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ABSTRACT

Mismeasured Household Size and Its Implications for the Identification of Economies of Scale*

We consider the possibility that demographic variables are measured with errors which arise because household surveys measure demographic structures at a point-in-time, whereas household composition evolves throughout the survey period. We construct and estimate sharp bounds on household size and find that the degree of these measurement errors is non-trivial. However, while these errors have the potential to resolve the Deaton-Paxson paradox, they fail to do so.

JEL Classification: J12, C14

Keywords: migration, measurement error, semi-parametric bounds, economies of scale

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1 Introduction

In this paper, we consider the possibility that household demographic variables are measured with error. Such measurement errors may arise because the bulk of consumption and income surveys in the world ask households to report their demographic structures at a point-in-time. These data are then typically used by researchers to proxy for the household's structure over the duration of the survey's recall period. However, if the household undergoes any number of demographic changes during the recall period then information reported at the time of data collection may deviate substantially from the household's average demographic structure during the recall period.

To provide readers with some notion of how fluid household structures may induce measurement errors in demographic variables, we use data from the BASIS Panel in El Salvador and the Panel Study of Income Dynamics (PSID) in the United States to calculate the change in household size across survey years.[1] Figures 1 and 2 show changes in El Salvador and the United States, respectively. The figures suggest that household structures are fluid. In El Salvador, fewer than 50% of households experienced no change in household size across survey years. In the United States, this figure rises to about 83%. This is consistent with evidence from intra-year panel surveys such as Gibson (2001) who shows that the correlation in household size (in adult equivalents) across a seven month period is 0.75 for rural households and 0.65 for urban households.

[1] The BASIS panel covers the years 1997, 1999 and 2001. The PSID data covers the years 1990, 1991, 1992 and 1993. It is important to point out that the difference in the BASIS data is across two years, whereas the difference in the PSID is across one year.

Unfortunately, typical surveys do not collect information on the household's demographic structure at all points-in-time during the recall period.[2] Consequently, researchers are unable to accurately calculate the household's average size over this period. In this paper, we try to gain some insight into the extent of this problem. We impose weak assumptions on the demographic processes which took place within the household during the survey period and then use these assumptions to derive bounds on household size. The derived bounds are sharp which is to say that they cannot be improved without stronger assumptions. We compute these bounds using the BASIS panel from El Salvador, a country in which a large amount of trans-national migration takes place.

Our computations have ramifications for a paradox posed by Deaton and Paxson (1998) (DP). The authors develop a test for economies of scale within the household which hinges on the observation that intra-household public goods become cheaper as households become larger; in effect, larger households are richer conditional on *per capita* expenditures. Accordingly, if we condition on *per capita* expenditures, we should observe that the consumption of goods with sufficiently high income effects (such as food in developing countries) increases with household size. However, DP present comprehensive evidence that directly contradicts this prediction. Moreover, the contradiction is the strongest for the poorest countries in which we would expect the income elasticity of food to be the highest.[3] Measurement errors in household size have the potential to resolve the DP paradox.

We proceed to answer two questions. First, how large are the measurement errors in house-

[2]Exceptions to this would be surveys that are used to measure the income elasticity of calories such as those used by Gibson and Rozelle (2002). Such surveys use a roster of meals for every day that consumption is measured. As pointed out by a referee, such surveys should adequately account for changing demographics within the recall period because they are filled out by all adults who are in the household during the recall period.

[3]Logan (2008) provides similar evidence using historical data from the US.

hold size? Second, can these measurement errors resolve the DP paradox?

2 The Problem

We assume that the household's decision process unfolds in continuous time. We let $N(s)$ denote the household's size at time s. We remain agnostic about the household's underlying decision process and we assume that $N(s) \geq 1$ for all s.

Survey instruments only collect data at discrete intervals such as yearly. As a consequence, researchers do not observe the quantity $N(s)$ for all s in any given time interval. This forces researchers to summarize $N(s)$ over discrete time intervals. One such interval could be $[t-1, t]$. However, a more reasonable interval would be $[t-\Delta, t]$ for $0 \leq \Delta \leq 1$. The reason is that many variables in surveys measure quantities over recall periods which can range from seven days to one year. The parameter Δ can be viewed as the survey's recall period.

In practice, surveys have many recall periods. Food typically has a recall period of one month or sometimes less and more durable items have recall periods of six months to a year. If one multiplies the recall period for a given item with its budget share and then sums across all items in the budget then one obtains what we call the *effective recall period*. In the consumption survey that we consider, the effective recall period is about 3 months.[4]

To help fix ideas, we define the object

$$N_t^* \equiv E[N(s)|s \in [t-\Delta, t]]$$

[4]Duflo and Banerjee (2007) report that food occupies between 50% and 75% of a household's budget in thirtteen poor countries. Given that food typically has a recall period of one month, this suggests that most other countries would have effective recall periods that are less than three months and probably more on the order on one to two months.

which denotes the average of the household's size over the interval $[t - \Delta, t]$.[5] Because most household surveys do not permit the precise measurement of N_t^*, researchers typically proxy for the household's size over the survey period with $N_t = N(t)$ where t is the time that the survey was administered. Unfortunately, this can be problematic as the household's structure often changes during the survey period and, so N_t^* and N_t may deviate from each other. When this occurs, household size will be measured with error which can be written as

$$\epsilon_t = N_t^* - N_t.$$

If the household's demographic structure is constant over the recall period so that $N(s) = N_t$ for all $s \in [t - \Delta, t]$, then there will be no measurement error and, $N_t^* = N_t$. Otherwise, errors will be present.

3 A Bounds Analysis

We now show how to construct bounds on N_t^*. If there are measurement errors in demographic variables then these bounds will be wide.[6] We let M_t denote the number of migrants in the household at the time of the survey, t. We define a migrant to be a household member residing outside of the household's dwelling. It is important to note that N_t only includes home dwellers and not migrants. We let B_t denote the number of births and D_t denote the number of deaths

[5] Note that this expectation is taken across time for a given household and, thus, there will be a distribution of N_t^* across households.

[6] Note, however, that the converse is not true. Wide bounds suggest, but do not imply, that measurement errors are problematic.

which took place in the household during the recall period. The following identity holds:

$$N_t = N_{t-1} - \Delta M_t + B_t - D_t. \quad (1)$$

Based on this simple identity, we propose some sensible assumptions which will allow us to construct our bounds.

