - \beta)w_U - \beta c \theta}{[1 - (1 - \beta)tL]} dG(\epsilon)
\end{aligned} \tag{42}$$

Let us now determine the conditions for job creation and job destruction. For job creation, we evaluating (42) at $\epsilon = \bar{\epsilon}$ and obtain:

$$(r + \delta) I_F(v) = (1 - \beta) \frac{(y + \bar{\epsilon}) \beta tL + (\bar{\epsilon} - \tilde{\epsilon})(1 - tL) + [(1 - \beta)w_U + \beta c \theta] tL}{[1 - (1 - \beta)tL]}$$

Combining this equation with (17), we obtain

$$\frac{c}{q(\theta)} = \left(\frac{1 - \beta}{r + \delta} \right) \frac{(y + \bar{\epsilon}) \beta tL + (\bar{\epsilon} - \tilde{\epsilon})(1 - tL) + [(1 - \beta)w_U + \beta c \theta] tL}{[1 - (1 - \beta)tL]}$$

which is (24). This is the job-creation condition.

Let us now determine the job-destruction condition. For that, we evaluate (42) at $\epsilon = \tilde{\epsilon}$ and using the reservation rule $I_F(\tilde{\epsilon}) = 0$, we get:

$$(y + \tilde{\epsilon})(1 - tL) - w_U - \frac{\beta c}{1 - \beta} \theta + \frac{\delta(1 - tL)}{(r + \delta)} \int_{\tilde{\epsilon}}^{\bar{\epsilon}} (\epsilon - \tilde{\epsilon}) dG(\epsilon) = 0 \tag{43}$$

Let us calculate this result for the case of a uniform distribution. We obtain:

$$(y + \tilde{\epsilon})(1 - tL) - w_U - \frac{\beta c}{1 - \beta} \theta + \frac{\delta(1 - tL)}{2(r + \delta)} (\bar{\epsilon} - \tilde{\epsilon}) = 0$$

which is (25). ■

Proof of Proposition 1

This proof follows that of Brueckner et al. (2002).

Existence: We establish existence by construction. In order for

$$\Psi_L(\epsilon(x), x, W_L^*(\epsilon(x))) = \max_{\epsilon} \Psi_L(\epsilon, x, W_L^*(\epsilon)),$$

it must be true that:

$$\frac{d\Psi_L(\epsilon, x, W_L^*(\epsilon))}{d\epsilon}\Big|_{\epsilon=\epsilon(x)} = 0 \quad (44)$$

We use this equation to solve for the unknown function $W_L^*(\epsilon)$, which ensures that the bid rent of a type $\epsilon(x)$ worker is maximal at location x . Using (40), the bid rent can be written as:

$$\begin{aligned} \Psi_L(\epsilon, x, W_L(\epsilon)) &= w_L(\epsilon) (1 - tx) - \tau x - W_L(\epsilon) \\ &= \left[\frac{(1 - \beta) w_U + \beta (y + \epsilon + c\theta)}{1 - (1 - \beta) tx} \right] (1 - tx) - \tau x - W_L(\epsilon) \end{aligned}$$

Thus (44) is equivalent to:

$$w'(\epsilon) (1 - tx) - W'_L(\epsilon) = 0 \quad (45)$$

that is

$$\frac{\beta (1 - tx)}{1 - (1 - \beta) tx} - W'_L(\epsilon) = 0 \quad (46)$$

Because we want to solve for $W_L^*(\epsilon)$, this equation must be rewritten in terms of ϵ . Since our mapping, given by (7), implies

$$x(\epsilon) = \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}} \right) L \quad \text{for } 0 \leq x \leq L$$

we have:

$$W'_L(\epsilon) = \frac{\beta [\bar{\epsilon} - \tilde{\epsilon} - tL (\bar{\epsilon} - \epsilon)]}{\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)} \quad (47)$$

Equation (47) constitutes a differential equation involving the unknown function $W_L^*(\cdot)$. Integrating, the solution is:

$$\begin{aligned} W_L^*(\epsilon) &= \int \frac{\beta [1 - tL (\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}})]}{1 - (1 - \beta) tL (\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \tilde{\epsilon}})} d\epsilon \\ &= \int \frac{\beta (\bar{\epsilon} - \tilde{\epsilon}) - \beta tL (\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)} d\epsilon \\ &= \beta (\bar{\epsilon} - \tilde{\epsilon}) \int \frac{1}{\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)} d\epsilon + \beta tL \int \frac{-(\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)} d\epsilon \\ &= \frac{\beta (\bar{\epsilon} - \tilde{\epsilon})}{(1 - \beta) tL} \log [\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)] \\ &\quad + \beta tL \left[- \left(\frac{\bar{\epsilon} - \epsilon}{(1 - \beta) tL} \right) - \frac{(\bar{\epsilon} - \tilde{\epsilon}) \log [\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)]}{[(1 - \beta) tL]^2} \right] + K \\ &= - \frac{\beta^2 (\bar{\epsilon} - \tilde{\epsilon})}{(1 - \beta)^2 tL} \log [\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)] - \frac{\beta tL (\bar{\epsilon} - \epsilon)}{(1 - \beta) tL} + K \end{aligned}$$

where K is a constant of integration. We thus have:

$$W_L^*(\epsilon) = -\frac{\beta^2 (\bar{\epsilon} - \tilde{\epsilon})}{(1 - \beta)^2 tL} \log [\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)] - \frac{\beta (\bar{\epsilon} - \epsilon)}{(1 - \beta)} + K \quad (48)$$

