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ABSTRACT

Collective Bargaining under Non-Binding Contracts[*]

We introduce collective bargaining in a static framework where the firm and its risk-neutral employees negotiate over wages in a non-binding contract setting. Our main result is the equivalence between the non-binding collective equilibrium wage-employment contract and the equilibrium contract under binding risk-neutral efficient bargaining. We also demonstrate that our non-cooperative equilibrium wages and profits coincide with the Owen values of the corresponding cooperative game with the coalitional structure that follows from unionization.

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1   Introduction

Equilibrium wage-employment contracts that result from worker-firm negotiations are determined by three distinct characteristics of the bargaining framework:

(1) Labor organization: bargaining can take place (i) collectively between workers organized in a union and the firm or (ii) at the individual level between each worker and the firm.

(2) Bargaining scope: bargaining issues involve (i) only wages, in which case the firm retains the right to determine employment unilaterally or (ii) wages and working conditions (e.g. employment, worker effort, capital-to-labor ratio).

(3) (In)completeness of the contract: the workers and the firm agree upon (i) a binding contract that commits either party to future wage and employment decisions or (ii) a non-binding contract where either party can call for renegotiations before production starts.

As a benchmark, consider a neoclassical (NC) (non-bargaining) firm that writes binding contracts with its workers at the reservation wage. The properties of two binding equilibrium wage-employment contracts are widely known in the collective bargaining literature. The right-to-manage bargaining (RTM) model postulates that the union bargains with the firm over wages (Nickell and Andrews, 1983). Compared to the NC firm, underemployment emerges. The union and the firm agree on a Pareto-inefficient contract. To obtain Pareto efficiency, the efficient bargaining (EB) model requires that the union and the firm negotiate simultaneously over wages and employment (McDonald and Solow, 1981). Under the assumption of a risk-neutral union, the underemployment result of the RTM model disappears. Stole and Zwiebel (SZ) (1996a, 1996b) formalize intrafirm wage bargaining between the firm and its individual risk-neutral employees who are irreplaceable and cannot be contractually tied to the firm. In equilibrium, the SZ firm overhires relative to the NC firm to such an extent that bargained wages are driven down to the reservation wage. Extending the SZ analysis, de Fontenay and Gans (FG) (2003) introduce an outside pool of ready-to-employ replacement workers. Such a finite pool makes it no longer optimal for the firm to overemploy. Moreover, insiders still capture a wage rent since losing an employee brings the firm closer to running out of replacement workers. Therefore, the FG firm underhires relative to the NC firm.

Whereas SZ and FG study individual wage bargaining, many real world labor markets are characterized by union wage bargaining. Capturing this institutional feature, our paper contributes to the collective bargaining literature by investigating how unionization affects equilibrium wages and profits in a non-binding contract setting. Following the binding collective bargaining literature, we assume that the conventional generalized Nash bargaining solution is the appropriate solution concept and that the union and the firm are risk neutral. A common assumption in binding collective bargaining models is that all employed union members return to the external labor market when negotiations fail. However, in the presence of non-binding contracts, it is not sensible to prevent workers from making individual employment decisions, even within a unionized firm. Therefore, the unique feature of our model is that, consistent with the SZ bargaining environment, a dissatisfied worker is free to quit and/or the firm is free to dismiss an individual worker. Hence, we allow the union to renegotiate with the firm in a finite sequence of bargaining sessions with each time one

employee less in the firm. As such, our analysis enables to verify whether the SZ overemployment and the FG underemployment results are robust to a change in the labor organization.

Our pronounced result is that equilibrium wages, profits and employment of our non-binding collective wage bargaining setting are equivalent to the corresponding equilibrium outcomes of the binding EB setting. The important corollary is that due to unionization, the SZ overemployment result disappears. We also show that the availability and the size of a finite replacement pool leaves the wage-employment equilibrium unchanged. These findings allow to conclude that due to unionization, the FG underemployment result disappears. Finally, to provide a game-theoretical characterization of our equilibrium wages and profits, we demonstrate that our non-cooperative equilibrium wages and profits coincide with the Owen values of the cooperative game with the coalitional structure that follows from unionization.