Suppose that the only demographic change that takes place in the household over the survey period is migration. Then, we will have that $N_{t-1} = N_t + \Delta M_t$. If $\Delta M_t > 0$, then this implies that

$$N_t < N_{t-1} = N_t + \Delta M_t.$$

We may reasonably assume that $N(s)$ was in the interval $[N_t, N_t + \Delta M_t]$ for all $s \in [t - \Delta, t]$. We use this logic to make three assumptions on the process for $N(s)$:

$$N(s) \in [N_t - B_t, N_t + D_t] \text{ for } \Delta M_t = 0 \text{ and } s \in [t - \Delta, t], \quad \text{(W1)}$$

$$N(s) \in [N_t - B_t, N_t + D_t + j] \text{ for } \Delta M_t = j > 0 \text{ and } s \in [t - \Delta, t] \quad \text{(W2)}$$

and

$$N(s) \in [\max\{N_t - B_t + j, 1\}, N_t + D_t] \text{ for } \Delta M_t = j < 0 \text{ and } s \in [t - \Delta, t]. \quad \text{(W3)}$$

The lower bound in W3 results from the assumption in Section 2 that the household size is always positive at any point-in-time. It is important to emphasize that these conditions are assumptions and are not simply implied by the identity in equation (1).[7] These assumptions

[7] While we concede that these assumptions may be unrealistic in certain circumstances, they are still far weaker

can easily be used to construct bounds on the conditional expectation of average household size: $E[N_t^*|W_t]$ where $W_t \equiv (N_t, M_t, D_t, B_t)$.[8]

We now derive the bounds in a series of steps. First, we note that the assumptions on $N(s)$ imply the following bounds on N_t^*:

$$N_t^* \in [N_t - B_t, N_t + D_t] \text{ for } \Delta M_t = 0 \tag{2}$$

$$N_t^* \in [N_t - B_t, N_t + D_t + j] \text{ for } \Delta M_t = j > 0 \tag{3}$$

and

$$N_t^* \in [\max\{N_t - B_t + j, 1\}, N_t + D_t] \text{ for } \Delta M_t = j < 0. \tag{4}$$

Second, we note that, by the Law of Iterated Expectations, we can write

$$E[N_t^*|W_t] = \sum_j E[N_t^*|\Delta M_t = j, W_t] P(\Delta M_t = j|W_t). \tag{5}$$

Third, conditions (2)-(4) imply that

$$N_t - B_t \leq E[N_t^*|\Delta M_t = j, W_t] \leq N_t + D_t + j \text{ for } j > 0,$$

$$N_t - B_t \leq E[N_t^*|\Delta M_t = j, W_t] \leq N_t + D_t \text{ for } j = 0$$

than the assumption that the household's demographic structure was constant over the survey period which is an assumption employed in the vast majority of studies. Thus, it is impossible to take exception to assumptions W1 through W3 without taking exception with the implicit assumptions in much of the literature.

[8] Note that the expectation, $E[N_t^*|W_t]$, is taken over households whereas the expectation, N_t^*, is taken over time for a given household.

and

$$\max\{N_t - B_t + j, 1\} \leq E[N_t^*|\Delta M_t = j, W_t] \leq N_t + D_t \text{ for } j < 0.$$

These bounds together with equation (5) give us Proposition 1.

Proposition 1 *If the process of $N(s)$ satisfies assumption W1 through W3 then we will have that*

$$L(W_t) \leq E[N_t^*|W_t] \leq U(W_t)$$

where

$$U(W_t) \equiv N_t \ + D_t + \sum_{j>0} j * P(\Delta M_t = j|W_t)$$

and

$$L(W_t) \equiv N_t - B_t + \sum_{j<0} \max\{j, 1 - N_t + B_t\} * P(\Delta M_t = j|W_t)$$

An important question is whether or not we can improve upon the bounds in Proposition 1 while only maintaining assumptions W1 through W3. In other words, is there any additional information contained in our assumptions which would enable us to construct smaller bounds? The answer is "no." This is summarized in Proposition 2. A proof can be found in the appendix.

Proposition 2 *The bounds in Proposition 1 are sharp in the sense that we can choose any point $Z \in [L(W_t), U(W_t)]$ and provide a process for $N(s)$ that satisfies W1 through W3 such that $E[N_t^*|W_t] = Z$.*

4 The Data

Our main data source is the BASIS panel which was administered by the Fundación Salvadoreña para el Desarollo Económico y Social (FUSADES) and The Ohio State University. These data are a longitudinal sample of rural dwellers who were sampled every two years. We primarily use the 1999 and 2001 waves of the panel and we employ data on household size as well as the number of migrants and infants (*i.e.* children under 12 months) in the household.[9] Descriptive statistics for these data can be found in Table 1.[10]

In addition, we use the *Encuesta de Hogares Propósitos Múltiples* (EHPM) which is a consumption survey that is administered annually by the Salvadoran Economic Ministry. In contrast to the BASIS data, this survey covers both rural and urban households. We use a total of 11696 households from the 2001 survey. These data are used to discuss the impact of mismeasured household size on the identification of economies of scale within the household. Summary statistics from the EHPM can also be found in Table 1. Additional detail on the consumption expenditure data can be found in Appendix 2.[11]

[9] We also used the 1997 wave to allow us to measure migration between 1997 and 1999.

[10] According to researchers at The Ohio State University, the BASIS survey has a stratified design with two strata: households with land and households without land. The sample sizes within strata were determined according to the 1992 census so as to (hopefully) ensure a representative sample. Consequently, no weighting scheme should be necessary. To the best of our knowledge, the survey contains no cluster design. However, we acknowledge the possibility that the observations in the sample are not independent of one another, particularly within small geographic units. Accordingly, we use the bootstrap to address any possible issues with the survey design. Additional detail about this procedure can be found the next section.

[11] The EHPM has a complex two-stage survey design. In the first stage, the country is divided into geographical strata. The Salvadoran Economic Ministry used the 1992 census to determine sample sizes within strata. In the second stage, primary sampling units or clusters were sampled within each strata. Because it is likely that observations within clusters will be correlated, it will also be necessary to adjust all standard errors when working with the EHPM.

5 Estimation and Inference

We use two methods to estimate the bounds in Section 3. The first method is the most straightforward and involves using the BASIS data to estimate the probabilities, $P(\Delta M_t = j|W_t)$, with ordered logit models. Note that because the BASIS data were fielded every other year, these bounds will summarize household size over two years. We include dummy variables for the household size as well as the number of migrants and infants in the household.[12] These fitted probabilities are then used to back out $U(W_t)$ and $L(W_t)$. One of the advantages of the ordered logit model is that it is easy to implement. Furthermore, the use of ancillary parameters for each migration category provides us with a flexible way of treating the regression function.[13] One of the disadvantages, however, is that it assumes the size and number of the ancillary parameters are the same for households of all sizes. This is potentially undesirable because it can produce positive probabilities of large positive values of ΔM_t for large households and large negative values for small households. In practice, however, these probabilities are typically small.

Nevertheless, to address this issue, we employ a simple alternative method where we split the sample into households with five or fewer members and households with more than five members and estimate the ordered logits separately for each sample. Doing this mitigates the problem of predicting large positive (negative) values of ΔM_t for larger (smaller) households since the procedure allows the ancillary parameters to vary in size and number with the household's size. After estimating the ordered logits on the split sample, we back out the migration probabilities and calculate the bounds just as before.

[12] We do not address mortality as the BASIS data do not have adequate information on it.
[13] We do not use non-parametric estimation due to small sample sizes within the "bins."

We calculate the standard errors using the bootstrap.[14] We do so for two reasons. First, calculating the analytical standard errors for these bounds is a rather cumbersome task due to the large number of ancillary parameters that are being estimated. Second, bootstrapping allows us to address any issues concerning the complex design of the survey.