To verify that the above solution indeed maximizes $\Psi_L(\cdot)$, we need to check that the second-order condition holds at any solution to the first-order condition (46). This condition requires that $d^2\Psi_L/d\epsilon^2 < 0$, which implies from (46) that $W_L''(\epsilon) > 0$. By differentiating (47), we obtain:

$$W_L''(\epsilon) = \frac{\beta^2 (\bar{\epsilon} - \tilde{\epsilon}) tL}{[\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)]^2} > 0$$

Let us check that $W_L^*(\epsilon) > 0$. Using (47), this is equivalent to:

$$\frac{\bar{\epsilon} - \tilde{\epsilon} - tL (\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tL (\bar{\epsilon} - \epsilon)} > 0$$

We thus need to show that both

$$tL < \frac{\bar{\epsilon} - \tilde{\epsilon}}{\bar{\epsilon} - \epsilon} \text{ and } tL < \frac{1}{(1 - \beta)} \left(\frac{\bar{\epsilon} - \tilde{\epsilon}}{\bar{\epsilon} - \epsilon} \right)$$

But since $\frac{\bar{\epsilon} - \tilde{\epsilon}}{\bar{\epsilon} - \epsilon} < \frac{1}{(1 - \beta)} \left(\frac{\bar{\epsilon} - \tilde{\epsilon}}{\bar{\epsilon} - \epsilon} \right)$, this reduces to

$$tL < \frac{\bar{\epsilon} - \tilde{\epsilon}}{\bar{\epsilon} - \epsilon}$$

and this is true using condition (32).

Substituting the value of $W_L^*(\epsilon)$, defined in (48), into (45), and using the assignment rule (7), we obtain the equilibrium land rent $R_L^*(x)$ at the given $x \in [0, L]$, which equals:

$$\begin{aligned} R_L^*(x) = & \left[\frac{(1 - \beta) w_U + \beta \left(y + \bar{\epsilon} - \frac{\bar{\epsilon}}{L} + \frac{\tilde{\epsilon}}{L} x + c\theta \right)}{1 - (1 - \beta) tx} \right] (1 - tx) - \tau x - K \\ & + \frac{\beta^2 (\bar{\epsilon} - \tilde{\epsilon})}{(1 - \beta)^2 tL} \log [\bar{\epsilon} - \tilde{\epsilon} - (1 - \beta) tx (\bar{\epsilon} - \tilde{\epsilon})] + \frac{\beta}{(1 - \beta) L} (\bar{\epsilon} - \tilde{\epsilon}) x \end{aligned}$$

We can calculate the equilibrium utility of the unemployed. Indeed, by solving (10) using (6), we obtain

$$W_U^* = w_U - \tau N$$

which is equation (34). By plugging this value in (6), we get the equilibrium land rent $R_U^*(x)$ at the given $x \in [L, N]$, that is:

$$R_U^*(x) = \tau (N - x)$$

which is (36). Then, using $R_U^*(x)$ evaluated at $x = L$ and $R_L^*(x)$, defined by (49), evaluated at $x = L$, equation (9) can be written as:

$$K = \left[\frac{(1-\beta)w_U + \beta(y + \tilde{\epsilon} + c\theta)}{1 - (1-\beta)tL} \right] (1-tL) - \tau N \quad (49)$$

$$+ \frac{\beta^2(\bar{\epsilon} - \tilde{\epsilon})}{(1-\beta)^2 tL} \log \{(\bar{\epsilon} - \tilde{\epsilon}) [1 - (1-\beta)tL]\} + \frac{\beta(\bar{\epsilon} - \tilde{\epsilon})}{(1-\beta)}$$

Substituting this value of K in (48), we obtain $W_L^*(\epsilon)$. Finally, by plugging this value of $W_L^*(\epsilon)$ in (45), we obtain the city's equilibrium land-rent function $R^*(x)$.

By plugging the value of the constant K defined by (49) into (48) and in (49), we finally obtain:

$$W_L^*(\epsilon) = \beta \frac{(\bar{\epsilon} - \tilde{\epsilon})}{(1-\beta)tL} \log \left[\frac{\bar{\epsilon} - \tilde{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \tilde{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \tilde{\epsilon})} \right] - \tau N$$

$$+ \beta tL \frac{(\bar{\epsilon} - \epsilon/2)(1-\epsilon)}{(\bar{\epsilon} - \tilde{\epsilon})} + \frac{[(1-\beta)w_U + \beta(y + \epsilon + c\theta)](1-tL)}{1 - (1-\beta)tL}$$

and

$$R_L^*(x) = \frac{[(1-\beta)w_U + \beta(y + \bar{\epsilon}(1-1/L) + x\tilde{\epsilon}/L + c\theta)](1-tx)}{1 - (1-\beta)tx}$$

$$- \frac{[(1-\beta)w_U + \beta(y + \tilde{\epsilon} + c\theta)](1-tL)}{1 - (1-\beta)tL} + \tau(N-x)$$

$$+ \frac{\beta(\bar{\epsilon} - \tilde{\epsilon})}{(1-\beta)L} \left[\frac{\beta}{(1-\beta)t} \log \left[\frac{1 - (1-\beta)tx}{1 - (1-\beta)tL} \right] - 1 + x \right]$$

which are (33) and (35), respectively.