The striking lesson that can be learned from our equivalence result is that starting from the RTM framework, a Pareto-efficient outcome can be obtained by changing the type of contract from binding to non-binding instead of changing the bargaining scope from wages to wages and employment. This alternative route of reaching Pareto efficiency is absent in standard labor economics textbooks that advocate changing the framework from RTM to EB.

The remaining part of the paper is organized as follows. Section 2 derives the equilibrium non-binding wage-employment contract under risk-neutrality in a collective bargaining setting, shows the equivalence between our outcome and the outcome of binding risk-neutral efficient bargaining and investigates the role of a replacement pool on the equilibrium wage-employment contract. The cooperative game-theoretical characterization of our equilibrium wages and profits is provided in Section 3. Section 4 concludes.

2 Collective bargaining under non-binding contracts

2.1 Model

In this section, we describe our model heuristically; in Appendix we provide a non-cooperative bargaining game in extensive form that analytically underpins our model. Essential for our analysis is the assumption that labor contracts are non-binding with no capability to bind either party to future wage and employment decisions. Hence, the union and the firm can engage in an arbitrary number of pairwise negotiations prior to production in which the union can costlessly re-open negotiations over the individual wages of its employed members with the firm and vice versa. Such a renegotiation occurs when a dissatisfied worker decides to quit and/or the firm decides to dismiss an individual worker. We allow the union to renegotiate with the firm on behalf of all remaining employees when any employee leaves the firm. An employee who returns to the external labor market can never re-enter the firm and stays a union member earning the reservation wage. In Section 2.2, we assume that employees are irreplaceable. We relax the irreplaceability assumption in Section 2.3. We assume risk-neutral employees with individual utilities equal to wages. Union preferences are represented by a utilitarian objective function. We assume generalized Nash bargaining. The bargaining scope is negotiation over wages alone.

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We present a discrete version of the model, but results easily extend when labor is assumed to be continuous. Consider a fixed-size union of $\mathcal{N} \in \mathbb{N}$ members. A subset of n union members (the employees) work in the firm. We assume that the union is sufficiently large to cover labor demand ($n \leq \mathcal{N}$). We endogenize the choice of n later on. Wages are generically denoted by w. The reservation wage is $\underline{w}$. We denote $\tilde{w}(n)$ the employee's wage in our non-binding setting when there are n employees. The firm utilizes a single asset, increasing and diminishing returns production function $F(n) : \mathbb{N} \to \mathbb{R}_+$. We assume that $F(i) \geq i\underline{w}$ for $i \in \{1, \ldots, n\}$ for reasons of incentive compatibility that will become clear later on. Denote $\Delta F(n) \equiv F(n) - F(n-1)$ the first difference operator. The profit function is generically denoted by $\pi(n) : \mathbb{N} \to \mathbb{R}$. The neoclassical firm's profit function equals $\pi_{NC}(n) \equiv F(n) - n\underline{w}$. The firm's profit function in this non-binding setting equals $\tilde{\pi}(n) \equiv F(n) - n\tilde{w}(n)$. We denote the bargaining power of the union by $\phi \in [0, 1]$.

2.2 Equivalence with efficient bargaining

We are looking for the bargaining outcome (or contract) that is (i) efficient, i.e. $i\tilde{w}(i) + \tilde{\pi}(i) = F(i)$ for all $i \leq n$, (ii) stable, i.e. for any given bargaining power, neither the union nor the firm can respectively improve wages or profits in a pairwise renegotiation and (iii) incentive compatible with respect to $\underline{w}$, i.e. $\tilde{w}(i) \geq \underline{w}$ for all $i \leq n$ implying that the employees' outside option constraint is not violated.

Our main result is stated in Proposition 1.