To allow us to make inferences about the unidentified parameter, $E[N_t^*|W_t]$, we construct confidence bands which were developed by Imbens and Manski (2004). The confidence intervals that we report cover $E[N_t^*|W_t]$ with at least 95% probability. Note that this is fundamentally different than covering the identified set, $(L(W_t), U(W_t))$, with 95% probability. In general, the intervals that cover the identified set will be larger than those that cover the unidentified parameter and, thus, the confidence intervals that we report should be viewed as conservative in the sense that they will tend to understate any problems associated with measurement errors. The confidence intervals that we report are

$$CI_{0.95} = \left[\widehat{L(W_t)} - C\widehat{\sigma}_{SE,L}, \widehat{U(W_t)} + C\widehat{\sigma}_{SE,U}\right]$$

where $\widehat{\sigma}_{SE,L}$ and $\widehat{\sigma}_{SE,U}$ are the respective standard errors of $\widehat{L(W_t)}$ and $\widehat{U(W_t)}$ and C satisfies

$$\Phi\left(C + \frac{\widehat{U(W_t)} - \widehat{L(W_t)}}{\max\langle\widehat{\sigma}_{SE,L}, \widehat{\sigma}_{SE,U}\rangle}\right) - \Phi(-C) = 0.95$$

[14] The bootstrapping procedure that we employ works as follows. First, we re-sampled from the data with replacement. To address the possibility of spatial correlation across households, we re-sampled *municipios* from the BASIS data. We re-sampled as many *municipios* as were present in the data. It is unclear from the survey's documentation and our communication with the Ohio State University whether or not the survey had a cluster design. Nevertheless, to the extent that there is spatial correlation across households in these data, our calculation of the standard errors will address it provided that there is only correlation across observations within *municipios*. Using the re-sampled data, we then calculated the bounds. After this, we re-sampled from the data again and repeated the process. After 500 replications, we calculated the standard errors of our estimated bounds.

where $\Phi(.)$ is the CDF of a standard normal random variable.[15] These confidence intervals have the desirable property that the probability that they cover $E[N_t^*|W_t]$ will converge uniformly to 95%.

6 Empirical Results

In this section, we discuss our results. Tables 2 and 3 report the estimated bounds and the 95% confidence intervals for the unidentified parameters. Table 2 reports the results for households that have no migrants and one migrant and Table 3 reports the results for households with two migrants and three or more migrants. In both tables, we only report the results for households with no infants. We report the results using both methods for estimating the bounds described in the previous section. We call the first, *Method 1*, and the second, *Method 2*. Finally, we graph the bound estimates in Figures 3 through 10. Figures 3 through 6 use *Method 1* and Figures 7 through 10 use *Method 2*.

The evidence suggests, not surprisingly, that the width of these bounds has a lot to do with the number of migrants in the household. In Figures 3 and 7, we see that the bounds are quite narrow for households that contain no migrants. The results in Table 3 show that the width of the confidence intervals for these households is on the order of 0.30 household members. Moving to households with one migrant each in Figures 4 and 8, we see that the bounds are wider. Calculations in Table 3 show that the width of these confidence intervals is somewhere between 0.60 and 0.90 household members. When we look at households with two migrants in Figures 5

[15] Some readers may note that our confidence intervals appear to be slightly different from those in the Imbens and Manski paper. This is because their intervals are defined in terms of the standard deviations of the estimated bounds, whereas ours are defined in terms of the standard errors.

and 9 and three or more migrants in Figures 6 and 10, we see that the situation gets much worse. In Table 3, the width of the confidence intervals is on the order of 1.5 people for households with 2 migrants and 2.8 people for households with 3 or more migrants.

7 Implications for Identifying Economies of Scale

Given that over 30% of the households in our data report having at least one migrant and over 15% report having at least two migrants, these bounds are wide for a large number of households in our data. This suggests that mismeasured household size could have implications for the identification of economies of scale within the household. This topic will occupy the rest of the paper.

We begin with an Engel curve at time s:

$$\omega_f(s) = \alpha + \beta \log\left(\frac{X(s)}{N(s)}\right) + \gamma \log(N(s)) + \varepsilon(s) = \alpha + \beta(x(s) - n(s)) + \gamma n(s) + \varepsilon(s)$$

where $\omega_f(s)$ is the share of food in the household's budget, $X(s)$ is total consumption expenditures over the survey period and $N(s)$ is household size. Lower-case variables denote the natural logarithms of relevant quantities. We assume that the residual in this equation is uncorrelated with all of the right-hand side regressors. For the remainder of the paper, we suppress all time subscripts.[16] If we take expectations conditional on the Engel curve at all points-in-time in the

[16] This specification was first estimated by Working (1943) and has been used extensively in the literature on household consumer behavior. See Lanjouw and Ravallion (1995), Deaton and Paxson (1998) and Deaton and Muellbauer (1986) for some examples. As pointed by Deaton (1997) and Deaton and Muellbauer (1980), this Engel curve has the advantage that it fits the data well and is consistent with optimizing household behavior.

recall period then we will have

$$\omega_f^* = \alpha + \beta (x^* - n^*) + \gamma n^* + \varepsilon^* \quad (6)$$

where we employ the notation from previous section that $z^* = E[z(s)|s \in [t - \Delta, t]]$.

Arguments put forth in DP suggest that γ is a measure of economies of scale within the household and should be positive in most circumstances. The foundation of their argument is that public goods within the household become cheaper as the household's size increases and, if we hold the household's *per capita* expenditures constant, this effectively makes the household richer. To better understand this consider a situation, discussed in DP, in which two people decide to move in together. Once these people are living under one roof, they no longer need to pay two separate rents. Provided that their incomes remain constant, each individual has in effect become richer. DP go on to argue that if the income elasticity of food is sufficiently high, as it is in the developing world, the household's consumption of food should increase and we should expect to see that γ is positive. However, using data from a variety of countries which run the whole gamut of living standards, they show that, contrary to the theory, the share of food in the household's budget actually *decreases* with household size holding *per capita* expenditures constant.[17] The authors consider numerous explanations for their puzzling finding but are ultimately unable to resolve the paradox.[18]

[17] Similar evidence has been presented using historical American data by Logan (2008).

[18] Attempts have been made to resolve the puzzle. In a comment on DP, Gan and Vernon (2003) claim that there may be relatively large economies of scale in food consumption and, consequently, it may be reasonable to see that the share of food expenditures in the household's budget decreases with household size. The main reason underlying this assertion is that total household expenditures may include goods that are potentially more private than food such as clothes. Gan and Vernon provide evidence that as the household's size rises, food expenditures as a share of food and housing expenditures also rise. They claim that this resolves the puzzle since housing is known to be more public than food. However, Deaton and Paxson (2003), in a response to the comment,

An explanation for the DP puzzle, that has yet to be pursued, concerns mismeasured household size. To better understand this, we first note that, because the household's size is measured with error, equation (6) cannot be estimated since n^* is never observed. Instead, researchers have to estimate

$$\omega_f^* = \alpha + \beta(x^* - n) + \gamma n + \upsilon \tag{7}$$

where $n = n^* - e$ and $\upsilon = \varepsilon^* + (\gamma - \beta)e$. Clearly, OLS will not yield consistent estimates of β and γ since υ is correlated with n. Next, we project e onto x^* and n and obtain

$$e = \kappa + \phi x^* + \lambda n + u \tag{8}$$

where u is uncorrelated with both x^* and n. Next, we substitute equation (8) into equation (7) and we obtain

$$\omega_f^* = \widetilde{\alpha} + \widetilde{\beta} x^* + \widetilde{\gamma} n + \widetilde{\upsilon}$$

where $\widetilde{\alpha} \equiv \alpha + (\gamma - \beta)\kappa$, $\widetilde{\beta} \equiv \beta + (\gamma - \beta)\phi$, $\widetilde{\gamma} \equiv \gamma + (\gamma - \beta)\lambda$ and $\widetilde{\upsilon} \equiv \varepsilon + (\gamma - \beta)u$.