Let us now calculate $R_L^*(x)$. For that, we use (4), that is:

$$R_L^*(x) = w_L(\epsilon)(1-tx) - \tau x - W_L(\epsilon)$$

with the assignment rule (7), i.e. $\epsilon(x) = \bar{\epsilon} - \left(\frac{\bar{\epsilon}-\tilde{\epsilon}}{L}\right)x$, that links negatively ϵ and x , $\epsilon'(x) < 0$, We have:

$$\frac{\partial R_L^*(x)}{\partial x} = w'(\epsilon)\epsilon'(x)(1-tx) - w_L(\epsilon)t - \tau - W_L'(\epsilon)\epsilon'(x)$$

$$= \epsilon'(x)[w'(\epsilon)(1-tx) - W_L'(\epsilon)] - w_L(\epsilon)t - \tau$$

$$= -w_L(\epsilon)t - \tau$$

which is always negative. To obtain the last equality, we use (45), which gives the value of $W_L'(\epsilon)$. Differentiating this expression leads to:

$$\frac{\partial R_L^*(x)}{\partial x} = -w'(\epsilon)\epsilon'(x)t \geq 0 \quad (50)$$

Since the land rent function is the upper envelope of downward-sloping bid-rent curves, we know that it must be downward sloping and convex. Convexity of $R_L^*(x)$ is clear from inspection of (50), and we have shown that the slope is negative, i.e. $R_L^{*'}(x) < 0$.

Uniqueness: To show uniqueness of the equilibrium, suppose that productivity types ϵ_0 and $\epsilon_1 > \epsilon_0$ reside at distances x_0 and x_1 where $x_1 > x_0$. This pattern differs from the mapping in (7). For workers of skill type ϵ_0 to reside at the close-in location x_0 , they must outbid workers of type ϵ_1 for land at this location. But since the bid rent curve of type ϵ_0 is flatter than that of type ϵ_1 , it follows that type ϵ_0 will also outbid type ϵ_1 for land at the more-distant location x_1 , where that type is assumed to live. This is a contradiction, and it rules out any location pattern in which productivity type and location distance are not perfectly correlated. ■