Proposition 1. *Under risk neutrality, the outcome of the non-binding collective wage bargaining framework coincides with the outcome of the binding efficient bargaining framework.*

Proof. Under utilitarian union preferences, the union's payoff when there are n employees equals $n\tilde{w}(n) + (\mathcal{N} - n)\underline{w}$. The union's payoff when there are $n - 1$ employees equals $(n-1)\tilde{w}(n-1) + (\mathcal{N} - n + 1)\underline{w}$. Hence, the union's net gain from reaching a bargaining agreement equals $n\tilde{w}(n) - (n-1)\tilde{w}(n-1) - \underline{w}$. The firm's net gain from reaching a bargaining agreement equals $\tilde{\pi}(n) - \tilde{\pi}(n-1)$. The outcome of the bargaining is the generalized Nash solution to

$$\max_{\tilde{w}}[n\tilde{w}(n) - (n-1)\tilde{w}(n-1) - \underline{w}]^{\phi}[\tilde{\pi}(n) - \tilde{\pi}(n-1)]^{1-\phi} \tag{1}$$

We derive the equilibrium contract inductively over the number of employees. Consider the case where only one employee is present. Let $F(0) = 0$. From the first-order condition of the logarithm of Eq. (1), we obtain

$$\tilde{\pi}(1) = \tfrac{1-\phi}{\phi}(\tilde{w}(1) - \underline{w})$$
$$\Leftrightarrow \Delta F(1) - \tilde{w}(1) = \tfrac{1-\phi}{\phi}(\tilde{w}(1) - \underline{w})$$
$$\Leftrightarrow \tilde{w}(1) = \phi\Delta F(1) + (1 - \phi)\underline{w}$$

Note that $\tilde{w}(1)$ is incentive compatible by assumption. Now consider the case where two employees are present. We obtain

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$$\tilde{\pi}(2) - \tilde{\pi}(1) = \frac{1-\phi}{\phi}(2\tilde{w}(2) - \tilde{w}(1) - \underline{w})$$

$$\Leftrightarrow \Delta F(2) - 2\tilde{w}(2) + \tilde{w}(1) = \frac{1-\phi}{\phi}(2\tilde{w}(2) - \tilde{w}(1) - \underline{w})$$

$$\Leftrightarrow \tilde{w}(2) = \frac{\phi}{2}\Delta F(2) + \frac{1}{2}\tilde{w}(1) + \frac{(1-\phi)}{2}\underline{w}$$

$$\Leftrightarrow \tilde{w}(2) = \frac{\phi}{2}[\Delta F(2) + \Delta F(1)] + (1-\phi)\underline{w}$$

Note that $\tilde{w}(2)$ is incentive compatible by assumption. Generalizing the above argument over any n by induction, we obtain as the solution to the first-order difference equation above the following expressions for $\tilde{w}(n)$ and $\tilde{\pi}(n)$:

$$\tilde{w}(n) = \frac{\phi}{n}\sum_{i=1}^{n}\Delta F(i) + (1-\phi)\underline{w} \tag{2}$$

and

$$\tilde{\pi}(n) = (1-\phi)\left[\sum_{i=1}^{n}\Delta F(i) - n\underline{w}\right] \tag{3}$$

Eqs. (2) and (3) easily rewrite when directly using the production function rather than the marginal products:

$$\tilde{w}(n) = \frac{\phi}{n}F(n) + (1-\phi)\underline{w} \tag{4}$$

and

$$\tilde{\pi}(n) = (1-\phi)\left[F(n) - n\underline{w}\right] = (1-\phi)\pi_{NC}(n) \tag{5}$$

Note that $\tilde{w}(n)$ is incentive compatible by assumption.[1] From Eq. (5), it follows that the optimal employment level in our setting, denoted by $\tilde{n}^*$, coincides with the optimal employment level of the neoclassical firm, denoted by n_{NC}^*.

Under EB, the outcome of the bargaining is the generalized Nash solution to

$$\max_{w,n}[n(w - \underline{w})]^{\phi}[\pi(n)]^{1-\phi} \tag{6}$$

Maximization of Eq. (6) with respect to the wage and employment gives the following two first-order conditions respectively:

$$w_{EB}(n) = (1-\phi)\underline{w} + \phi\frac{F(n)}{n} \tag{7}$$

$$w_{EB}(n) = \Delta F(n) + \phi\left(\frac{F(n) - \Delta F(n)n}{n}\right) \tag{8}$$

Solving Eqs. (7) and (8) simultaneously gives the expression for the contract curve: $\Delta F(n) = \underline{w}$. Hence, the optimal level of employment under risk-neutral efficient bargaining, denoted n_{EB}^*,