Because $n = n^* - e$, it is reasonable to expect that $\lambda < 0$ since the covariance between n^* and e will be given by $\sigma_{ne} = \sigma_{n^*e} - \sigma_e^2$. If the measurement errors are classical in the sense that they are uncorrelated with the true value of the household's size, we will have $\sigma_{ne} = -\sigma_e^2 < 0$. A perusal of our estimated bounds from the previous section does not show that the bounds increase in width with household size suggesting that these measurement errors may be uncorrelated with

assert that Gan and Vernon's findings are consistent with empirical results in their original piece, but do nothing to resolve the puzzle. Their fundamental contention with Gan and Vernon's comment is that it provides little evidence that there are substantial economies of scale in food consumption. An alternative explanation for the puzzle has been proposed by Gibson (2002) and Gibson and Kim (2007) who claim that non-classical measurement error in food expenditures may be correlated with household size and that this may result in a negative bias in the economies of scale coefficient estimate.

n^*. We do concede, however, that casual empiricism would suggest that larger households have more scope for demographic change which would induce some positive correlation between e and n^*, but our calculations suggest that this correlation would be small.

The probability limit of the OLS estimate of the economies of scale parameter is $\widetilde{\gamma}$. Accordingly, we can write

$$p\lim \widehat{\widetilde{\gamma}} = (1+\lambda)\gamma - \lambda\beta. \tag{9}$$

This equation illustrates how mismeasured household size can lead to a failure to identify economies of scale even when they are present. To better see this, first note that if λ is negative, the first term on the right-hand side of the equation will be less than γ (if $\gamma > 0$). Second, Engel's Law says that the share of food in the household's budget will fall as the household becomes richer and, thus, β will be negative. Accordingly, if λ is negative, the second term in the probability limit will be negative and potentially large. This calculation suggests that negative estimates of γ may occur even when economies of scale are present. Finally, DP find that their puzzle is deepest (*i.e.* the estimates of γ are the most negative) for the poorest countries. It is interesting that the poorest countries are also likely to be the ones where household demographic structures are the most pliable as is suggested by Figures 1 and 2.

We conclude this section with some *prima facie* evidence which suggests that OLS estimates of γ are positively related to β as is suggested by equation (9). To do this, we estimate

$$\omega_f^j = \alpha^j + \beta^j x + \gamma^j n + \sum_{k=1}^{K-1} \eta_k^j \frac{N_k}{N} + \upsilon^j \text{ for } j = 1, ..., J.$$

For the sake of simplicity, we have dropped the $*$ super-script on the budget share for food and total consumption. The dependent variable in this equation is the budget share of a particular food item. The food items that we use are tortillas, bread, rice, milk, beans, chicken, beef, pork, vegetables, fruit and eggs. The term $\frac{N_k}{N}$ is the share of the total number of household members in a particular age and gender category. We report the estimates of γ^j and β^j in Table 4. What can be seen in the table is that the estimates of γ^j are related to the estimates of β^j. Generally, we see that food items with higher income elasticities also have higher estimates of γ^j. To better see this, we plot the pairs $\left(\widehat{\gamma^j}, \widehat{\beta^j}\right)$ in Figure 11 which clearly illustrates a strong positive relationship between the two parameter estimates.[19]

8 Bounds on the Economies of Scale

In the previous section, we provided some calculations suggesting that mismeasured household size might explain the DP paradox. In this section, we provide additional insight into this issue by calculating an upper bound on the economies of scale parameter in Working's Engel curve. A positive upper bound would indicate that the true economies of scale parameter may plausibly be positive as the theory indicates. This would be strong evidence that mismeasured household

[19] There are two alternative explanations for the positive relationship in Figure 11. The first is that goods that have higher income elasticities also have fewer economies of scale associated with them than the other goods in the household's budget. If this were, in fact, the case, then we would see that, as the household's size increases, the prices of the other goods in the budget would decrease more rapidly than the goods with the higher income elasticities. However, if this were true, then these results suggest that there are fewer economies of scale in beef consumption than in pork consumption. It is unclear to us why this would be the case. The second explanation for the relationship in the figure has to do with the theory in Deaton and Paxson's original work. Specifically, they show that the consumption of a good should increase with the household's size when the income elasticity of that good is high relative to the absolute value of its price elasticity. The fact that we find positive estimates of the economies of scale parameter for goods that are luxuries (or almost luxuries) like beef or pork suggests that there may be some credence to this. However, working against this explanation is the presumption that the price elasticity of beef or pork is higher than the price elasticity of staples like tortillas. Unfortunately, without data on unit prices, there is no way of verifying this presumption. In addition, this argument suggests that the negative estimates of the economies of scale parameter for tortillas is the result of the absolute value of the price elasticity of a staple being high relative to its income elasticity which we find to be somewhat hard to believe.

size could resolve the DP paradox.

We begin by bounding the variance of the measurement error in log household size. To do this, we use sharp bounds on the expectation of n^* conditional on W which we denote by $(l(W), u(W))$. These bounds can calculated by following the steps from Section 3.[20] The following lemma, which we prove in the appendix, summarizes this bound.

Lemma 3 *If* $E(n^*|W) \in [l(W), u(W)]$ *then* $\sigma_e^2 \leq \overline{\sigma}_e^2$ *where*

$$\overline{\sigma}_e^2 \equiv \sum_W \overline{\sigma}_e^2(W)\, p(W)$$

and

$$\overline{\sigma}_e^2(W) \equiv \max\left\langle (n - l(W))^2, (n - u(W))^2 \right\rangle.$$

We can now use this lemma to derive a bound on the economies of scale parameter. For simplicity's sake, suppose that $\phi = 0$ in equation (8) so that the measurement error does not vary with total consumption expenditures. This implies that

$$\lambda = \frac{\sigma_{n,e}}{\sigma_n^2} = \frac{\sigma_{n^*,e} - \sigma_e^2}{\sigma_n^2}.$$

If the measurement error is either classical or positively correlated with the truth (as we argued earlier) then we will have that $\sigma_{n^*,e} \geq 0$. Thus, we obtain that

$$\lambda \geq -\frac{\sigma_e^2}{\sigma_n^2} \geq -\frac{\overline{\sigma}_e^2}{\sigma_n^2} \equiv \underline{\lambda}.$$

[20] For the sake of brevity, we omit the details, but the details can be found in an earlier draft of this paper which we will provide upon request.

Provided that $-1 \leq \underline{\lambda} \leq \lambda \leq 0$ and $\widehat{\widetilde{\gamma}} < 0$, we will have that

$$\frac{\widehat{\widetilde{\gamma}} + \lambda\beta}{1+\lambda} \leq \widehat{\widetilde{\gamma}} + \underline{\lambda}\beta \equiv \widehat{\overline{\gamma}} \text{ if } \widehat{\widetilde{\gamma}} + \underline{\lambda}\beta < 0$$

and

$$\frac{\widehat{\widetilde{\gamma}} + \lambda\beta}{1+\lambda} \leq \frac{\widehat{\widetilde{\gamma}} + \underline{\lambda}\beta}{1+\underline{\lambda}} \equiv \widehat{\overline{\overline{\gamma}}} \text{ if } \widehat{\widetilde{\gamma}} + \underline{\lambda}\beta \geq 0$$

which will asymptotically provide an upper bound on γ. The key of this derivation is that the sign of $\widehat{\widetilde{\gamma}} + \underline{\lambda}\beta$ will tell us if a positive bound on γ is possible.