Figure 1. Equilibrium assignment rule

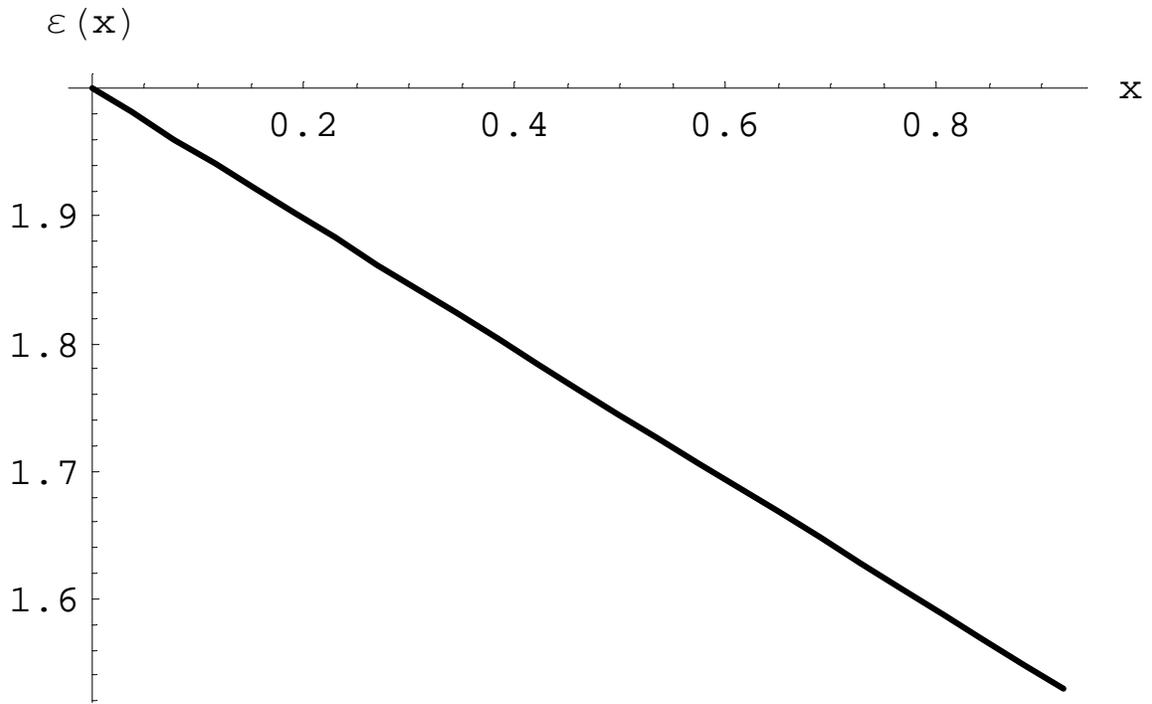


Figure 2a. Equilibrium wage distribution in the urban space

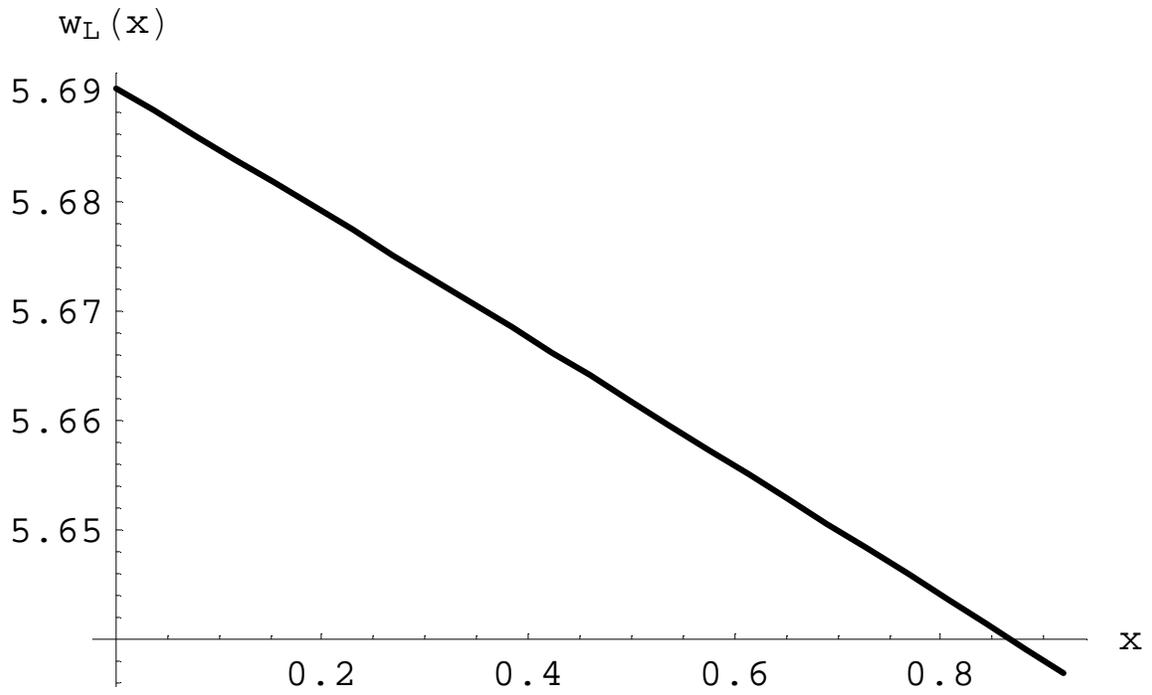


Figure 2b. Equilibrium wage distribution for the spatial and non-spatial models in the productivity space

