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ABSTRACT

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The consequences of business cycle contingencies in unemployment insurance systems are considered in a search-matching model allowing for shifts between "good" and "bad" states of nature. We show that not only is there an insurance argument for such contingencies, but there may also be an incentive argument. Since benefits may be less distortionary in a recession than a boom, it follows that counter-cyclical benefits reduce average distortions compared to state independent benefits. We show that optimal (utilitarian) benefits are counter-cyclical and may reduce the structural (average) unemployment rate, although the variability of unemployment may increase.

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1 Introduction

Optimal unemployment insurance systems trade off incentives and insurance. Since unemployment risk is intimately related to the business cycle situation, one often finds calls for improved benefit generosity in periods with slack labour markets[1]. The standard argument against is that this will reinforce distortions and therefore come at the costs of an increase in structural unemployment. However, as the gains from unemployment insurance in general are dependent on the state of nature, so are distortions. We show in a standard search-matching framework that the distortions of search incentives created by unemployment insurance may be larger in periods with low than high unemployment. Hence, there may both be an insurance and incentive argument for making benefits depending on the state of nature in a counter-cyclical fashion; that is, benefit generosity is high when unemployment is high, and low when unemployment is low.

However, the key parameters of unemployment schemes are business cycle independent in most countries. Though, there are cases where elements of the unemployment insurance system are explicitly linked to the state of the labour market. Probably the most sophisticated scheme is found in Canada where benefit eligibility, levels, and duration depend on the level of unemployment according to pre-determined rules[2]. The US has a system of extended benefit duration in high unemployment periods (see Committee on Way and Means (2004)). Some countries have pursued a more discretionary approach - in some cases in a semi automatic fashion[3] - by adjusting labour market policies to the state of the labour market; i.e. extending benefits or labour market policies in general in high unemployment periods, and tightening the schemes in periods with low unemployment.

There is a large literature on the design of unemployment insurance schemes. Since Baily (1978) it is well-known that the optimal benefit level trades off insurance and incentives. Recent work has extended these insights in various directions (for a survey see e.g. Frederiksson and Holmlund (2006)). Surprisingly, there is neither a large theoretical literature on the effects of business cycle dependent unemployment insurance nor an empirical

[1]The issue of business cycle contingencies in unemployment insurance has gained further interest in perspective of the downturn induced by the financial crisis. Calls for increases in unemployment benefits or extension of benefit duration have been made by e.g. the IMF and the OECD (see Spilimbergo et al. (2008) and OECD (2009)), and if such changes are made, it is an important issue whether they should be made contingent on the business cycle to prevent that these changes become permanent. OECD (2009) reports that 15 countries have made unemployment benefits more generous as a response to the crisis, and in many countries additional steps have been taken towards support for jobless.

[2]See http://www.hrsdc.gc.ca/eng/ei/menu/eihome.shtml.

[3]Sweden is an example of a country which has used labour market policies in this way.

literature[4] exploring the state of nature dependencies in the effects of various labour market policies including the benefit level. Kiley (2003) and Sanchez (2008) argue within a search framework that the initial benefit level should be higher and its negative duration dependence weaker in a business cycle downturn compared to an upturn. Both models are partial and rely on the assumption that benefits are more distortionary in a boom[5]. In Andersen and Svarer (2010), it is shown that the optimality of counter-cyclical benefit levels depends not only on the possibility of using the public budget as a buffer but also on the extent to which distortions move pro-cyclically. If this is the case, counter-cyclical unemployment benefits may also contribute to lower the structural (average) unemployment rate. However, the model is static and does not allow for changes in the business cycle situation.

This paper develops a general equilibrium search-matching model in which the business cycle situation may change between "good" and "bad" states of nature[6]. Matching frictions imply a co-existence of unemployed persons and vacant jobs, but the underlying job separation rates and job finding rates are business cycle dependent. The unemployment benefit scheme is tax financed, and benefits are allowed to be business cycle dependent. Since the main issue in this paper is the trade-off between insurance and incentive, the model is cast in such a way that it focuses on how unemployment benefits affect job search incentives. The paper addresses both the positive issue of how such state contingencies affect labour market performance and the normative issue of the optimal (utilitarian) state of nature contingencies to build into unemployment insurance schemes[7].

In the search context considered in this paper, the response of job search to both unemployment benefits and the business cycle situation plays a crucial role. It is shown that the distortion of search incentives caused by benefits tends to be business cycle dependent in a pro-cyclical way; i.e. a high benefit level distorts incentives more in a good than a bad business cycle situation. At the same time, insurance arguments may call for counter-cyclical

[4]The few exceptions are: Moffitt (1985), Arulampalam and Stewart (1995), Jurajda and Tannery (2003), and Røed and Zhang (2005), see section 2.

[5]In a related study, Costain and Reiter (2005) analyse a business cycle model with exogenous search state, allowing for contingencies in social security contributions levied on firms and unemployment benefits. In this model the public budget does not need to balance in each state due to contingent assets traded with risk neutral capitalists. It is shown that it is optimal to have pro-cyclical social security contributions, while benefits are almost state invariant.

[6]The main modelling difficulty here is to ensure stationarity of public finances under a tax financed unemployment insurance scheme. This is done by the specific assumptions concerning state transitions and the tax policy.

[7]In addition, business cycle dependent unemployment benefits would also strengthen automatic stabilizers, which may have effects via aggregate demand effects. Such effects do not arise in the present framework, which focuses on the structural consequences of business cycle dependent benefit levels.

benefit levels. This has two important implications, namely, first that optimal benefits may be counter-cyclical, and second that the structural (average) unemployment rate could be lower with business cycle contingent compared to business cycle independent benefits. However, as a consequence the actual unemployment rate may become more variable.

In addition, it is shown that accounting explicitly for business cycle fluctuations has an important effect on search behaviour and therefore on unemployment and other key variables. The reason is that agents perceive the possibility of a change in the business cycle situation, and this affects the search behaviour of the unemployed. Clearly, this effect depends on both the difference between the two states of nature and the likelihood of a change in the business cycle situation. This may even imply that counter-cyclical benefits increase search effort in both states of nature, and therefore cause a fall in unemployment in both states. This arises if the business cycle situation is not too persistent, in which case agents in a downturn expect a shift to an upturn with a higher job finding rate.

The paper is organized as follows: In section 2 we introduce a search model with business cycle fluctuations. The issue of business cycle dependent incentive and insurance effects are analysed in section 3. The consequences of business cycle dependent benefits are addressed in section 4, while section 5 considers optimal benefits for a utilitarian policy maker. A few concluding remarks are given in section 6.

2 A search matching model with business cycles

Consider a standard search matching model of the Pissarides-Mortensen type in discrete time (see e.g. Mortensen and Pissarides (1994) and Pissarides (2000)). To economize on notation, we suppress the time index, which is possible since interest is confined to stationary equilibria (see Appendix A). All workers are ex-ante identical and have the same productivity. Workers search for jobs, but a matching friction implies that unemployment and vacancies coexist. Firms create vacancies, and filled jobs are destructed by some probability.

We assume that the economy shifts between two states, *good* (G) and *bad* (B), according to a Markow process with the following symmetric and stationary transition probability matrix[8]

present\past state	B	G
B	π	$1-\pi$
G	$1-\pi$	π

where $0 \leq \pi \leq 1$.[9] If the economy is in a boom (recession), this state of nature may continue with probability π and terminate and turn into a recession (boom) with probability $1-\pi$. Hence, π is also a measure of the persistence in the current business cycle situation. Given this assumption we consider a stationary Markov equilibrium to the model.

The job separation rate p is in the four possible states of nature given as follows

present\past state	B	G
B	p_{BB}	p_{BG}
G	p_{GB}	$p_{GG} < p_{BB}$

i.e. the basic transition is between a regime with a low level (p_{GG}) or a high level ($p_{BB} > p_{GG}$) of job separations[10]. Upon transition there is an extraordinarily high ($p_{BG} > p_{BB}$) or low ($p_{GB} < p_{GG}$) level of job separations (see below)[11].

There is an unemployment benefit scheme providing a flow benefit b to unemployed workers, and it is financed by a proportional wage income tax (τ) and a lump sum tax (T) (see below). The inclusion of lump sum taxes facilitates the analysis involving four possible states of nature in a setting which does not impose a balanced budget requirement for each state of nature. The key problem is that the debt level in general will display path dependence violating the possibility of having a stationary Markov equilibrium[12]. To cope

[8]We assume a symmetric transition matrix to simplify the analysis. Empirical evidence indicates some asymmetry with more persistence in good than in bad business cycle situations. The estimated value of π in discrete models on quarterly data is in the range 0.7 to 0.9, see Hamilton (1994). In a three state model (recession, normal and high growth), somewhat higher levels of persistence are found, see Artis et al. (2004).

[9]Note that the unconditional stationary probability of being in a given state B or G is $\Pr(G) = \Pr(B) = \frac{1}{2}$. The unconditional probabilities of the four possible states are: $\Pr(BB) = \Pr(GG) = \frac{1}{2}\pi$ and $\Pr(GB) = \Pr(BG) = \frac{1}{2}(1-\pi)$.

[10]Differences in the business cycle situation may be generated by changing other variables in the model like job creation, the costs of vacancies, matching efficiency etc., but the qualitative results would be the same, see Andersen and Svarer (2010).

[11]There has been some debate on the extent to which changes in the job separation rate are a driver of unemployment fluctuations, especially in the US (see Shimer (2005)). Elsby et al. (2008) find that the US is an outlier compared to other OECD countries where fluctuations in both inflow and outflow rates are found to be important.

[12]This is so since the initial debt level depends on the past history of the economy if the budget is allowed not to balance in each state of nature. The budget requirements to ensure debt sustainability are in general path dependent.

with this and to ensure stable debt levels, policies will in general have to be path dependent. This problem is addressed via the lump sum tax. The income tax rate is assumed state independent, while the benefit level may depend on whether the state is "good" or "bad". Note that there are no marginal labour supply decisions (intensive margin) in the following, hence the use of lump sum taxation does not affect any results, but serves the purpose of making the analysis more simple and transparent. Search is affected by the gains from employment and thus net taxes and benefits.

2.1 Individual utility and search effort

Consider an infinite number of identical households, and normalize the population size to unity. Employed workers receive a wage w and work l hours. Both w and l are business cycle independent, and the instantaneous utility is assumed to be separable in the utility from consumption (first term) and leisure (second term), i.e.

$$\nu(w, \tau, T_{ij}) \equiv g\left(w\left[1-\tau\right]-T_{ij}\right)+f(1-l)$$

where τ is the income tax rate, and T_{ij} is the lump sum tax paid if the current state is i and the previous state j. Working hours l are exogenous, and the time endowment has been normalized to 1. Both $g()$ and $f()$ are concave increasing functions. The instantaneous utility for unemployed is similarly assumed separable over consumption and leisure and given by

$$\mu(b_i, T_{ij}, s_{ij}) \equiv g\left(b_i - T_{ij}\right)+f(1-s_{ij})$$

where s_{ij} is time spent searching for a job if the current state is i and the previous state j.[13] Note that the separability assumption ensures that search is not dependent on current income (see below)[14]. In addition, note that the benefit level only takes two values conditional on the current state, whereas the lump sum tax also depends on the past state. This results in four different levels of net compensation to the unemployed.