We can now calculate the bound. First, we compute $\overline{\sigma}_e^2$ in two ways. The first uses direct estimates of the interval $(l(W), u(W))$ from the BASIS data and we obtain that $\overline{\sigma}_e^2 = 0.015760$.[21] One problem with this estimate is that the time between the BASIS surveys is about two years, whereas the effective recall period in our consumption survey is approximately three months. This suggests that we might reasonably assume that the intervals over the recall period were $\frac{1}{8}$ the size of $(l(W), u(W))$. Under this assumption, we obtain that $\overline{\sigma}_e^2 = 0.000246$. Next, in the EHPM, we obtain that $\sigma_n^2 = 0.323600$ and so, we then have that $\underline{\lambda} = -0.048702$ or $\underline{\lambda} = -0.000760$ when we adjust for effective recall period. Finally, in the EHPM, we obtain that $\widehat{\widetilde{\gamma}} = -0.079611$ and an estimate of β of -0.088187. If we do not adjust for the effective recall period, we obtain that $\widehat{\overline{\gamma}} = -0.075316$ and if we do, we obtain that $\widehat{\overline{\gamma}} = -0.079544$. Thus, even in the most conservative scenario where we do not account for the effective recall period, we do not obtain a positive bound on the economies of scale parameter.

[21] These calculations are from an earlier draft of the paper and are available upon request.

9 Conclusions

We can now answer our research questions. First, how large are the measurement errors in household size? Using a bounds analysis, we show that over the course of one or two years, these errors can be large, particularly, for households with a history of migration. This suggests that a cheap way in which the measurement errors in household size could be partially addressed is to ask the survey respondent to retrospectively report their household size at points during the survey year corresponding to the major recall periods. This would allow researchers to compute measures of the household's effective size. Second, can these measurement errors resolve the DP paradox? Probably not. Despite providing evidence that there are potentially considerable measurements errors in household size in El Salvador, the errors are not sufficient to resolve the DP paradox. Using our bounds on household size, we derive an upper bound on the economies of scale parameter and show that it is not positive.

In order for mismeasured household size to explain the DP paradox, two conditions would have to be met. First, the effective recall period would have to be high. This will tend to occur in richer countries where durables are a higher share of the household's consumption. Second, the household structure would have to be highly fluid which is more likely to occur in poorer countries. Accordingly, these two conditions are unlikely to obtain at the same time. Moreover, even in a country with fluid household structures such as El Salvador, an *upper* bound on the measurement error in log household size is only 4.8% of the variance in log household size when we consider household fluctuations over *two* years. Overall, we are skeptical that mismeasured household size can explain the DP paradox.

10 Appendix 1 - Proof of Proposition 2

Proof. The goal of this proof is to produce a set of processes for $N(s)$ for all households that satisfy assumptions W1 through W3 such that $Z = E[N_t^*|W_t]$ for $Z \in [L_t(W_t), U(W_t)]$. For the sake of simplicity, we consider the case where the only source of demographic change is migration. We begin by writing

$$E[N_t^*|W_t] = e_{-J} * p_{-J} + ... + e_{-1} * p_{-1} + e_0 * (1 - p_{-J} - ... - p_{-1} - p_1 - ...p_J) + e_1 * p_1 + ... + e_J * p_J$$

where $e_i \equiv E[N_t^*|\Delta M_t = i, W_t]$ and $p_i \equiv P(\Delta M_t = i|W_t)$. Next, we consider the case where $Z \in [U_t(W_t) - p_J J, U(W_t)]$. We now choose the following processes for $N(s)$:

$$N(s) = \begin{matrix} N_t \text{ for } s \in (t-1, t] \\ N_t + j \text{ for } s = t-1 \end{matrix} \quad \text{and } \Delta M_t = j \leq 0$$

$$N(s) = \begin{matrix} N_t \text{ for } s = t \\ N_t + j \text{ for } s \in [t-1, t) \end{matrix} \quad \text{and } 0 < \Delta M_t = j < J$$

We assume that these two conditions hold for *all* households. These conditions assume that, for $\Delta M_t < J$, the household's size is constant over a time interval of measure one. Clearly, these conditions satisfy W1 through W3. These conditions on the $N(s)$ process then give us that

$$E[N_t^*|W_t] = N_t * (1 - p_J) + p_1 + 2p_2 + ... + (J-1)p_{J-1} + p_J * e_J$$

since they hold for all households. If we set the above expression equal to Z, we can then write e_J as

$$e_J = N_t + p_J^{-1} * [Z - p_1 - ... - (J-1)p_{J-1} - N_t].$$

Next, noting that

$$U(W_t) = N_t + p_1 + ... + Jp_J,$$

and recalling that $Z \in [U_t(W_t) - p_J J, U(W_t)]$, we will have that

$$e_J = E[N_t^* | \Delta M_t = J, W_t] \in [N_t, N_t + J].$$

Finally, we choose

$$N(s) = \begin{matrix} N_t \text{ for } s \in (t-\delta, t] \\ \\ N_t + J \text{ for } s \in [t-1, t-\delta] \end{matrix} \qquad \text{for } \Delta M_t = J$$

where $\delta \equiv 1 + \frac{N_t - e_J}{J}$. Note that $\delta \in [0,1]$ since $e_J \in [N_t, N_t + J]$. The proof for the other values of Z is completely analogous. ■

11 Appendix 2 - Proof of Lemma 3

Proof. Note that

$$\begin{aligned} \sigma_e^2 &= E\left[(n-n^*)^2\right] \\ &= E\left[E\left[(n-n^*)^2 | W\right]\right] \\ &= \sum_W E\left[(n-n^*)^2 | W\right] P(W). \end{aligned}$$

By the definition of the bounds, we will have that

$$|n-n^*| \leq \max \langle |u(W)-n|, |l(W)-n| \rangle$$

and so, we obtain that

$$E\left[(n-n^*)^2 | w\right] \leq \sigma_e^2(W)$$

which proves the lemma. ■

12 Appendix 3 - Consumption Expenditures in the EHPM

The EHPM contains detailed information on consumption expenditures which is summarized in Table 5. The data on food expenditures as well all expenditures in consumption categories 1 and 2 includes all items purchased on the market, produced at home and received as aid. Total consumption is the sum of all expenditures in categories 1 and 2 plus expenditures on food, utilities, schooling and medical care. We did not include expenditures on housing as these data were suspect.[22]

[22] Discussions with a researcher at FUSADES, a Salvadoran think tank, corroborated these suspicions. In addition, it is important to note that the lack of data on housing expenditures does not impact the analysis

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in Section 7. The reason is that our analysis focuses on the biases that measurement error can create when estimating Engel curves. These biases will be present with or without the housing expenditure data.