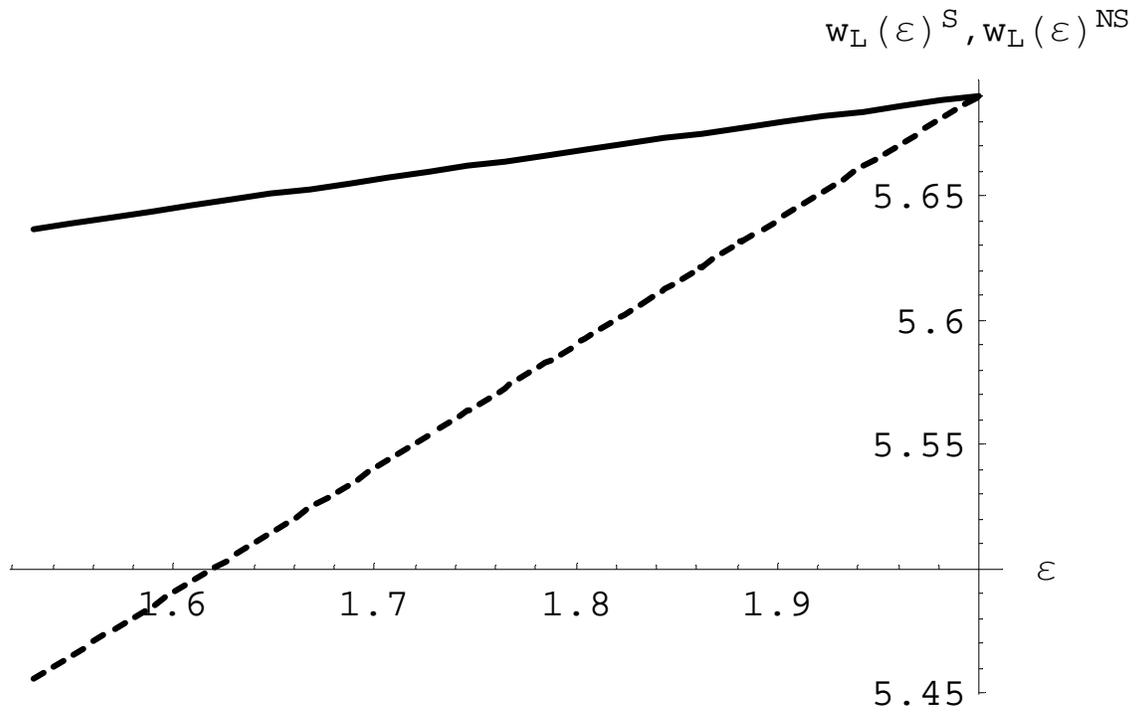


Figure 2c. Equilibrium distribution of lifetime expected utility of firms with filled jobs in the productivity space

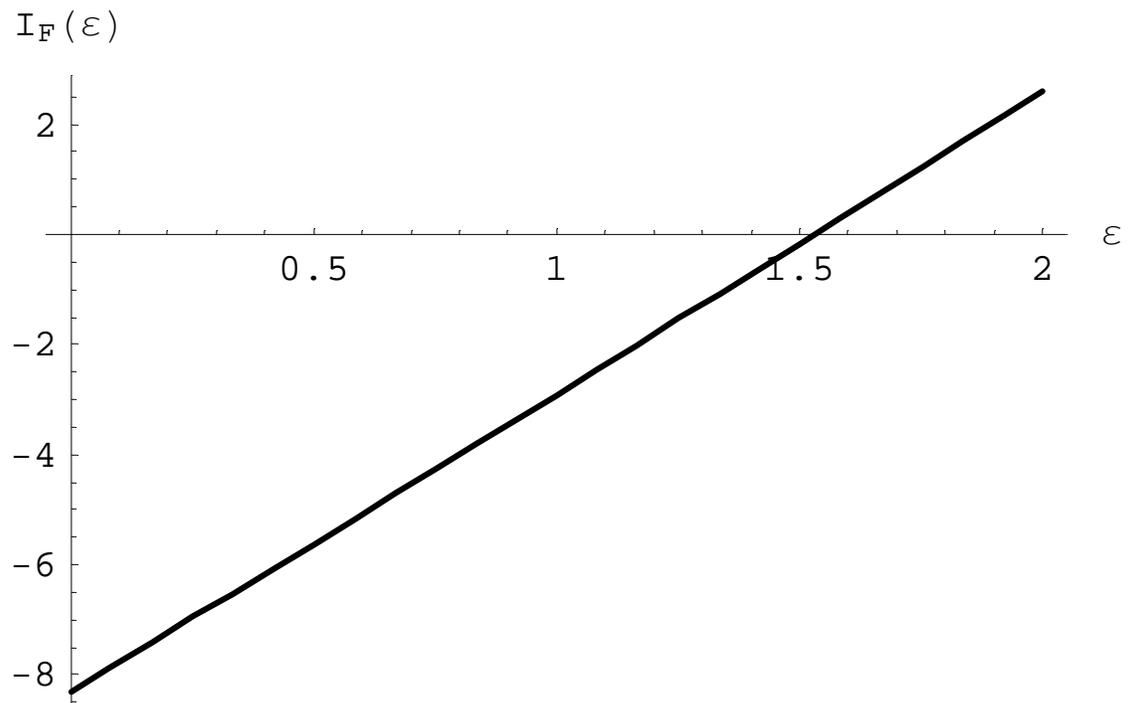


Figure 2d. Comparison of lifetime expected utilities of firms with filled jobs between the spatial and non-spatial models