Value functions

The value functions for being employed when the current state is bad and the past state was either good or bad are in stationary equilibrium given as (see Appendix A)

[13] The underlying utility function is assumed to be the same for employed and unemployed workers. In a more general formulation, stigmatization and other factors may cause both the utility from income to depend on its source and the disutility from work to depend on the type of time use. In an earlier version, we allowed for such differences, but they did not have any qualitative implications for the results.

[14] There is no on-the-job search since all jobs are assumed identical and have the same wage.

$$\frac{\rho}{1+\rho}W_{BB}^{E} = \nu(w,\tau,T_{BB})+\frac{\pi}{1+\rho}p_{BB}\left[W_{BB}^{U}-W_{BB}^{E}\right] + \frac{1-\pi}{1+\rho}\left[(1-p_{GB})\left[W_{GB}^{E}-W_{BB}^{E}\right]+p_{GB}\left[W_{GB}^{U}-W_{BB}^{E}\right]\right]$$

$$\frac{\rho}{1+\rho}W_{BG}^{E} = \nu(w,\tau,T_{BG})+\frac{\left[W_{BB}^{E}-W_{BG}^{E}\right]}{1+\rho}+\frac{\pi p_{BB}}{1+\rho}\left[W_{BB}^{U}-W_{BB}^{E}\right] + \frac{(1-\pi)}{1+\rho}\left[(1-p_{GB})\left[W_{GB}^{E}-W_{BB}^{E}\right]+p_{GB}\left[W_{GB}^{U}-W_{BB}^{E}\right]\right]$$

where ρ is the subjective discount rate. Similar value functions exist when the current state is good. To save space these are delegated to the appendix A.

Similarly, the value functions for current unemployed workers when the current state is bad and the past state was either good or bad are:

$$\frac{\rho}{1+\rho}W_{BB}^{U} = \mu(b_B,T_{BB},s_{BB})+\frac{\pi}{1+\rho}\alpha_B s_{BB}\left[W_{BB}^{E}-W_{BB}^{U}\right] + \frac{1-\pi}{1+\rho}\left[(1-\alpha_G s_{BB})\left[W_{GB}^{U}-W_{BB}^{U}\right]+\alpha_G s_{BB}\left[W_{GB}^{E}-W_{BB}^{U}\right]\right]$$

$$\frac{\rho}{1+\rho}W_{BG}^{U} = \mu(b_B,T_{BG},s_{BG})+\frac{W_{BB}^{U}-W_{BG}^{U}}{1+\rho}+\frac{\pi}{1+\rho}\alpha_B s_{BG}\left[W_{BB}^{E}-W_{BB}^{U}\right] + \frac{1-\pi}{1+\rho}\left[(1-\alpha_G s_{BG})\left[W_{GB}^{U}-W_{BB}^{U}\right]+\alpha_G s_{BG}\left[W_{GB}^{E}-W_{BB}^{U}\right]\right].$$

Again similar value functions for when the current state is good can be found in the appendix.

We focus solely on risk sharing via the unemployment insurance scheme. One issue is the role private savings may play as a buffer and thus self-insurance mechanism[15]. Allowing for interaction between different forms of insurance will complicate the analysis, and since risk diversification offered by savings is incomplete[16], we focus only on the unemployment insurance scheme[17].

[15] The issue of how individual savings can be a buffer and thus a form of self-insurance in the case of unemployment has been analysed in relation to unemployment insurance benefits in e.g. Lenz and Tranæs (2005) and the wider context of so-called welfare accounts by Bovenberg, Hansen and Sørensen (2008).

[16] The scope for self-insurance via savings is restricted both due to capital market imperfections affecting the scope for intertemporal diversification and the fact that savings and accumulation of wealth do not provide much insurance for young workers (see e.g. Bailey (1976) and Chetty (2008)). Empirical evidence shows that unemployment is associated with reductions in consumption, and that a large fraction of unemployed are liquidity constrained, see e.g. Gruber (1997) and Bloemen and Stancanelli (2005). The argument that risk diversification via savings is incomplete is here taken to the limit.

[17] However, note that in the special case where utility functions over consumption are linear ($g\left(w\left[1-\tau\right]-T_{ij}\right)=w\left[1-\tau\right]-T_{ij}$ and $g(b_i-T_{ij})=b_i-T_{ij}$) and the discount rate ρ is interpreted as the market rate of interest, the value functions equal the expected present value of income (net of disutility from work/search). This special case can therefore be interpreted as reflecting a situation with a perfect capital market allowing individuals to smooth consumption via saving/dissaving.

Job Search

Individuals choose search effort s_{ij} to maximize W_{ij}^U, taking all "macro" variables as given. Current search may result in a job match the next period. The first order conditions to the search problem when the current state is bad read[18]

$$f'(1-s_{BB}) = \frac{\pi}{1+\rho}\alpha_B\left[W_{BB}^E - W_{BB}^U\right] + \frac{1-\pi}{1+\rho}\alpha_G\left[W_{GB}^E - W_{GB}^U\right] \tag{1}$$
$$f'(1-s_{BG}) = \frac{\pi}{1+\rho}\alpha_B\left[W_{BB}^E - W_{BB}^U\right] + \frac{1-\pi}{1+\rho}\alpha_G\left[W_{GB}^E - W_{GB}^U\right]. \tag{2}$$

Similar relations hold for the good state.

Note that search depends, in the usual way, on the gain from shifting from unemployment into a job in the next period in the case of as successful job match. However, since the business cycle situation may change, job search depends on the gain from finding a job if remaining in the current state (probability π) and the gain if there is a shift in the state of nature (probability $1-\pi$). The higher π, the more search is affected by the current state, and vice versa.

It follows immediately that search is independent of the past state of nature, and hence there are only two levels of search, i.e.

$$s_{BB} = s_{BG} = s_B$$
$$s_{GG} = s_{GB} = s_G$$

The intuition is that the search decision is forward-looking since current search influences the future labour market status, and therefore it is independent of the past state[19].

2.2 Firms

A filled job generates an output (exogenous) y, and firms can create job vacancies at a flow cost of ky ($k > 0$). A filled job may be destroyed in the next period if there is a job separation. The value of a filled job in a given state of nature is

[18] Concavity of the f function ensures that the second order conditions are fulfilled.

[19] Note that the separability assumption is crucial for this property.

$$\frac{\rho}{1+\rho}J_B^E = y - w + \frac{\pi}{1+\rho}\left[p_{BB}(J_B^V - J_B^E)\right] + \frac{1-\pi}{1+\rho}\left[p_{GB}(J_G^V - J_B^E) + (1-p_{GB})(J_G^E - J_B^E)\right] \tag{3}$$

$$\frac{\rho}{1+\rho}J_G^E = y - w + \frac{\pi}{1+\rho}\left[p_{GG}(J_G^V - J_G^E)\right] + \frac{1-\pi}{1+\rho}\left[p_{BG}(J_B^V - J_G^E) + (1-p_{BG})(J_B^E - J_G^E)\right] \tag{4}$$

Note that the value of a filled job does not depend on the past state. A vacant job may be filled in the future if there is a job match, and hence the current value of a vacant job in a given state is

$$\frac{\rho}{1+\rho}J_B^V = -ky + \frac{\pi}{1+\rho}q_B(J_B^E - J_B^V) + \frac{1-\pi}{1+\rho}q_G(J_G^E - J_G^V) + \frac{1-\pi}{1+\rho}\left(J_B^V - J_G^V\right)$$
$$\frac{\rho}{1+\rho}J_G^V = -ky + \frac{\pi}{1+\rho}q_G(J_G^E - J_G^V) + \frac{1-\pi}{1+\rho}q_B(J_B^E - J_B^V) + \frac{1-\pi}{1+\rho}\left(J_G^V - J_B^V\right)$$

where q_i denotes the probability of filling a vacant job in state i (see below). Vacancies are created up to the point where the value of a vacancy is zero, i.e. $J_G^V = J_B^V = 0$. From this it follows that

$$J_B^E = \frac{q_G}{q_B}J_G^E \tag{5}$$

i.e. the relative value of having a filled job in either state (B or G) depends on the ratio of the job finding rates, and $J_G^E > J_B^E$ if $q_B > q_G$, or vice versa. Hence, the value of a filled job is higher in the G state than in the B state, provided the job filling rate is lower $q_G < q_B$. The intuition is that the more difficult it is to fill a vacant job, the higher is the value of a filled job. The value of a filled job is

$$J_B^E = \frac{ky(1+\rho)}{q_B}$$
$$J_G^E = \frac{ky(1+\rho)}{q_G}$$

2.3 Wages

Wages are assumed to be set in a Nash-bargain after a match has been made. Employed workers are represented by unions having the objective of maximizing wages for employed workers. As has been argued in non-cooperative approaches to justify this bargaining model, the relevant outside option is what can be achieved during delay in reaching an agreement (see Binmore, Rubinstein and Wolinski (1986)). This outside option is assumed to be zero for both workers and firms, and hence the wage setting problem is given as the solution to

$$Max_w \quad [w]^{\beta}\,[y-w]^{1-\beta}$$

where $0 < \beta < 1$. The bargaining power of firms is thus β, and for workers $(1-\beta)$. This wage setting model implies that the wage is given as

$$w = \beta y \tag{6}$$

The main attraction of this approach is that it gives a simple wage relation which implies that the wage is rigid across states of nature[20]. Alternative routes may be pursued in modelling wage rigidities (see e.g. Hall (2005) and Hall and Milgrom (2008) for recent work in a search matching context), and the specific formulation adopted here is to be considered as an illustrative workhorse model. The crucial property is that wages do not respond to variations in unemployment (job separations etc.)[21].

2.4 Public sector

The public sector provides the benefit level b_i to unemployed in a given state of nature i and finances this by a proportional tax rate τ and a (path dependent) lump sum tax T_{ij}. The income tax rate τ is assumed to be constant across states of nature; i.e. any business cycle dependency runs via the benefit level and the lump sum tax.