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Figure 1

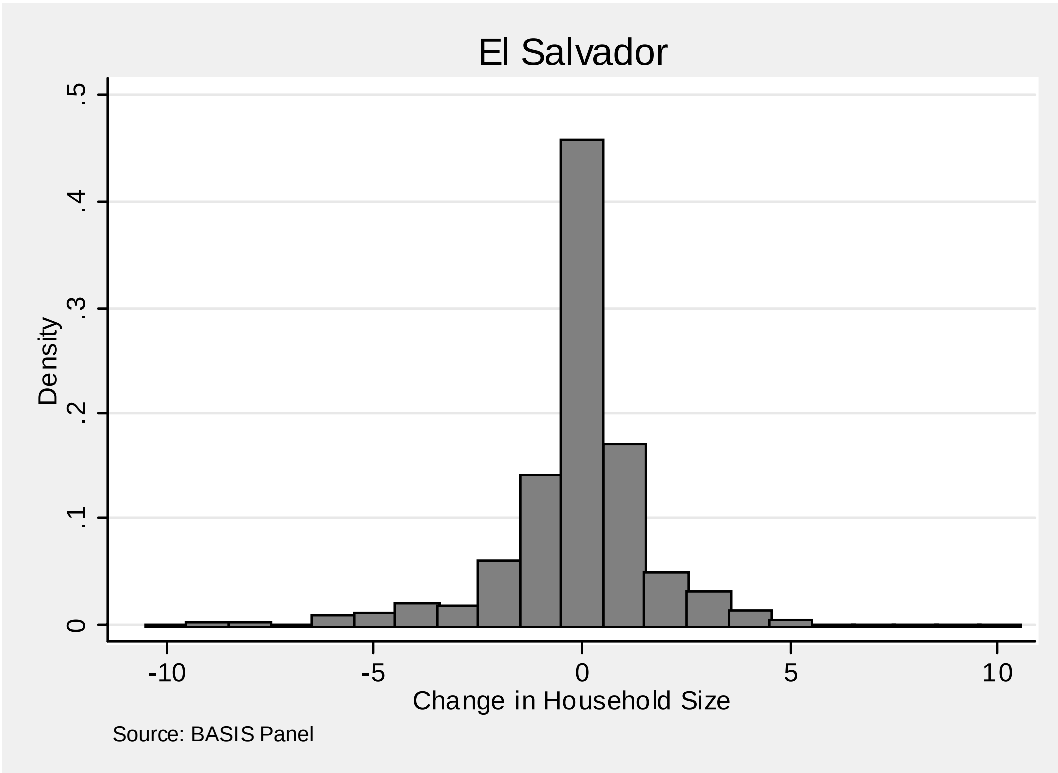


Figure 2

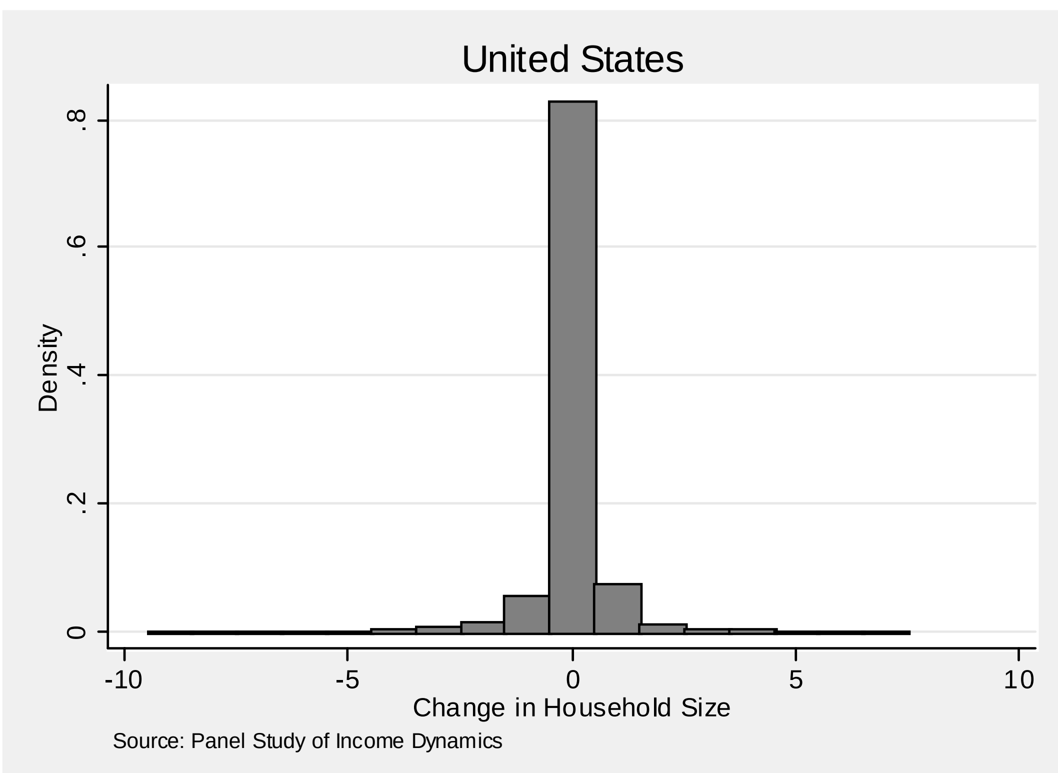


Figure 3

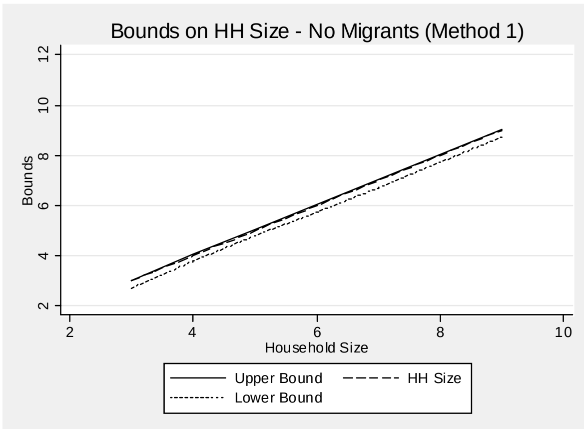


Figure 5

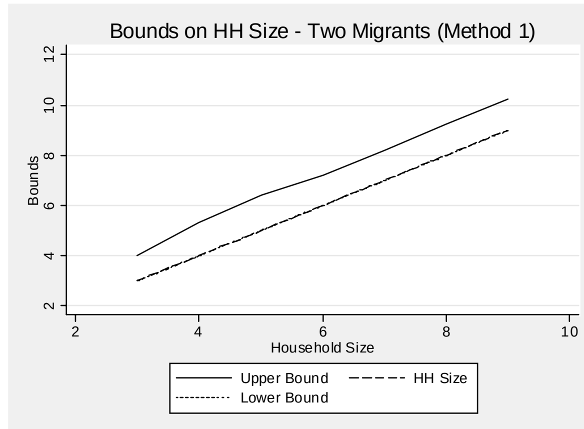
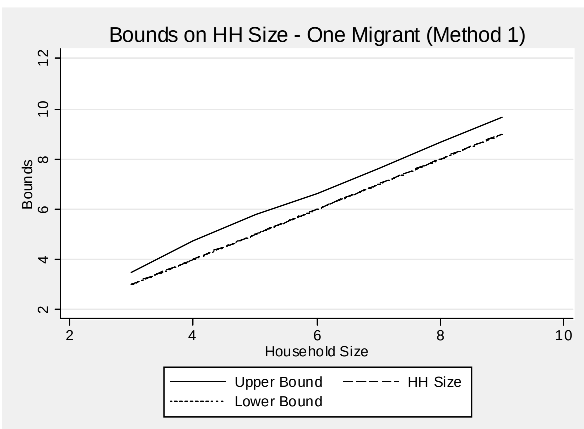