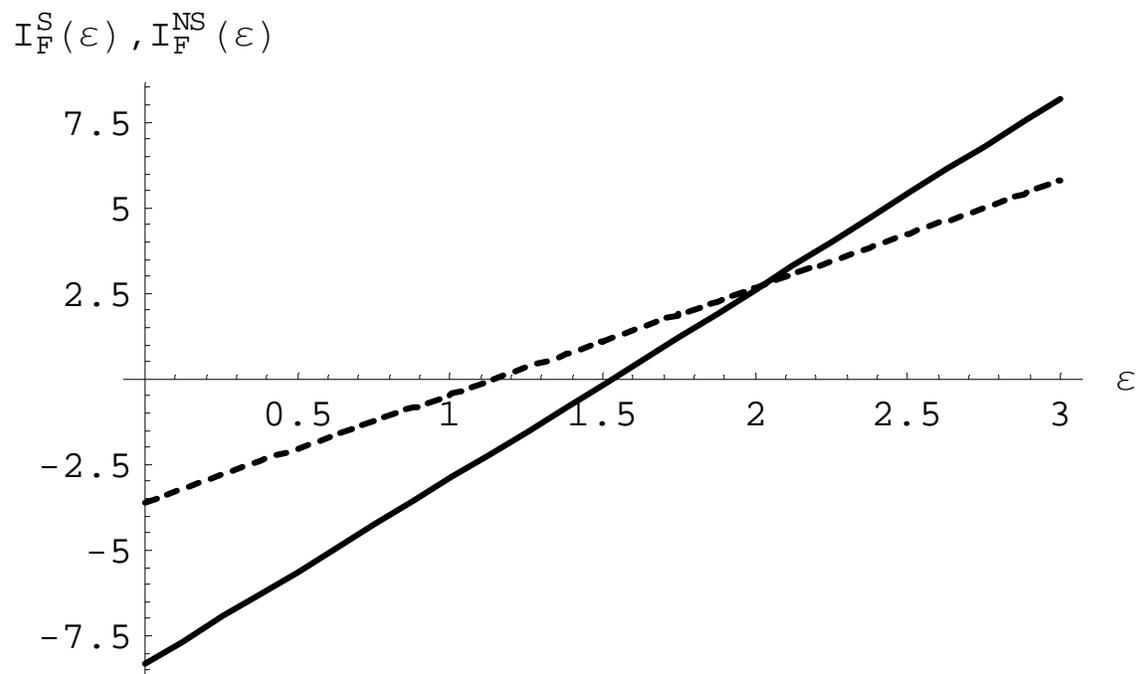


Figure 3a. Equilibrium instantaneous utility distribution of the employed workers in the urban space

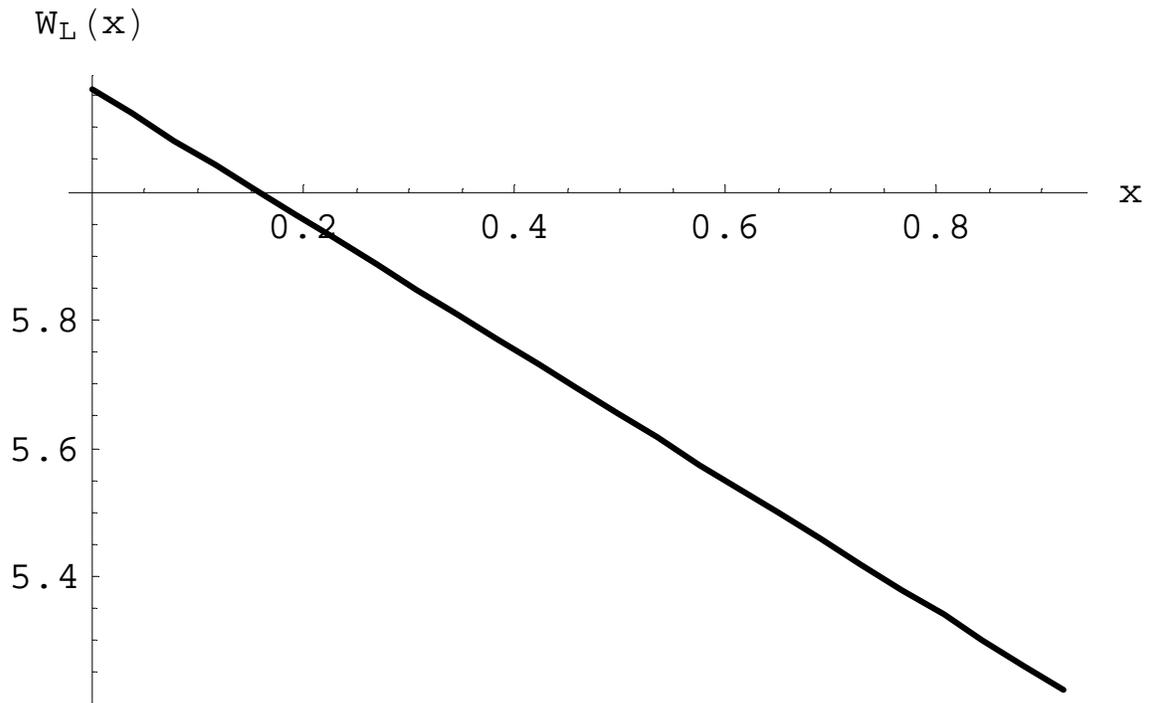


Figure 3b. Comparison of equilibrium instantaneous utilities between the spatial and non-spatial models