The primary budget balance in any state is

$$B_{ij} = (1-u_{ij})\tau w + T_{ij} - b_i u_{ij}.$$

Hence, the debt level D when the current state is bad (similar expressions when the current state is good are easily derived) is:

$$\frac{\rho}{1+\rho}D_{BB} = b_B u_{BB} - \tau w(1-u_{BB}) - T_{BB} + \frac{1-\pi}{1+\rho}\left[D_{GB} - D_{BB}\right]$$
$$\frac{\rho}{1+\rho}D_{BG} = b_B u_{BG} - \tau w(1-u_{BG}) - T_{BG} + \frac{\pi}{1+\rho}\left[D_{BB} - D_{BG}\right] + \frac{1-\pi}{1+\rho}\left[D_{GB} - D_{BG}\right]$$

Since the primary budget is dependent on the current state of nature, nothing ensures that the debt level is stationary. A sequence of bad draws in combination with debt servicing may lead to a non-sustainable debt level. Several budget policies contingent on the debt level could be conceived to ensure that the intertemporal budget constraint for the public sector is fulfilled. To find a stationary Markov equilibrium, we choose here a simpler procedure and

[20] The issue of the cyclical properties of wages is a controversial question in macroeconomics. However, the empirical evidence on cyclical properties of wages is inconclusive (see e.g. Abraham & Haltivanger (1995) and Messina et al. (2009)).

[21] Allowing for wages to be different across states of nature may contribute to dampen unemployment variations via lower wages in downturns and higher wages in upturns, see e.g. Coles and Masters (2007).

propose a tax policy rule and an implied budget profile, ensuring that each state of nature is associated with a well-defined and finite debt level. Therefore this policy is sustainable and introduces risk sharing via the public budget, but clearly it is dominated by more sophisticated policies. Accordingly the gains from risk sharing across states of nature are downward biased in the following.

Specifically, consider the following rule for the state contingent lump sum taxes

$$T_{ij} = b_i u_{ij} - \tau w(1 - u_{ij}) \text{ for } i, j = \{B, G\}.$$

It can be shown that this policy ensures a stationary debt level in all states of nature, and thus satisfies the no-Ponzi condition (see Appendix B).

The policy rule outlined above implies that the primary balance is given as

$$\begin{aligned}
B_{BB} &= 0 \\
B_{BG} &= [b_G u_{GB} - \tau w(1 - u_{GB})] - [b_B u_{BG} - \tau w(1 - u_{BG})] \\
B_{GB} &= [b_B u_{BG} - \tau w(1 - u_{BG})] - [b_G u_{GB} - \tau w(1 - u_{GB})] \\
B_{GG} &= 0
\end{aligned}$$

Hence if $u_{BG} > u_{GB}$ and/or $b_B > b_G$ (see below), a budget deficit arises when a bad state follows a good state ($B_{BG} < 0$), and a budget surplus arises when a good state follows a bad state ($B_{GB} > 0$). If the state of nature is unchanged, the budget is in balance. It is thus implied that there is an across state of nature insurance mechanism when the state of nature changes, but not when it persists. Broadly speaking, this captures that transitory shocks can be diversified, while persistent shocks can not.

2.5 Matching

Vacant job may be filled with a one period lag, and matches are determined by a standard constant returns to scale matching function; i.e. the number of matches in state i are given as

$$m(S_{ij}, V_{ij}) \equiv A S_{ij}^{\varepsilon} V_{ij}^{1-\varepsilon} \quad , 0 < \varepsilon < 1$$

where V_i is the number of vacancies in state i, and aggregate search is given as

$$S_{ij} = s_i u_{ij}$$

The job finding rate is therefore

$$\alpha_{ij} \equiv \frac{m(S_{ij}, V_{ij})}{S_{ij}} = m(1, \theta_{ij}) = A\theta_{ij}^{1-\varepsilon}$$

where $\theta_{ij} \equiv \frac{V_{ij}}{S_{ij}}$ measures market tightness, and $\alpha(\theta_{ij})$, $\alpha'_\theta(\theta_{ij}) > 0$.

Firms fill vacancies at the rate

$$q_{ij} \equiv \frac{m(S_{ij}, V_{ij})}{V_{ij}} = m(\theta_{ij}^{-1}, 1) = A\theta_{ij}^{-\varepsilon}$$

where $q'_\theta(\theta) < 0$.

2.6 Inflows and outflows

The unemployment rate is a stock variable displaying inertia due to the matching friction. Hence, in general the unemployment rate adjusts sluggishly to changes in the state of nature[22], and therefore it displays path dependence. A stationary Markov equilibrium is ensured if it is assumed that job separation rates differ at state transitions, so as to ensure that the unemployment rate only takes on two values, u_B and u_G. The intuition is that if there is a shift from the "good" to the "bad" state, there is an extraordinarily high job separation rate, and vice versa when shifting from a "bad" to a "good" equilibrium. Hence,

$$u_{BG} = u_{BB} = u_B$$

$$u_{GB} = u_{GG} = u_G$$

The change in unemployment is given as the difference between job separations and hires. Hence, to ensure that the economy fluctuates between two levels of unemployment u_B and u_G for given exogenous job separation rates p_{BB} and p_{GG}, it is required that the following restrictions are met

$$0 = (1 - u_B)p_{BB} - \alpha_B s_B u_B \tag{7}$$

$$u_G - u_B = (1 - u_B)p_{GB} - \alpha_G s_G u_G \tag{8}$$

$$u_B - u_G = (1 - u_G)p_{BG} - \alpha_B s_B u_B \tag{9}$$

$$0 = (1 - u_G)p_{GG} - \alpha_G s_G u_G \tag{10}$$

Note that α and s only depend on the current state, and u_i is the unemployment rate in state $i(= B, G)$. It is an implication that the above conditions determine p_{GB} and p_{BG}[23]. From (7) and (9) we have

$$p_{BG} = \frac{u_B - u_G}{(1 - u_G)} + \frac{(1 - u_B)}{(1 - u_G)} p_{BB} \tag{11}$$

[22] See e.g. Pissarides and Mortensen (1994) and Shimer (2005) for business cycle versions of the search model in which the unemployment rate evolves from the initial unemployment rate conditional on the realization of shocks.

[23] Note that this makes the job separations at "switching" states a jump variable to ensure that unemployment only varies between two levels.

and from (8) and (10) that

$$p_{GB} = \frac{u_G - u_B}{(1 - u_B)} + \frac{(1 - u_G)}{(1 - u_B)} p_{GG} \tag{12}$$

It follows that $u_G - u_B < 0$ implies that a shift from the G-state to the B-state is associated with extraordinarily high job separations, i.e. $p_{BG} > p_{BB}$, and a shift from the B-state to the G-state is associated with an extraordinarily low level of job separations[24], i.e. $p_{GB} < p_{GG}$.

2.7 Equilibrium

In Appendix C it is proved that there exists a stationary Markov equilibrium in which market tightness is larger in a good than a bad state of nature $\theta_G > \theta_B$. This implies that i) unemployment is higher in a bad state than a good state, i.e. $u_B > u_G$, ii) the job finding rate is lower in a bad state $\alpha_B < \alpha_G$, iii) the job filling rate is higher in a bad state $q_B > q_G$, and therefore iv) the value of a filled job is higher in a good state $J_G^E > J_B^E$.

2.8 Numerical illustrations

Below we present some numerical results to clarify various mechanisms, and they are based on the following functional forms. The utility from income is

$$g(y) = \frac{(y)^{1-\varkappa}}{1-\varkappa}$$

and from leisure

$$f(1-l) = \log(1-l)$$

where $\varkappa = 4$. Following Frederiksson & Holmlund (2006), among others, the matching function is assumed to be Cobb-Douglas of the form $m = As^{1-\varepsilon}v^{\varepsilon}$, with $\varepsilon = 0.5$ and $A = 0.29$. Time is quarterly, and we discount utility at $\rho = 0.003$ and assume that workers spend 30% of their time at work, $l = 0.3$. The tax rate is $t = 0.01$ and $\beta = 0.5$. Finally, output is set to $y = 1$, vacancy costs are set to $k = 0.2$.

3 Optimal business cycle dependent benefits

We now turn to the issue of how unemployment benefits should depend on the business cycle situation. We follow standard practice and consider a utilitarian social welfare function. In

[24] Conditions ensuring that $p_{GB} > 0$ are assumed fulfilled.

the general case, we have that total utility can be written

$$\Psi = \sum_{i,j=B,G} \sigma_{ij} \left[(1-u_{ij}) W_{ij}^E + u_{ij} W_{ij}^U \right]$$

where σ_{ij} is the ex ante unconditional probability of being in state (i,j) ($\sum_{i,j=B,G} \sigma_{ij} = 1$), and the value functions are evaluated for the tax payments implied by the budget constraints given above. Solving for the optimal benefit levels (b_B and b_G), we have the following first order condition

$$\sum_{i,j=B,G} \sigma_{ij} \left[(1-u_{ij}) \frac{\partial W_{ij}^E}{\partial b_k} + u_{ij} \frac{\partial W_{ij}^U}{\partial b_k} + \left[W_{ij}^E - W_{ij}^U \right] \frac{\partial u_{ij}}{\partial b_k} \right] = 0 \text{ for } k = B, G. \tag{13}$$

2nd order conditions are assumed fulfilled.

3.1 One state model

Although the main interest is to analyse the design of unemployment benefits over the cycle, it is useful to start by considering the one state version of the model to bring out some basic points. This applies both in terms of interpreting the expression for optimal benefits (13) and in relation to stressing why an explicit modelling of business cycle shifts makes a crucial difference.

Consider the one state version of the model, i.e. there is no shift in the state of nature ($\pi = 1$), or alternatively that the job separation rate is state invariant ($p_{BB} = p_{GG} = p$) (for details see Appendix D). In this case there exists a stationary equilibrium (see Appendix D) with a given unemployment rate u and the budget balances. Equilibrium unemployment is larger, the higher the job separation rate ($\frac{\partial u}{\partial p} > 0$), and the higher the benefit level ($\frac{\partial u}{\partial b} > 0$).

In this case the condition for the optimal benefit level (13) reads

$$(1-u) \frac{\partial W^E}{\partial b} + u \frac{\partial W^U}{\partial b} + \frac{\partial u}{\partial b} \left[W^U - W^E \right] = 0$$

Note that in the one state case there is only one policy decision since if the compensation to unemployed is determined, then the tax payment for the employed follows directly from the budget constraint. The first order condition for the optimal benefit level can be rewritten as (see Appendix E)

$$u \left[g'(b) - g'(w - \frac{u}{1-u} b) \right] = \frac{\partial u}{\partial b} \left[W^E - W^U \right]. \tag{14}$$

This expression has a straightforward interpretation in terms of marginal gains and costs of providing unemployment benefits both measured in units of utilities. The LHS gives the marginal benefit as the difference in marginal utility of consumption for unemployed relative to employed times the unemployment rate. The larger the difference in marginal utilities or the unemployment rate, the larger the marginal gains from providing higher benefits. The RHS gives the marginal costs as the effects of benefits on unemployment (the distortion) times the utility gain from being employed rather than unemployed. If either the distortion is large or the utility loss from being unemployed is large, the marginal costs of providing benefits are high.