Figure 4



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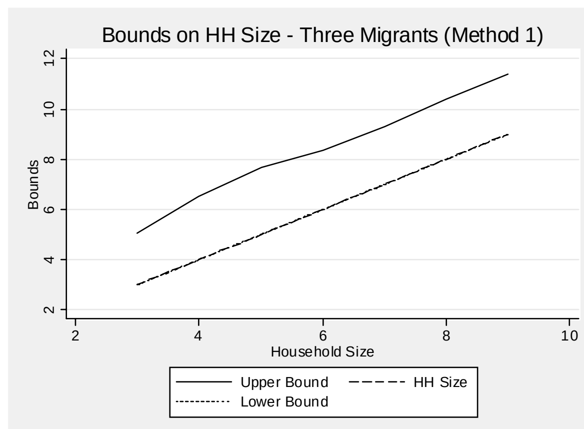


Figure 7

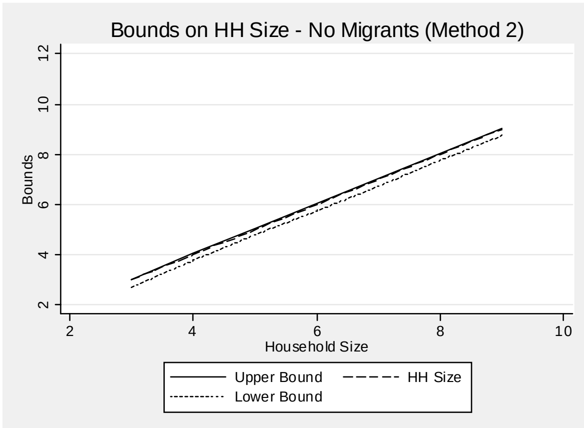


Figure 9

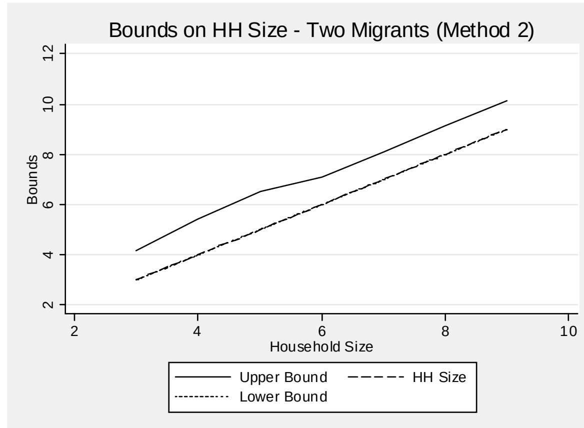


Figure 8

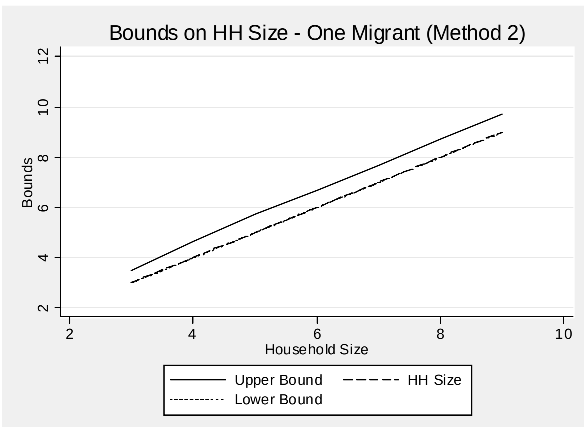


Figure 10

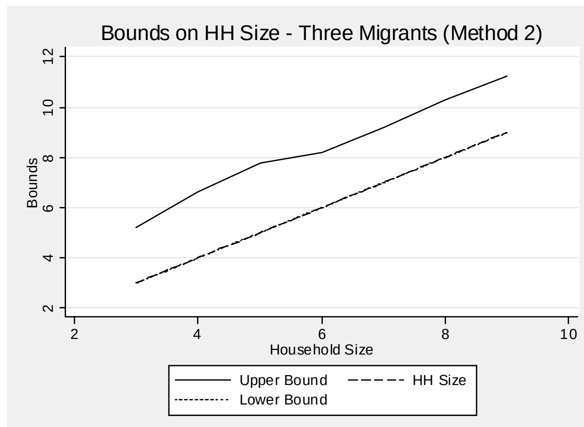
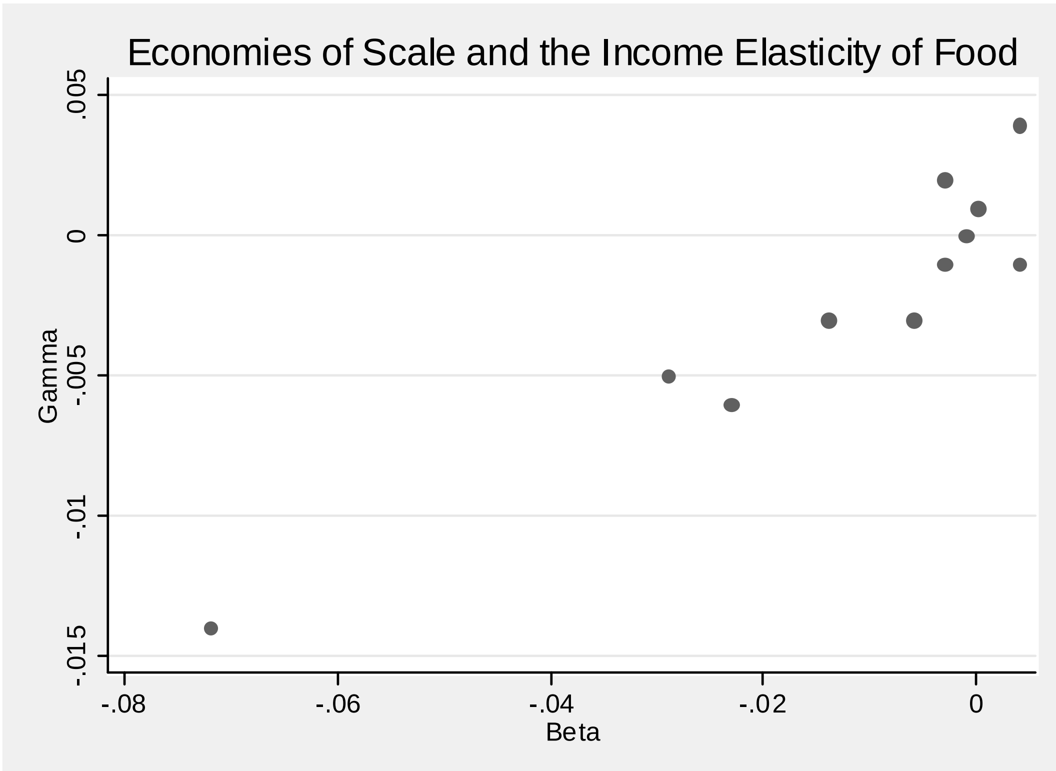


Figure 11



Economies of Scale and the Income Elasticity of Food
Gamma
.005
0
-.005
-.01
-.015
-.08
-.06
-.04
-.02
0
Beta

Table 1: Descriptive Statistics

	Mean (Standard Deviation)
BASIS[1]	
Household Size	5.96 (2.68)
Migrants	0.64 (1.32)
Infants	0.06 (0.25)
EHPM[2]	
Total Consumption Expenditures	3044.98 (2223.48)
Household Size	4.44 (2.26)

[1]The sample size for these data is 1265 households.
[2]The sample size for these data is 11696.