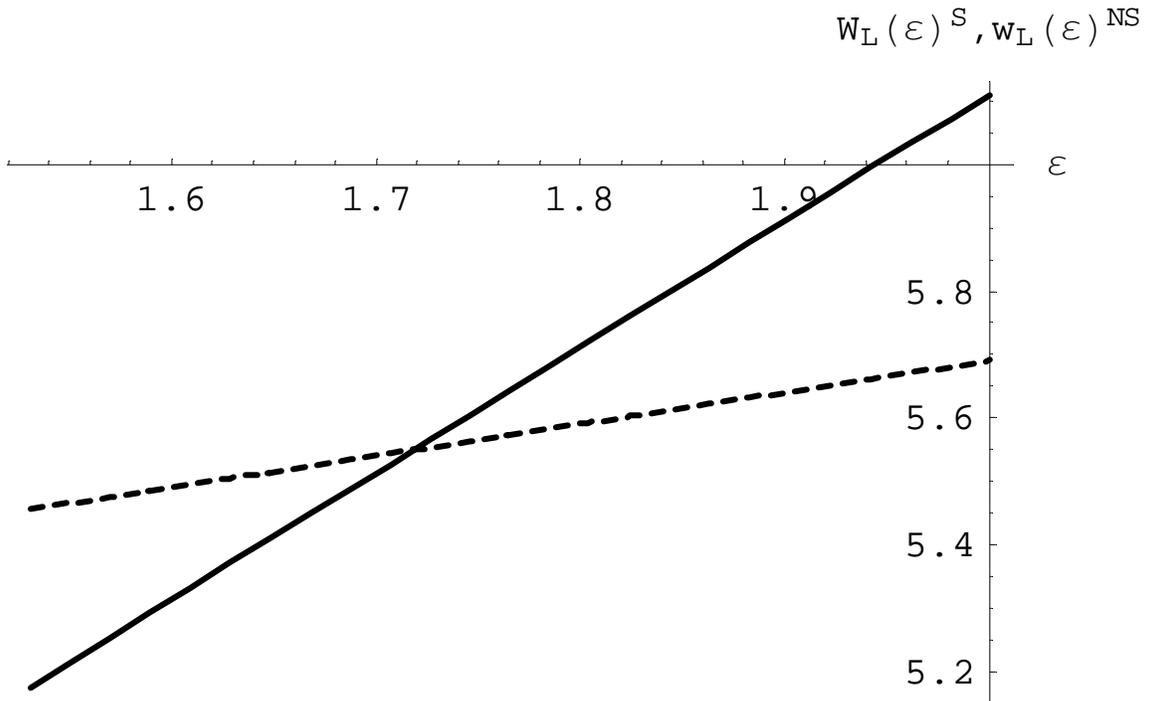


Figure 4. Equilibrium land rent of the employed workers of type $\tilde{\varepsilon}$