The expression (14) thus implies that the marginal gains from unemployment benefits tend to be large in a state of nature with high unemployment, while the marginal costs are low if the gain from being employed is small, and vice versa. These effects turn out to be crucial when we allow for different business cycle situations below.

To gain more insight into the effects involved, it is useful to consider the special case where there is no distortion, i.e. $\frac{\partial u}{\partial b} = 0$ (follows if $\frac{\partial s}{\partial b} = 0$, i.e. no incentive effects of unemployment benefits). In this special case, optimal benefits are determined by the condition, cf.(14),

$$g'(b) = g'(w - \frac{u}{1-u}b) \tag{15}$$

i.e. the optimal benefit level ensures that the marginal utility of income is the same for employed and unemployed[25]. This is known as the "Borch condition" for full insurance (Borch (1960)). The insurance effect is not directly related to the unemployment rate in this case but depends on the conditions prevailing as either unemployed or employed. However, there is a budget effect since the benefits are financed by taxes levied on the employed, and we have

$$\frac{\partial b}{\partial u} = -\frac{g''(w - \frac{u}{1-u}b)\frac{b}{(1-u)^2}}{g''(b) + g''(w - \frac{u}{1-u}b)\frac{u}{1-u}} < 0$$

i.e. a higher unemployment rate is accompanied by lower benefits. The intuition is that higher unemployment raises the financing requirements to maintain a given benefit level, which in turn reduces the disposable income of employed and thus raises their marginal utility of income. To rebalance the marginal utility of consumption between the two groups, it is necessary to lower benefits. While non-distortionary benefits are a special case, this shows that a one state model (implying a balanced budget requirement) tends to imply pro-cyclical benefits.

[25] Note that the participation constraint is implicitly assumed fulfilled. Otherwise there is an additional constraint, in which case the benefit level is determined by the "corner" condition that

$$\left[h(w - \frac{u}{1-u}b) - e(1-l)\right] - [g(b) - f(1-s_u)] = 0$$

Returning to the role of distortions, the question is whether benefits are more or less distortionary in a situation with a high unemployment rate. The driver of the unemployment rate in this model is the job separation rate (p), and hence the sign of $\frac{\partial}{\partial p}\frac{\partial u}{\partial b}$ is crucial. If $\frac{\partial}{\partial p}\frac{\partial u}{\partial b} > 0$, it follows that benefits are more distortionary with a high job separation rate and thus unemployment rate, and this goes in the direction of making optimal benefits pro-cyclical, and vice versa for $\frac{\partial}{\partial p}\frac{\partial u}{\partial b} < 0$. The distortion arises in this model via search, which is seen by noting that (see Appendix E)

$$\frac{\partial u}{\partial b}\frac{b}{u} = \frac{-b}{s}\frac{\partial s}{\partial b}\left[1-u\right].$$

Hence, benefits tend to increase unemployment ($\frac{\partial u}{\partial b} > 0$) because they make individuals search less ($\frac{\partial s}{\partial b} < 0$). Note that the distortion of search matters more for the unemployment effect if the unemployment rate is low. The sensitivity of the search distortion to the job separation rate turns out to depend crucially on how search is affected by the job separation rate. In Appendix E it is shown that for $\eta(s) \equiv \frac{f''(\cdot)(1-s)}{f'(\cdot)}$ to be constant (as assumed in the numerical illustrations), we have for a given tax rate

$$sign\left(\frac{\partial}{\partial p}\left[\frac{\partial s}{\partial b}\frac{b}{s}\right]\right) = sign\left(\frac{\partial s}{\partial p}\right).$$

Since $\frac{\partial s}{\partial p} < 0$, i.e. individuals search less, the higher the job separation rate, this implies $\frac{\partial}{\partial p}\left[\frac{\partial s}{\partial b}\frac{b}{s}\right] < 0$, and therefore search is more distorted by a marginal benefit increase at a high job separation rate (high unemployment level) than at a low job separation rate (low unemployment rate). The fact that the one state model implies that job search moves pro-cyclically therefore tends to imply that distortions move counter-cyclically. As is pointed out below, when allowing for changes in the business cycle situation the search response may be different, and this has important implications for how the optimal benefit level depends on the business cycle situation.

In sum, we find in the one state version of the model that both the budget effect and the distortion effect tend to call for pro-cyclical benefit levels. However, this result arises by comparing steady states, and as shown in the next section this changes when allowing explicitly for changes in the business cycle situation, which both opens for risk diversification across states of nature and a different response of search and thus distortions to the benefit level.

In Figure 1 we show the optimal benefit level in the one state model for the parameter values presented above. The figure shows as expected that equilibrium unemployment is higher, the higher the job separation rate. The optimal benefit level (net compensation) is seen to be decreasing in the job separation rate and thus falling in the unemployment rate. Hence, in the one state case optimal benefits are pro-cyclical; if unemployment is high, net compensation is low, and vice versa. The main driver behind this is the budget effect discussed earlier.

Insert figure 1 here

3.2 Two state model - insurance and distortions

Crucial in the two state model is the explicit modelling of changes in the business cycle situation. The possibility of a business cycle change captured by π $(0 < \pi < 1)$ affects behaviour since these possible changes are anticipated by individuals. The following considers this in detail both to explain the difference to the one state model and to work out the implications for business cycle contingencies in unemployment benefits.

Business cycles and search Job search is the key behaviourable variable, and its response to the business cycle situation is crucial. The standard version of the matching model with a stationary equilibrium (one state of nature) implies that a higher job separation rate and thus unemployment rate is associated with less search (see above). Making inferences from a comparison of stationary equilibria would thus lead to the conclusion that search is lower in bad than in good states of nature. This conclusion does not hold when business cycle changes are explicitly accounted for, and this underlines the need to model fluctuations explicitly.

To see how changes in the business cycle affect job search, consider for the sake of argument search in the bad state determined by (10)

$$f'(1 - s_{BB}) = \pi\alpha_B \left[W_{BB}^E - W_{BB}^U\right] + (1 - \pi)\alpha_G \left[W_{GB}^E - W_{GB}^U\right].$$

Two factors determine the return to job search, namely, the probability of finding a job and the gain from finding a job. Both of these effects go in the direction of strengthening job search in the bad state and weakening job search in the good state. To see this consider first the ex ante perceived job finding probability, which is given as the probability of being in a given state of nature in the future times the job finding rate in that state of nature. Suppose for the sake of argument that $W_{BB}^E - W_{BB}^U = W_{GB}^E - W_{GB}^U$, in which case it follows that the possibility of shifting to the "good" state $(0 < \pi < 1)$ will increase search in the "bad" state compared to a situation with no chance of a change in the business cycle situation $(\pi = 1)$. Since $\alpha_G > \alpha_B$, we have

$$\pi\alpha_B + (1 - \pi)\alpha_G > \alpha_B \text{ for all } \pi < 1$$

i.e. the possibility of a shift to a state with a higher job finding rate increases, other things being equal, the search level, and the effect is stronger, the larger the difference in job finding rates between the two states. The effect is obviously the opposite for search in the good state of nature, i.e.

$$\pi\alpha_G + (1-\pi)\alpha_B < \alpha_G \text{ for all } \pi < 1.$$

Moreover, shifting business cycle situations affect the gain from having a job $(W^E - W^U)$. We have from the value functions that

$$\left[W_{BB}^E - W_{BB}^U\right] = \frac{\Delta + [1-\pi]\,(1 - p_{GB} - \alpha_G s_{BB})\left[W_{GB}^E - W_{GB}^U\right]}{\rho + 1 + \pi\left[p_{BB} + \alpha_B s_{BB} - 1\right]}$$

where

$$\Delta \equiv g(w\,[1-\tau] - T_{BB}) + f(1-l) - g(b_B - T_{BB}) + f(1 - s_{BB})$$

is the instantaneous utility gain from being employed rather than unemployed. If there is no chance of a change in the business cycle situation ($\pi = 1$), we have

$$\left[W_{BB}^E - W_{BB}^U\right]|_{\pi=1} = \frac{\Delta}{\rho + p_{BB} + \alpha_B s_{BB}}.$$

Hence, using that $W_{GB}^E - W_{GB}^U > 0$

$$\left[W_{BB}^E - W_{BB}^U\right] > \left[W_{BB}^E - W_{BB}^U\right]|_{\pi=1}.$$

By similar reasoning it can be shown as follows

$$\left[W_{GG}^E - W_{GG}^U\right] < \left[W_{GG}^E - W_{GG}^U\right]|_{\pi=1}.$$

Hence, the possibility that the business cycle situation might change tends to increase the gain from having a job in the bad state of nature, and to decrease it in the god state of nature. This goes in the direction of increasing search in the bad state and lowering it in the good state[26]. In sum both the difference in the job finding rates and the gains from employment induced by shifts in the business cycle situation tend to induce more search in the bad state, and less search in the good state.

The role of the business cycle situation for job search is illustrated in Figure 2 showing on the x axis a widening of the difference in the job separation rate between the two states of nature (zero difference corresponds to a one state model). It is seen that job search is higher in bad states of nature. The difference widens as expected as the two states become more different.

Insert figure 2 here

[26] Shimer (2004) similarly argues that search intensity need not be pro-cyclical in a discrete time setting, focussing on the fact that job search is affected by how easy it is to find a job.

3.2.1 Business cycles and insurance

Turning to the insurance aspects, there are two dimensions of insurance. One is between the employed and unemployed in a given state of nature. The other dimension is across states of nature. To see this, note that disposable income for the employed (y_{ij}^E) is

$$y_{ij}^E = w(1-\tau) - T_{ji} = w - (b_B + \tau w)\, u_{ij} \text{ for } i,j = \{B,G\}$$

and for the unemployed

$$y_{ij}^U = b_i - T_{ji} = b_i + \tau w - (b_B + \tau w) u_{ij} \text{ for } i,j = \{B,G\}.$$

It is seen that in a given state of nature an increase in the benefit level increases the disposable income of the unemployed and decreases it for the employed. By changing the benefit level, it is thus possible to provide insurance (redistribute) between employed and unemployed[27]. Second, by running a non-balanced budget in the swing states (GB and BG), it is possible to insure across states of nature. In the present context, this possibility arises when the state of nature changes, and it is seen that for $b_B > b_G$ and $u_B > u_G$ both employed and unemployed are compensated when the state shifts from G to B, and vice versa. The latter is also seen by considering how a change in the state of nature affects the overall position of employed, where we have

$$\frac{\rho}{1+\rho}\left[W_{BG}^E - W_{BB}^E\right] = h(w\,[1-\tau] - T_{BG}) - h(w\,[1-\tau] - T_{BB})$$
$$\frac{\rho}{1+\rho}\left[W_{GB}^E - W_{GG}^E\right] = h(w\,[1-\tau] - T_{GB}) - h(w\,[1-\tau] - T_{GG}).$$

Hence, if $T_{BB} > T_{BG}$ and $T_{GB} > T_{GG}$, it follows that $W_{BG}^E > W_{BB}^E$ and $W_{GB}^E < W_{GG}^E$; i.e. employed are better off when a bad state follows a good state than when it follows a bad state, and they are worse off when a good state follows a bad state rather than a good state. To put it differently, a shift from a good to a bad state is compensated, whereas a shift from a bad to a good state implies a contribution.