Table 2: Bounds - Zero or One Migrant, No Infants

HH Size*	$\left(\widehat{L(W_t)}, \widehat{U(W_t)}\right)$	95% CI for $E[N_t^*\|W_t]$	$\left(\widehat{L(W_t)}, \widehat{U(W_t)}\right)$	95% CI for $E[N_t^*\|W_t]$
	Method 1		Method 2	
No Migrants				
3	$(2.708, 3.029)$	$[2.633, 3.043]$	$(2.702, 3.030)$	$[2.615, 3.047]$
4	$(3.788, 4.049)$	$[3.728, 4.071]$	$(3.777, 4.050)$	$[3.711, 4.077]$
5	$(4.808, 5.059)$	$[4.752, 5.082]$	$(4.794, 5.060)$	$[4.723, 5.087]$
6	$(5.725, 6.042)$	$[5.646, 6.062]$	$(5.747, 6.042)$	$[5.672, 6.060]$
7	$(6.710, 7.041)$	$[6.628, 7.057]$	$(6.731, 7.040)$	$[6.657, 7.057]$
8	$(7.733, 8.045)$	$[7.636, 8.065]$	$(7.756, 8.044)$	$[7.682, 8.062]$
9	$(8.726, 9.044)$	$[8.632, 9.070]$	$(8.745, 9.043)$	$[8.668, 9.060]$
One Migrant				
3	$(2.986, 3.502)$	$[2.975, 3.613]$	$(2.983, 3.463)$	$[2.967, 3.576]$
4	$(3.990, 4.711)$	$[3.982, 4.883]$	$(3.988, 4.650)$	$[3.976, 4.822]$
5	$(4.992, 5.792)$	$[4.985, 5.944]$	$(4.990, 5.724)$	$[4.978, 5.903]$
6	$(5.988, 6.640)$	$[5.977, 6.791]$	$(5.990, 6.705)$	$[5.980, 6.864]$
7	$(6.987, 7.625)$	$[6.976, 7.751]$	$(6.989, 7.684)$	$[6.979, 7.833]$
8	$(7.988, 8.667)$	$[7.977.8.833]$	$(7.990, 8.736)$	$[7.980, 8.878]$
9	$(8.988, 9.662)$	$[8.978, 9.822]$	$(8.990, 9.717)$	$[8.979, 9.856]$

*Refers to the household's reported size at the time of the survey.

Table 3: Bounds - Two or More than Three Migrants, No Infants

HH Size*	$\left(\widehat{L(W_t)}, \widehat{U(W_t)}\right)$	95% CI for $E[N_t^*\|W_t]$	$\left(\widehat{L(W_t)}, \widehat{U(W_t)}\right)$	95% CI for $E[N_t^*\|W_t]$
	Method 1		Method 2	
		Two Migrants		
3	$(2.996, 4.008)$	$[2.991, 4.229]$	$(2.997, 4.144)$	$[2.992, 4.474]$
4	$(3.997, 5.304)$	$[3.994, 5.557]$	$(3.998, 5.431)$	$[3.995, 5.772]$
5	$(4.997, 6.415)$	$[4.995, 6.638]$	$(4.998, 6.539)$	$[4.995, 6.848]$
6	$(5.996, 7.206)$	$[5.993, 7.436]$	$(5.995, 7.104)$	$[5.990, 7.363]$
7	$(6.996, 8.185)$	$[6.992, 8.400]$	$(6.995, 8.077)$	$[6.990, 8.325]$
8	$(7.996, 9.244)$	$[7.992, 9.511]$	$(7.996, 9.145)$	$[7.990, 9.387]$
9	$(8.996, 10.236)$	$[8.993, 10.483]$	$(8.995, 10.120)$	$[8.990, 10.360]$
		Three or More Migrants		
3	$(2.999, 5.053)$	$[2.998, 5.470]$	$(2.999, 5.204)$	$[2.999, 5.708]$
4	$(3.999, 6.505)$	$[3.999, 6.969]$	$(3.999, 6.635)$	$[3.999, 7.187]$
5	$(4.999, 7.675)$	$[4.999, 8.120]$	$(4.999, 7.799)$	$[4.999, 8.313]$
6	$(5.999, 8.355)$	$[5.999, 8.831]$	$(5.999, 8.223)$	$[5.998, 8.759]$
7	$(6.999, 9.323)$	$[6.999, 9.750]$	$(6.999, 9.181)$	$[6.998, 9.714]$
8	$(7.999, 10.413)$	$[7.999, 10.945]$	$(7.999, 10.286)$	$[7.998, 10.820]$
9	$(8.999, 11.401)$	$[8.999, 11.965]$	$(8.999, 11.248)$	$[8.998, 11.787]$

*Refers to the household's reported size at the time of the survey.

Table 4: Engel Curve Estimates

	$\widehat{\beta}_j$	$\widehat{\gamma}_j$
Tortillas	−0.072	−0.014
	(−30.71)	(−8.03)
Beans	−0.029	−0.005
	(−26.72)	(−8.20)
Eggs	−0.023	−0.006
	(−31.85)	(−7.85)
Rice	−0.014	−0.003
	(−28.28)	(−7.68)
Vegetables	−0.006	−0.003
	(−9.38)	(−5.34)
Bread	−0.003	−0.001
	(−2.34)	(−0.87)
Chicken	−0.003	0.002
	(−2.91)	(2.26)
Milk	−0.001	−0.000
	(−1.28)	(−0.36)
Pork	0.000	0.001
	(0.91)	(3.37)
Beef	0.004	0.004
	(4.60)	(5.02)
Fruit	0.004	−0.001
	(7.64)	(−1.58)

*This table contains OLS estimates of the Engel curves described in Section 8. All standard errors allow for clustering on municipios.

Table 5: Constituents of Consumption Expenditures: EHPM

Component	Mean (Standard Deviation)	Contents
Food	1423.33 (880.54)	tortillas, bread, rice, beans, salt, sugar, grains, chicken, beef, pork, fish, eggs, fruits, restaurant meals, prepared meals, alcohol, vegetables, aceite, drinks, coffee, milk, cheese, other items
Category 1	244.51 (252.65)	toiletries, soap, cleaning products, newspapers, cosmetics, fuel, babysitting, magazines, transportation
Category 2	167.26 (343.92)	travel, jewelry, pots, towels, other repairs, appliances, furniture, clothes, glasses, car repairs
Utilities	461.41 (541.76)	water, electricity, kerosene, propane, candles, carbon, leña, telephone, cell phone, cable, garbage
School	677.80 (984.91)	tuition, supplies, uniforms, textbooks
Medical	70.68 (320.00)	doctor's visits, lab work, x-rays, hospital days, medicine
Total	3044.98 (2223.48)	