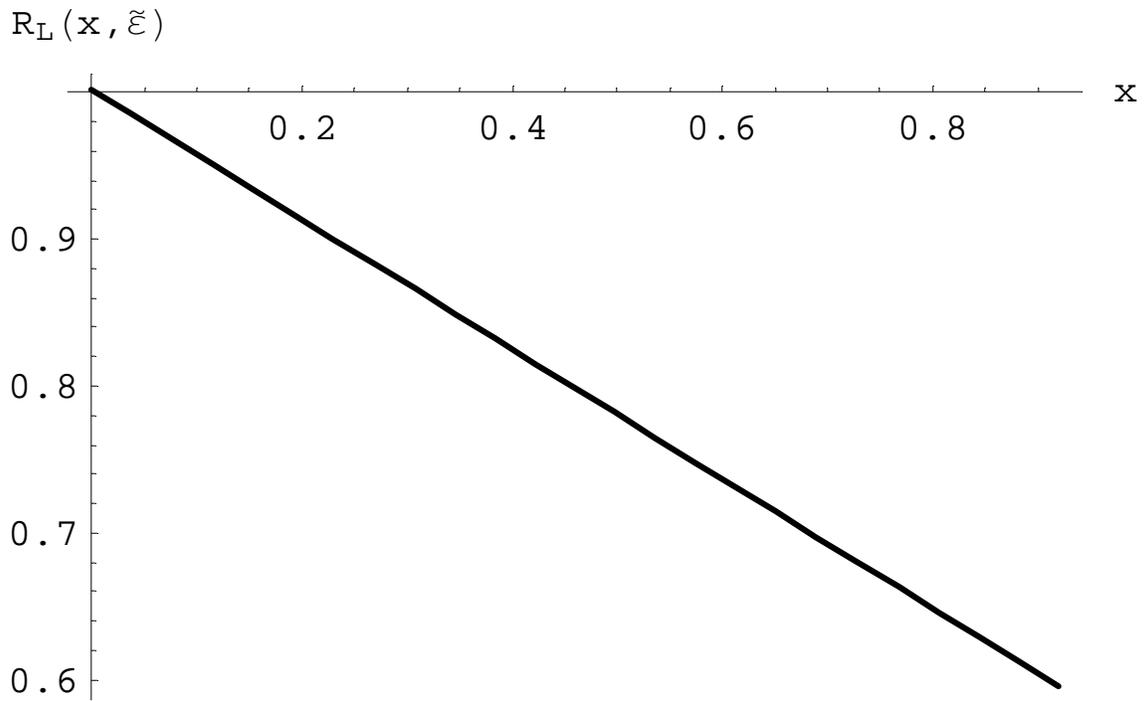


Figure 5a. Impact of the commuting cost on the equilibrium unemployment

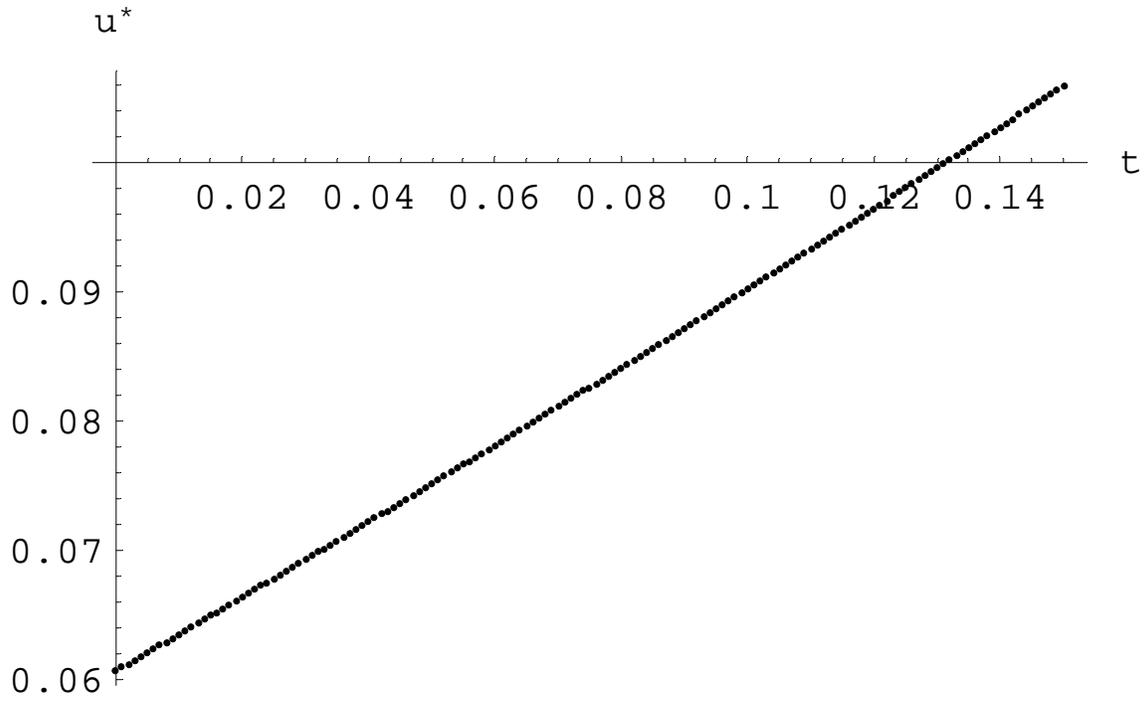


Figure 5b. Impact of the commuting cost on the equilibrium labor-market tightness

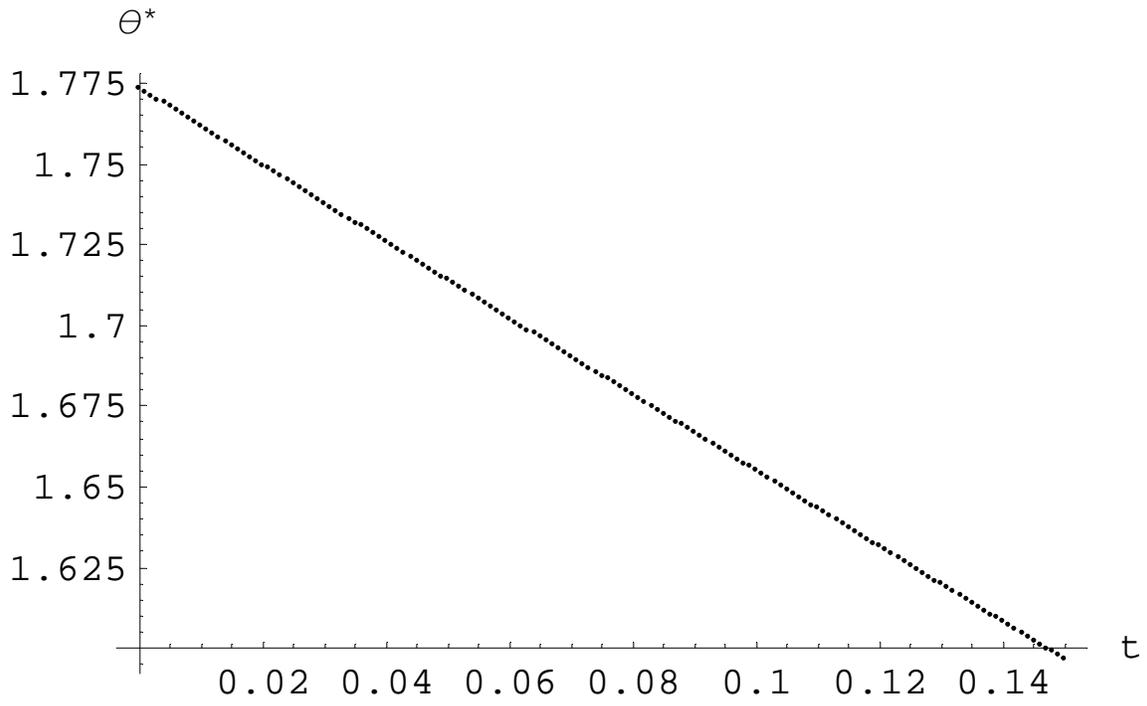


Figure 5c. Impact of the commuting cost on the equilibrium reservation productivity