Similarly, a change in the state of nature affects the overall position of the unemployed by

$$W_{BG}^U - W_{BB}^U = \frac{g(b_H - T_{BG}) - g(b_B - T_{BB})}{\rho + 1}$$

and

$$W_{GB}^U - W_{GG}^U = \frac{g(b_G - T_{GB}) - g(b_G - T_{GG})}{\rho + 1}$$

[27] It is easily verified that it is not possible with the state dependent policy to achieve complete insurance as defined by the Borch condition for employed and unemployed across the four different possible states of nature.

and if $T_{BB} > T_{BG}$ and $T_{GB} > T_{GG}$, it follows that $W_{BG}^U > W_{BB}^U$ and $W_{GB}^U < W_{GG}^U$; i.e. unemployed are better off when a bad state follows a good rather than a bad state, and worse off when a good state follows a bad rather than a good state.

3.2.2 Business cycle dependent distortions

The distortionary effects of the benefit level on unemployment are crucial for the optimal benefit level (see also below). Intuitively, one would expect the benefit level to be more distortionary in good states of nature with higher job finding rates than in bad states of nature. To address this issue, we can rewrite optimal search in a given state i from (1) and (2) by the implicit function

$$s_i = \phi(z_{ij}) \qquad \phi' > 0$$

where the expected gain from shifting from unemployment into employment is given as

$$z_{ij} \equiv \frac{\pi}{1+\rho}\alpha_i \left[W_{ii}^E - W_{ii}^U\right] + \frac{1-\pi}{1+\rho}\alpha_j \left[W_{ji}^E - W_{ji}^U\right]$$

i.e. search is increasing in the expected gain from becoming employed. It follows that

$$\frac{\partial s_i}{\partial z_{ij}}\frac{z_{ij}}{s_i} = \frac{1}{\epsilon(s_i)}\frac{1-s_i}{s_i}$$

where $\eta(s_i) \equiv -\frac{f''(1-s)}{f'(1-s)}(1-s) > 0$. Assuming that the latter elasticity is constant (as is the case in the numerical illustrations), we have that if unemployed search more in a bad than a good state $s_B > s_G$, then it follows that

$$\frac{\partial s_B}{\partial z_{BG}}\frac{z_{BG}}{s_B} < \frac{\partial s_G}{\partial z_{GB}}\frac{z_{GB}}{s_G}$$

i.e. the elasticity of search wrt. the expected gain from becoming employed is smaller in a bad than a good state; i.e. search tends to be less distorted in a bad than in a good state of nature (see also section 3.1)

The following tables consider this issue and report the elasticities of search and unemployment, respectively, with respect to the benefit level in the two possible states of nature. Consider first search. As expected, higher benefits lower search. There is both a direct effect in the state of nature for which the change applies and an effect in the alternate state since agents perceive the possible shift in the business cycle situation. The direct effect is numerically larger in the good than in the bad state; i.e. search is affected more by benefits in good than in bad states of nature.

Table 1: Effects of changing benefits: elasticity of search intensity wrt. benefit level

	$\pi = 0.7$		$\pi = 0.9$	
	b_B	b_G	b_B	b_G
Elasticity of search, bad state: s_B	-1.58	-0.87	-1.87	-0.33
Elasticity of search, good state: s_G	-0.90	-1.69	-0.34	-1.92

Note: $p_{BB} = 0.042$ and $p_{GG} = 0.038$.

The effect of benefits on the unemployment rate derives from its effect on job search, and we have

$$\frac{\partial u_B}{\partial b_B}\frac{b_B}{u_B} = -(1-u_B)\frac{\partial s_B}{\partial b_B}\frac{b_B}{s_B}$$

and a similar relation holds for the good state (see Appendix E). Using this we can easily characterize distortions in terms of unemployment effects, and table 2 provides numerical illustrations. As should be expected, the direct effect is stronger, the more persistent the business cycle situation, whereas the indirect effect on the alternate state is stronger, the less persistent the business cycle situation. It is seen that the direct effect of benefit increases is larger in good than in bad states of nature; i.e. the distortions are business cycle dependent, and we have that they are larger in good than in bad states. This goes in the direction of making optimal benefit levels business cycle dependent, and we explore this issue in the next section.

Table 2: Effects of changing benefits: elasticity of unemployment rate wrt. benefit level

	$\pi = 0.7$		$\pi = 0.9$	
	b_B	b_G	b_B	b_G
Elasticity of unemployment, bad state: u_B	1.47	0.83	1.72	0.35
Elasticity of unemployment, good state: u_G	0.88	1.61	0.36	1.79
Elasticity of mean unemployment: u	1.20	1.18	1.07	1.04

Note: p_{BB}= 0.042 and p_{GG}= 0.038.

3.3 Optimal business cycle contingent benefits

We can now return to the issue of how optimal benefits depend on the business cycle situation; that is, should they be counter-cyclical or pro-cyclical? Figure 3 shows how the optimal net compensation (benefits less taxes paid) for the four possible states of nature depends on the underlying persistence in the business cycle situation[28]. It is seen that the net compensation is highest when a bad state follows a good state, and the intuition is that unemployed are compensated for the more bleak outlooks and lower possibilities of finding a job. Oppositely, we have the lowest net compensation when a good state follows a bad state. The net compensation offered when the bad state persists (BB) is higher than when the good state persists (GG). It is seen that the differences in net compensation are largest for intermediary levels of persistence. The intuition is that the expected gains from shifting status become lower in bad states and higher in good states of nature.

Insert figure 3 here

Figure 4 shows that optimal business cycle dependent benefits imply more variability in unemployment rates than business cycle independent benefits. The reason is that benefits are increased in bad times with high unemployment, and decreased in good times with low employment. Hence, the optimal state contingent policy shifts compensation from good to bad times, and search effort from bad to good times. In this way insurance and incentives are better aligned with the business cycle situation.

[28] We present the optimal net compensation imposing a symmetry condition; that is, increases in bad states equal decreases in good states. Considering whether optimal policies imply asymmetric adjustments, we found only small differences to the symmetric case.

Insert figure 4 here

This shows that it is possible to improve the insurance properties by making benefit levels business cycle dependent without causing an increase in the structural (average) unemployment rate. However, this gain may be achieved at the cost of more variability in unemployment.

4 Conclusion

In this paper the effects of making unemployment benefits conditional on the business cycle situation have been shown to depend not only on an insurance effect but also a budget and an incentive (distortion) effect. We have shown in a stylized business cycle model that the benefit level tends to distort more in good than in bad times, and this strengthens the argument for counter-cyclical benefit levels. It is an important implication that such a dependency is welfare improving (utilitarian) since it shifts utility for unemployed from good to bad times. Moreover, it can reduce structural (average) unemployment, but it may imply that the unemployment rate becomes more sensitive to the business cycle situation. The present analysis therefore shows that a business cycle dependent unemployment insurance system may provide better insurance without resulting in higher structural unemployment.

The preceding analysis considers a very stylized unemployment insurance scheme focussing entirely on the benefit level. In practice, it may be an equally important dimension of the unemployment insurance to make the benefit duration business cycle contingent. We conjecture that the case for such a business cycle dependency is qualitatively the same as the one found in this paper for the benefit level.

There are several possible extensions of the current analysis. First, we completely ignore aggregate demand effects (automatic stabilizers) of running a business cycle dependent policy. We conjecture that incorporation of this aspect will strengthen the case for having a state dependent benefit level. Second, the model used in this paper relies on a very stylized description of the business cycle and a somewhat rudimentary policy rule for diversification across states of nature. It would be interesting to extend the model in these two dimensions - something which we leave for future work.

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4.1 Appendix A: Value functions

Consider first the value functions for currently employed workers (W_{ij}^E) in a given current state (i) and past state (j).

$$\begin{aligned} W_{BB}^E(t) &= \nu(w,\tau,T_{BB}) + \frac{\pi}{1+\rho}\left[p_{BB}W_{BB}^U(t+1) + (1-p_{BB})W_{BB}^E(t+1)\right] \\ &+ \frac{1-\pi}{1+\rho}\left[(1-p_{GB})W_{GB}^E(t+1) + p_{GB}W_{GB}^U(t+1)\right] \end{aligned}$$

Using that in stationary state $W_{BB}^E(t) = W_{BB}^E$ for all t, and similarly for all other value functions $W_{i,j}^x$ ($x = e,u; i,j = B,G$) we have

$$\begin{aligned} W_{BB}^E &= \nu(w,\tau,T_{BB}) + \frac{\pi}{1+\rho}\left[p_{BB}\left[W_{BB}^U - W_{BB}^E\right] + W_{BB}^E\right] \\ &+ \frac{1-\pi}{1+\rho}\left[(1-p_{GB})\left[W_{GB}^E - W_{BB}^E\right] + p_{GB}\left[W_{GB}^U - W_{BB}^E\right] + W_{BB}^E\right] \end{aligned}$$

or

$$\begin{aligned} \frac{\rho}{1+\rho}W_{BB}^E &= \nu(w,\tau,T_{BB}) + \frac{\pi}{1+\rho}p_{BB}\left[W_{BB}^U - W_{BB}^E\right] \\ &+ \frac{1-\pi}{1+\rho}\left[(1-p_{GB})\left[W_{GB}^E - W_{BB}^E\right] + p_{GB}\left[W_{GB}^U - W_{BB}^E\right]\right] \end{aligned}$$

Similarly we have

$$\begin{aligned} W_{BG}^E(t) &= \nu(w,\tau,T_{BG}) + \frac{\pi}{1+\rho}\left[(1-p_{BB})W_{BB}^E(t+1) + p_{BB}W_{BB}^U(t+1)\right] \\ &+ \frac{1-\pi}{1+\rho}\left[(1-p_{GB})W_{GB}^E(t+1) + p_{GB}W_{GB}^U(t+1)\right] \end{aligned}$$

which in stationary state can be written

$$\begin{aligned} W_{BG}^E &= \nu(w,\tau,T_{BG}) + \frac{\pi}{1+\rho}\left[p_{BB}\left[W_{BB}^U - W_{BB}^E\right] + W_{BB}^E\right] \\ &+ \frac{1-\pi}{1+\rho}\left[(1-p_{GB})\left[W_{GB}^E - W_{BB}^E\right] + p_{GB}\left[W_{GB}^U - W_{BB}^E\right] + W_{BB}^E\right] \end{aligned}$$

and hence

$$\begin{aligned} \frac{\rho}{1+\rho}W_{BG}^E &= \nu(w,\tau,T_{BG}) + \frac{\pi}{1+\rho}\left[p_{BB}\left[W_{BB}^U - W_{BB}^E\right]\right] \\ &+ \frac{1-\pi}{1+\rho}\left[(1-p_{GB})\left[W_{GB}^E - W_{BB}^E\right] + p_{GB}\left[W_{GB}^U - W_{BB}^E\right]\right] + \frac{1}{1+\rho}\left[W_{BB}^E - W_{BG}^E\right] \end{aligned}$$

A similar procedure follows straightforwardly for the remaining value functions.

4.2 Appendix B: Stationary debt levels

To see that the proposed policy rule ensures stationary debt levels in all states, note that the primary budget balance now can be written

$$\begin{aligned}
B_{BB} &= 0 \\
B_{BG} &= [b_G u_{GB} - \tau w(1 - u_{GB})] - [b_B u_{BG} - \tau w(1 - u_{BG})] \\
B_{GB} &= [b_B u_{BG} - \tau w(1 - u_{BG})] - [b_G u_{GB} - \tau w(1 - u_{GB})] \\
B_{GG} &= 0
\end{aligned}$$

implying

$$B_{BG} = -B_{GB}$$

i.e. if the public sector is running a budget deficit when a bad state of nature with high job separations ($B_{BG} < 0$) replaces a good state of nature with low job separations, then it will run a similar surplus when a good state of nature replaces a bad state of nature. In this way the scheme allows some risk diversification. To see that this is consistent with a stationary debt level in any state of nature, observe further that

$$\begin{aligned}
\rho D_{GB} &= b_G u_{GB} - \tau w(1 - u_{GB}) - [b_B u_{BG} - \tau w(1 - u_{BG})] \\
&\quad + \pi [D_{GG} - D_{GB}] + (1 - \pi) [D_{BG} - D_{GB}] \\
\rho D_{BG} &= b_B u_{BG} - \tau w(1 - u_{BG}) - [b_G u_{GB} - \tau w(1 - u_{GB})] \\
&\quad + \pi [D_{BB} - D_{BG}] + (1 - \pi) [D_{GB} - D_{BG}]
\end{aligned}$$

implying that

$$(\rho + \pi) [D_{GB} + D_{BG}] = \pi [D_{GG} + D_{BB}]$$

and since we have from the debt level equation for D_{GG} and D_{BB} that

$$(\rho + 1 - \pi) [D_{GG} + D_{BB}] = (1 - \pi) [D_{GB} + D_{BG}]$$

it follows that

$$\begin{aligned}
D_{GB} + D_{BG} &= 0 \\
D_{GG} + D_{BB} &= 0.
\end{aligned}$$

The debt levels in the different states of nature can be found by using that

$$\begin{aligned}
\rho D_{BB} &= b_B u_{BB} - \tau w(1 - u_{BB}) - T_{BB} + (1 - \pi) [D_{GB} - D_{BB}] \\
\rho D_{GB} &= b_G u_{GB} - \tau w(1 - u_{GB}) - T_{GB} + \pi [D_{GG} - D_{GB}] + (1 - \pi) [D_{BG} - D_{GB}]
\end{aligned}$$

which implies (by use of $b_B u_{BB} - \tau w(1-u_{BB}) - T_{BB} = 0$)

$$\begin{aligned}(\rho + 1 - \pi)\, D_{BB} &= (1-\pi) D_{GB} \\ (\rho + \pi + 2(1-\pi))\, D_{GB} &= b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB} - \pi D_{BB}\end{aligned}$$

Hence

$$D_{GB} = \left[\rho + \pi + 2(1-\pi) + \pi \frac{1-\pi}{\rho+1-\pi}\right] \left[b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB}\right]^{-1}$$

which is finite, and hence D_{BB}, D_{BG}, and D_{GG} are finite.

4.3 Appendix C: Proof of equilibrium to the two state model

Note that from (7) and (10), we have

$$\begin{aligned}\frac{m(1,\theta_B)}{(1-u_B)} &= p_{BB} \\ \frac{m(1,\theta_G)}{(1-u_G)} &= p_{GG}\end{aligned}$$

and hence

$$\frac{(1-u_G)}{(1-u_B)} \frac{m(1,\theta_B)}{m(1,\theta_G)} = \frac{p_{BB}}{p_{GG}} \tag{16}$$

Since $\frac{p_{BB}}{p_{GG}} > 1$, it follows from a sufficient condition that $u_B > u_G$ is $\frac{m(1,\theta_B)}{m(1,\theta_G)} < 1$ or $\theta_B < \theta_G$.

From the value functions for a filled job (3) and (4), we have by use of $J_G^V = J_B^V = 0$ that

$$\begin{aligned}\frac{\rho}{1+\rho} J_B^E &= y - w + \frac{\pi}{1+\rho}\left[p_{BB}(-J_B^E)\right] + \frac{1-\pi}{1+\rho}\left[p_{GB}(-J_B^E) + (1-p_{GB})(J_G^E - J_B^E)\right] \\ \frac{\rho}{1+\rho} J_G^E &= y - w + \frac{\pi}{1+\rho}\left[p_{GG}(-J_G^E)\right] + \frac{1-\pi}{1+\rho}\left[p_{BG}(-J_G^E) + (1-p_{BG})(J_B^E - J_G^E)\right]\end{aligned}$$

Hence

$$\begin{aligned}\left[\frac{\rho}{1+\rho} + \frac{\pi}{1+\rho} p_{BB} + \frac{1-\pi}{1+\rho}\left[p_{GB} + (1-p_{GB})(1 - \frac{q_B}{q_G})\right]\right] J_B^E &= y - w \\ \left[\frac{\rho}{1+\rho} + \frac{\pi}{1+\rho} p_{GG} + \frac{1-\pi}{1+\rho}\left[p_{BG} + (1-p_{BG})(1 - \frac{q_G}{q_B})\right]\right] J_G^E &= y - w\end{aligned}$$

and

$$\frac{\left[\rho + \pi p_{BB} + (1-\pi)\left[1 - (1-p_{GB})\frac{q_B}{q_G}\right]\right]}{\left[\rho + \pi p_{GG} + (1-\pi)\left[1 - (1-p_{BG})\frac{q_G}{q_B})\right]\right]} = \frac{J_G^E}{J_B^E} = \frac{q_B}{q_G} \tag{17}$$

where the last equality follows from (5).

Using (11) and (12), we have

$$1 - p_{BG} = \frac{(1-u_B)}{(1-u_G)}(1-p_{BB})$$

$$1 - p_{GB} = \frac{(1-u_G)}{(1-u_B)}(1-p_{GG})$$

Implying that (17) can be written

$$\frac{\left[\rho + \pi p_{BB} + (1-\pi)\left[1 - \frac{(1-u_G)}{(1-u_B)}(1-p_{GG})\frac{q_B}{q_G}\right]\right]}{\left[\rho + \pi p_{GG} + (1-\pi)\left[1 - \frac{(1-u_B)}{(1-u_G)}(1-p_{BB})\frac{q_G}{q_B})\right]\right]} = \frac{q_B}{q_G}$$

and using (16), we get

$$\frac{\left[\rho + \pi p_{BB} + (1-\pi)\left[1 - \frac{p_{BB}}{p_{GG}}\frac{m(1,\theta_G)}{m(1,\theta_B)}(1-p_{GG})\frac{q_B}{q_G}\right]\right]}{\left[\rho + \pi p_{GG} + (1-\pi)\left[1 - \frac{p_{GG}}{p_{BB}}\frac{m(1,\theta_B)}{m(1,\theta_G)}(1-p_{BB})\frac{q_G}{q_B})\right]\right]} = \frac{q_B}{q_G} \tag{18}$$

We have that

$$\frac{q_B}{q_G} = \frac{m(\theta_B^{-1},1)}{m(\theta_G^{-1},1)} = \frac{\theta_B^{-\alpha}}{\theta_G^{-\alpha}} = \left[\frac{\theta_G}{\theta_B}\right]^{\alpha}$$

and

$$\frac{q_B}{q_G}\frac{m(1,\theta_G)}{m(1,\theta_B)} = \frac{m(\theta_B^{-1},1)}{m(\theta_G^{-1},1)}\frac{m(1,\theta_G)}{m(1,\theta_B)} = \frac{\theta_B^{-\alpha}}{\theta_G^{-\alpha}}\frac{\theta_G^{1-\alpha}}{\theta_B^{1-\alpha}} = \frac{\theta_G}{\theta_B}$$

Condition (18) can now be written

$$\frac{\left[\rho + \pi p_{BB} + (1-\pi)\left[1 - \frac{p_{BB}}{p_{GG}}(1-p_{GG})\frac{\theta_G}{\theta_B}\right]\right]}{\left[\rho + \pi p_{GG} + (1-\pi)\left[1 - \frac{p_{GG}}{p_{BB}}(1-p_{BB})\frac{\theta_B}{\theta_G})\right]\right]} = \left[\frac{\theta_G}{\theta_B}\right]^{\alpha} \tag{19}$$

It is seen that the LHS of (19) is decreasing in $\frac{\theta_G}{\theta_B}$, and the RHS is increasing in $\frac{\theta_G}{\theta_B}$. It follows that there is a unique solution to $\frac{\theta_G}{\theta_B}$, from which all other variables can be found. To prove that $\frac{\theta_G}{\theta_B} > 1$, observe that for $\frac{\theta_G}{\theta_B} = 1$ we have that the RHS of (19) equals one, whereas the LHS is larger than one. Hence, it follows that $\frac{\theta_G}{\theta_B} > 1$. Note that this implies $\frac{q_G}{q_B} < 1$, and hence $u_G < u_B$.

4.4 Appendix D: One state model

In the one state case ($p_{BB} = p_{GG} = p$), we have that the model is summarized by

Value function employed	$\frac{\rho}{1+\rho}W^E = g(w-T) + g(1-l) + \frac{p}{1+\rho}\left[W^U - W^E\right]$
Value function unemployed	$\frac{\rho}{1+\rho}W^U = g(b) + f(1-s) + \frac{\alpha s}{1+\rho}\left[W^E - W^U\right]$
Search	$f'(1-s) = \frac{\alpha}{1+\rho}\left[W^E - W^U\right]$
Inflow outflow	$0 = (1-u)p - \alpha(\theta)su$
Job filling rate	$[\rho + p]\frac{k}{q(\theta)} = 1 - \beta$
Budget balance	$(1-u)T = ub$

Note that the job filling rate is found from (3), which in the one state case reads

$$\frac{\rho}{1+\rho}J^E = y - w - \frac{p}{1+\rho}J^E$$

and using (22) we have

$$[\rho + p]\,\frac{k}{q} = 1 - \beta$$

This determines the job filling rate (q) and thus also the job finding rate (α). It follows straightforwardly that $\frac{\partial q}{\partial b} = \frac{\partial \alpha}{\partial b} = 0$ and $\frac{\partial q}{\partial p}\frac{p}{q} = \frac{p}{\rho+p} \in [0,1]$ and $\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} = \frac{\epsilon-1}{\epsilon}\frac{p}{\rho+p} < 0$.

Note for later reference that

$$\frac{\rho}{1+\rho}\left[W^E - W^U\right] = g(w-T) + g(1-l) - [g(b) + f(1-s)] + \frac{p+\alpha s}{1+\rho}\left[W^U - W^E\right]$$

and hence

$$\left[W^E - W^U\right] = \left(\frac{1+\rho}{\rho+p+\alpha s}\right)[g(w-T) + f(1-l) - [g(b) + f(1-s)]] \tag{20}$$

From the inflow-outflow relation, we have

$$\frac{u}{1-u} = \frac{p}{\alpha s} \tag{21}$$

Job separation

First consider the response of the unemployment rate to the job separation rate. From (21) we have

$$\frac{\partial u}{\partial p} = [1-u]^2\,\frac{1 - \left[\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} + \frac{\partial s}{\partial p}\frac{p}{s}\right]}{s\alpha}$$

where $\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} < 0$ and $\frac{\partial s}{\partial p}\frac{p}{s}$ is found from (1) implying

$$-f''(1-s)\frac{\partial s}{\partial p} = \left[W^E - W^U\right]\frac{\partial \alpha}{\partial p} + \alpha\frac{\partial\left[W^E - W^U\right]}{\partial p}$$

and hence

$$\frac{\partial s}{\partial p}\frac{p}{s} = \frac{1}{\epsilon(s)}\frac{1-s}{s}\left[\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} + \frac{\partial\left[W^E - W^U\right]}{\partial p}\frac{p}{\left[W^E - W^U\right]}\right]$$

where $\eta(s) \equiv -\frac{f''()(1-s)}{f'()} > 0$.

From (20) we have

$$\begin{aligned}
\frac{\partial\left[W^E - W^U\right]}{\partial p} &= \left(\frac{1+\rho}{\rho+p+\alpha s}\right)f'(1-s)\frac{\partial s}{\partial p} - \frac{\partial(p+\alpha s)}{\partial p}\frac{(1+\rho)\left[h(w-T) + e(1-l) - [g(b) + f(\right.}{(\rho+p+\alpha s)^2} \\
&= \left(\frac{1+\rho}{\rho+p+\alpha s}\right)\frac{\alpha}{1+\rho}\left[W^E - W^U\right]\frac{\partial s}{\partial p} - \frac{\partial(p+\alpha s)}{\partial p}\frac{\left[W^E - W^U\right]}{(\rho+p+\alpha s)}
\end{aligned}$$

Hence, using that $f'() = \frac{\alpha}{1+\rho}\left[W^E - W^U\right]$ we have

$$\frac{\partial\left[W^E - W^U\right]}{\partial p}\frac{p}{\left[W^E - W^U\right]} = \frac{1}{\rho + p + \alpha s}\left[-p\left[1 + s\frac{\partial \alpha}{\partial p}\right]\right]$$

It follows that

$$\begin{aligned}\frac{\partial s}{\partial p}\frac{p}{s} &= \frac{1}{\epsilon(s)}\frac{1-s}{s}\left[\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} - \frac{1}{\rho + p + \alpha s}\left[p + s\alpha\frac{\partial \alpha}{\partial p}\frac{p}{\alpha}\right]\right] \\ &= \frac{1}{\epsilon(s)}\frac{1-s}{s}\left[\left[\frac{\rho + p}{\rho + p + \alpha s}\right]\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} - \frac{p}{\rho + p + \alpha s}\right] < 0\end{aligned}$$

It is an implication that $s\left[1 - \left[\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} + \frac{\partial s}{\partial p}\frac{p}{s}\right]\right] > 0$ and hence $\frac{\partial u}{\partial p} < 0$.

Benefits

From (21) it follows that

$$\frac{\partial u}{\partial b}\frac{b}{u} = \frac{-b}{s}\frac{\partial s}{\partial b}\left[1 - u\right]$$

i.e. the elasticity of unemployment wrt. the benefit level depends on the elasticity of search wrt. the benefit level times the employment rate. To find the latter, we have from the search equation (1) that

$$-f''()\frac{\partial s}{\partial b} = \alpha\frac{\partial\left[W^E - W^U\right]}{\partial b}$$

and hence

$$\frac{\partial s}{\partial b}\frac{b}{s} = \frac{\alpha b}{f''()\left[\rho + p + \alpha s\right]s}\left[h'()\frac{u}{1-u}\frac{\partial \tau}{\partial b}\frac{b}{\tau} + g'(b)\right] < 0$$

In the special case where $\frac{\partial \tau}{\partial b} = 0$, we have

$$\begin{aligned}\frac{\partial s}{\partial b}\frac{b}{s} &= \frac{\alpha b}{f''()\left[\rho + p + \alpha s\right]s}g'(b) \\ &= \frac{f'()}{f''()(1-s)}\frac{1-s}{s}\frac{bg'(b)}{\left[h(w-T) + e(1-l) - \left[g(b) + f(1-s)\right]\right]}\end{aligned}$$

Assuming $\eta(s) \equiv -\frac{f''()(1-s)}{f'()}$ to be constant, we get

$$\begin{aligned}\frac{\partial}{\partial p}\left[\frac{\partial s}{\partial b}\frac{b}{s}\right] &= -\eta(s)\frac{bg'(b)}{\left[h(w-T) + e(1-l) - \left[g(b) + f(1-s)\right]\right]}\left[\frac{f'()}{h(w-T) + e(1-l) - [g(b) + f(1-\right. \\ &= -\eta(s)\frac{bg'(b)}{\left[h(w-T) + e(1-l) - \left[g(b) + f(1-s)\right]\right]}\left[\frac{\alpha s(s-1) - \rho - p}{s^2\left(\rho + p + \alpha s\right)}\right]\frac{\partial s}{\partial p}\end{aligned}$$

Hence, since $0 < s < 1$, we have

$$sign\frac{\partial}{\partial p}\left[\frac{\partial s}{\partial b}\frac{b}{s}\right] = sign\left[\frac{\partial s}{\partial p}\right]$$

Returning to the general case, we have from the budget constraint

$$\frac{\partial T}{\partial b} = \frac{u}{1-u} + \frac{\frac{\partial u}{\partial b}}{(1-u)^2} b > 0$$

or

$$\frac{\partial T}{\partial b}\frac{b}{T} = 1 + \frac{\partial u}{\partial b}\frac{b}{u}\frac{1}{1-u} = 1 - \frac{\partial s}{\partial b}\frac{b}{s}$$

Hence,

$$\begin{aligned}\frac{\partial s}{\partial b}\frac{b}{s} &= \frac{\alpha b}{f''()\left[\rho + p + \alpha s\right] s}\left[f'()\frac{u}{1-u}\left[1 + \frac{\partial u}{\partial b}\frac{b}{u}\frac{1}{1-u}\right] + g'(b)\right] \\ &= \frac{\alpha b}{f''()\left[\rho + p + \alpha s\right] s}\left[f'()\frac{u}{1-u}\left[1 - \frac{\partial s}{\partial b}\frac{b}{s}\right] + g'(b)\right]\end{aligned}$$

and

$$\frac{\partial s}{\partial b}\frac{b}{s} = \frac{\frac{\alpha b}{f''()[\rho+p+\alpha s]s}\left[f'()\frac{u}{1-u} + g'(b)\right]}{1 + \frac{\alpha b}{f''()[\rho+p+\alpha s]s} f'()\frac{u}{1-u}} < 0$$

The sign follows by noting that $1 + \frac{\alpha b}{f''()[\rho+p+\alpha s]s} f'()\frac{u}{1-u} > 0$ is required for stability. To see the latter, note that search is a decreasing function of the tax rate, and that the tax rate is a decreasing function of the search level. The former gives the chosen search level for a given tax rate, and the latter is giving the required search to balance the budget for a given tax rate.

Specifically we have from (1)

$$\frac{\partial s}{\partial T}\,|_{\text{behaviour}} = \frac{\alpha f'()}{f''()\left[\rho + p + \alpha s\right]} < 0$$

and $T = \frac{u}{1-u}b = \frac{p}{\alpha s}b$, and hence

$$\frac{\partial \tau}{\partial s}\,|_{\text{budget}} = \frac{-1}{s^2}\frac{p}{\alpha}b = \frac{-1}{s}\frac{u}{1-u}b < 0$$

Stability requires that the required search level exceeds the chosen search level for a tax rate below the equilibrium value, and vice versa, and this is ensured if

$$\frac{\partial s}{\partial \tau}\,|_{\text{behaviour}} < \frac{\partial s}{\partial \tau}\,|_{\text{budget}}$$

or

$$\frac{\alpha f'() w}{f''()\left[\rho + p + \alpha s\right]} < \frac{-1}{\frac{1}{s}\frac{u}{1-u}\frac{b}{w}}$$

and hence

$$\frac{\alpha h'() b}{f''()\left[\rho + p + \alpha s\right] s}\frac{u}{1-u} > -1$$

4.5 Appendix E: Distortions

First, notice a recursive structure of the model. We have from (3), (4), and (6) that

$$J_B^E = \frac{(1-\beta)\,y + \frac{1-\pi}{1+\rho}(1-p_{GB})J_G^E}{\left[\frac{\rho}{1+\rho} + \frac{\pi}{1+\rho}p_{BB} + \frac{1-\pi}{1+\rho}\right]} \tag{22}$$

$$J_G^E = \frac{(1-\beta)\,y + \frac{1-\pi}{1+\rho}(1-p_{BG})J_B^E}{\frac{\rho}{1+\rho} + \frac{\pi}{1+\rho}p_{GG} + \frac{1-\pi}{1+\rho}} \tag{23}$$

and using that in equilibrium that

$$J_B^E = \frac{ky(1+\rho)}{q_B}$$
$$J_G^E = \frac{ky(1+\rho)}{q_G}$$

we get (using that $q = q(\theta)$)

$$\frac{ky(1+\rho)}{q(\theta_B)} = \frac{(1-\beta)\,y + \frac{1-\pi}{1+\rho}(1-p_{GB})\frac{ky(1+\rho)}{q(\theta_G)}}{\left[\frac{\rho}{1+\rho} + \frac{\pi}{1+\rho}p_{BB} + \frac{1-\pi}{1+\rho}\right]}$$

$$\frac{ky(1+\rho)}{q(\theta_G)} = \frac{(1-\beta)\,y + \frac{1-\pi}{1+\rho}(1-p_{BG})\frac{ky(1+\rho)}{q(\theta_B)}}{\frac{\rho}{1+\rho} + \frac{\pi}{1+\rho}p_{GG} + \frac{1-\pi}{1+\rho}}$$

From Appendix B we have

$$\frac{m(1,\theta_B)}{(1-u_B)} = p_{BB}$$
$$\frac{m(1,\theta_G)}{(1-u_G)} = p_{GG}$$

and from (11) and (12) that

$$p_{BG} = \frac{u_B - u_G}{(1-u_G)} + \frac{(1-u_B)}{(1-u_G)}p_{BB}$$
$$p_{GB} = \frac{u_G - u_B}{(1-u_B)} + \frac{(1-u_G)}{(1-u_B)}p_{GG}$$

The last six equations determine the six endogenous variables: $\theta_B, \theta_G, u_B, u_G, p_{BG}$ and p_{GB}, given the exogenous: p_{BB} and p_{GG}.

Using this and from (7) that

$$u_B = \frac{p_{BB}}{p_{BB} + \alpha_B s_B}$$

we have

$$\frac{\partial u_B}{\partial b_B}\frac{b_B}{u_B} = -(1-u_B)\frac{\partial s_B}{\partial b_B}\frac{b_B}{s_B}$$

and similarly for the good state of nature.

4.6 Appendix F: Optimal benefits

The optimal benefit level solves

$$Max_b \ \Psi \equiv (1-u)\, W^E + uW^U$$

This problem has the first order condition

$$\digamma \equiv (1-u)\,\frac{\partial W^E}{\partial b} + u\frac{\partial W^U}{\partial b} + \frac{\partial u}{\partial b}\left[W^U - W^E\right] = 0 \tag{24}$$

and the second order condition

$$\digamma_b < 0$$

Using the envelope theorem, we have from the value functions for employed and unemployed, respectively

$$\begin{aligned}
\frac{\rho}{1+\rho}\frac{\partial W^E}{\partial b} &= -g'(w-T)\frac{u}{1-u} + \frac{p}{1+\rho}\left[\frac{\partial W^U}{\partial b} - \frac{\partial W^E}{\partial b}\right] \\
\frac{\rho}{1+\rho}\frac{\partial W^U}{\partial b} &= g'(b) + \frac{\alpha s}{1+\rho}\left[\frac{\partial W^E}{\partial b} - \frac{\partial W^U}{\partial b}\right]
\end{aligned}$$

and hence

$$\begin{aligned}
\frac{\rho}{1+\rho}(1-u)\frac{\partial W^E}{\partial b} &= -ug'(w-T) + \frac{p}{1+\rho}(1-u)\left[\frac{\partial W^U}{\partial b} - \frac{\partial W^E}{\partial b}\right] \\
\frac{\rho}{1+\rho}u\frac{\partial W^U}{\partial b} &= ug'(b) + \frac{\alpha s}{1+\rho}u\left[\frac{\partial W^E}{\partial b} - \frac{\partial W^U}{\partial b}\right]
\end{aligned}$$

It follows that

$$\frac{\rho}{1+\rho}(1-u)\frac{\partial W^E}{\partial b} + \frac{\rho}{1+\rho}u\frac{\partial W^U}{\partial b} = ug'(b) - ug'(w(1-\tau))$$

which implies

$$\digamma = u\left[g'(b) - g'(w(1-\tau))\right] + \frac{\partial u}{\partial b}\left[W^U - W^E\right] = 0$$

Figure 1: One state model: unemployment and net compensation to unemployed

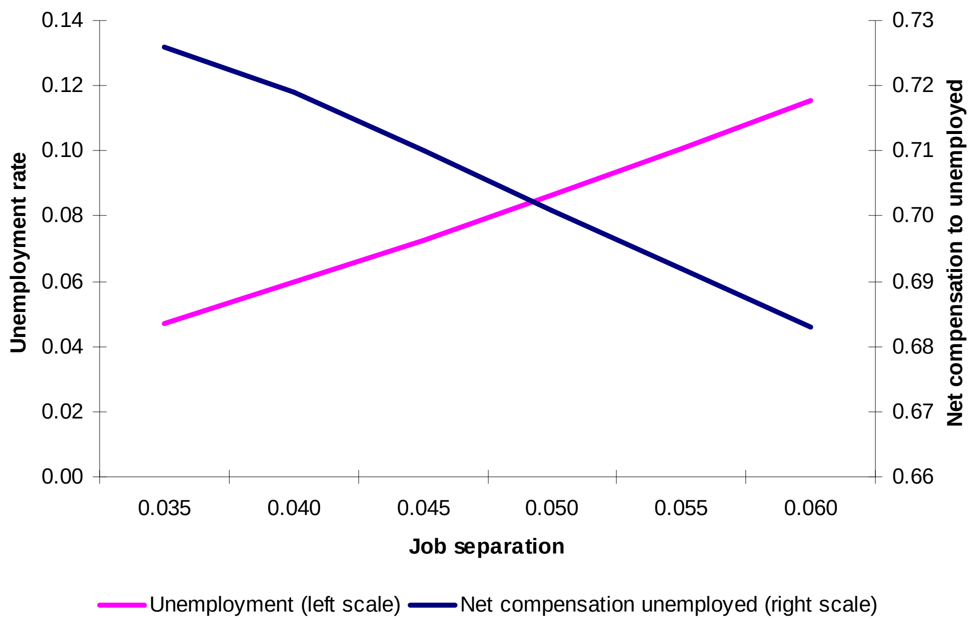
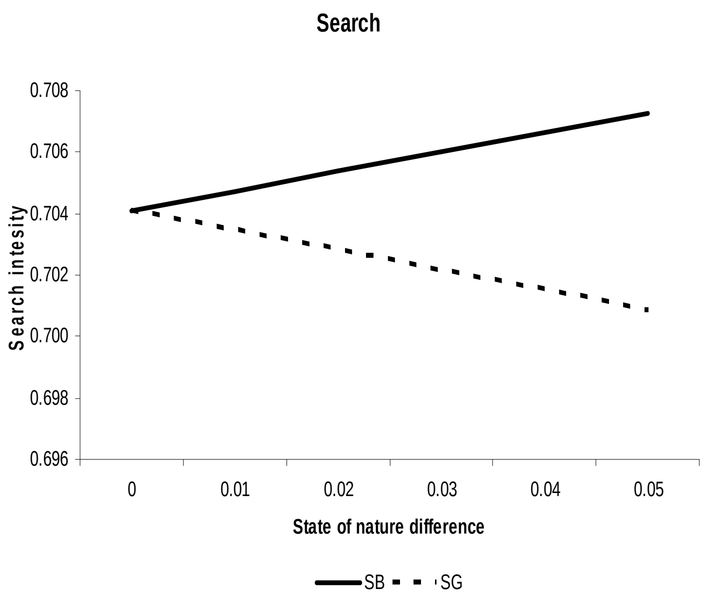
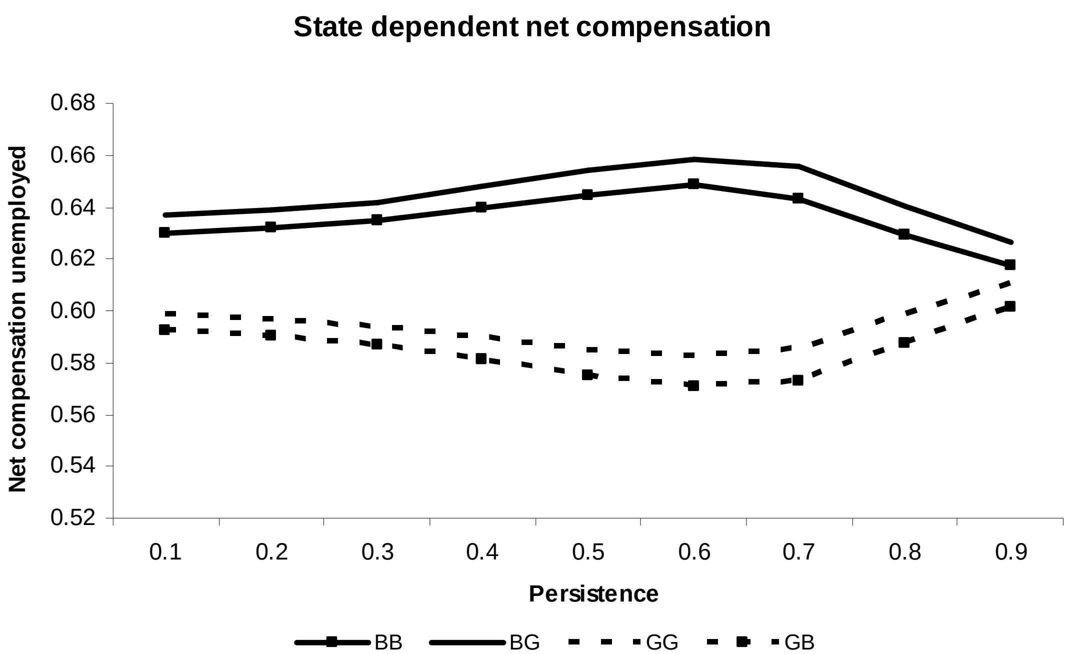


Figure 2: Widening business cycle differences: search



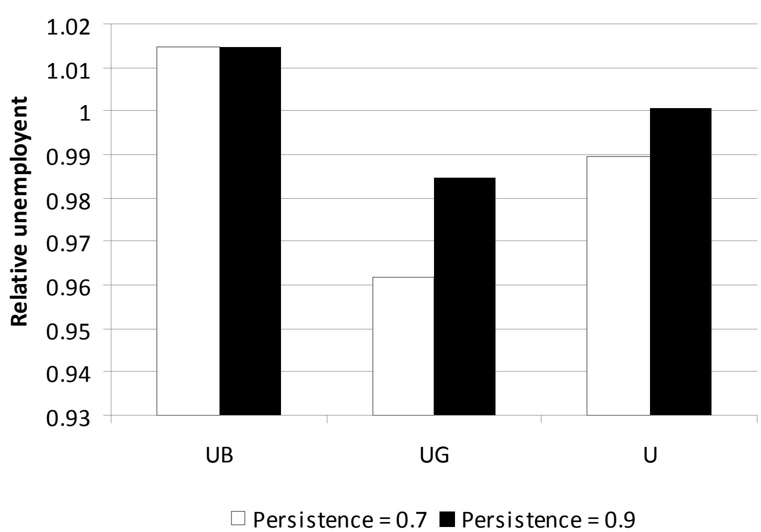
Note: For 0 the job separation rates are $p_{BB} = p_{GG} = 0.04$, and for each step 0.01 is added to p_{BB} and subtracted from p_{GG}, and the persistence is $\pi = 0.5$.

Figure 3: Business cycle dependent net compensation to unemployed and persistence



Note: The net compensation is given as $b_i - T_{ij}$. The optimal level is found in the class of symmetric business cycle dependencies in benefit levels; i.e. the increase in the bad state equals the decrease in the good state.

Figure 4: Relative unemployment: constant vs business cycle dependent benefits



Note: The figure shows the unemployment with business cycle dependent benefits relative to the level of unemployment in a model with business cycle independent benefits. The level of unemployment in the latter model is normalized to 1.