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[1] If $\tilde{w}(n) \geq \underline{w}$, it cannot happen that $\tilde{w}(i) < \underline{w}$ for some $i < n$ when the firm optimally chooses its input level and the underlying neoclassical profit function is quasi-concave. Furthermore, our analysis shows that it is never optimal for the firm to hire workers beyond the point where $\tilde{w}(n) = \underline{w}$. Nevertheless, beyond this employment level, wages in our bargaining game would be given by $\underline{w}$ and not by $\tilde{w}(n) < \underline{w}$.

coincides with n^*_{NC} and, as we just showed, with $\tilde{n}^*$. From Eqs. (4) and (7), it follows that $\tilde{w}(\tilde{n}^*) = w_{EB}(n^*_{EB})$. As a result, $\tilde{\pi}(\tilde{n}^*) = \pi_{EB}(n^*_{EB})$. ∎

We provide an intuitive interpretation for Proposition 1. The driving force behind the result is that when a firm bargains with a union in a non-binding contract setting over wages alone, the firm cannot determine employment afterwards unilaterally anymore. This is due to the stability requirement of the contract. Suppose that the union and the firm agree upon a wage $\tilde{w}(\tilde{n}^*)$. However, assume that $\max_{n} \tilde{\pi}(n) = n' > \tilde{n}^*$. The firm chooses employment level n', after which the union and the firm want to renegotiate $\tilde{w}(n')$, contradicting that $\tilde{w}(\tilde{n}^*)$ was stable. In other words, although at the outset the union and the firm bargain only over wages in a non-binding contract setting, they implicitly have to reach a binding agreement on wages *and* employment in the end. The latter is exactly the objective of union-firm bargaining in an EB framework.

Proposition 1 allows to investigate whether the SZ overemployment result is robust to a change in the labor organization. The answer is negative since the optimal level of employment in our setting coincides with the one of an NC firm. Hence, we obtain the following corollary.

Corollary 1. *Due to unionization, the SZ overemployment result disappears in a non-binding contract setting.*

2.3 The role of a replacement pool

In the previous section, we assume that employees are irreplaceable. Alternatively, the union could deploy its union members outside the firm to provide the firm with a finite, ready-to-employ replacement pool. More specifically, we again assume that an employee who leaves the firm can never return to the firm and stays a union member earning the reservation wage. However, the firm can now immediately draw upon the replacement pool to substitute the latter. The question in terms of application is whether firms have such a replacement pool available. The answer is most likely affirmative when untrained or low-skilled employees are involved. It is well documented that for such employees, negotiations with the firm typically occur collectively rather than individually, making our setting most relevant.

We obtain Proposition 2.

Proposition 2. *In a non-binding collective wage bargaining framework, the availability and the size of a finite replacement pool leaves the wage-employment equilibrium unchanged.*

Proof. It is sufficient to show that the equilibrium wage in a non-binding collective bargaining setting is not affected by the availability and the size of a replacement pool. Denote the employee's wage by $\tilde{w}_{\mathcal{N}-n}(n)$ where the subscript indicates the number of unionized ready-to-employ workers outside the firm and the number in parentheses indicates the number of employees. Similarly, the firm's profit equals $\tilde{\pi}_{\mathcal{N}-n}(n)$. The union's payoff in case the firm does not draw upon the replacement pool equals $n\tilde{w}_{\mathcal{N}-n}(n) + (\mathcal{N}-n)\underline{w}$. The union's payoff in case the firm replaces an employee equals $n\tilde{w}_{\mathcal{N}-n-1}(n) + (\mathcal{N}-n)\underline{w}$. Hence, the union's net gain from reaching a bargaining agreement equals $n(\tilde{w}_{\mathcal{N}-n}(n) - \tilde{w}_{\mathcal{N}-n-1}(n))$. The firm's net gain from reaching a bargaining agreement

equals $\tilde{\pi}_{\mathcal{N}-n}(n) - \tilde{\pi}_{\mathcal{N}-n-1}(n)$. The outcome of the bargaining is the generalized Nash solution to

$$\max_{\tilde{w}}[n\left(\tilde{w}_{\mathcal{N}-n}(n) - \tilde{w}_{\mathcal{N}-n-1}(n)\right)]^{\phi}[\tilde{\pi}_{\mathcal{N}-n}(n) - \tilde{\pi}_{\mathcal{N}-n-1}(n)]^{1-\phi} \tag{9}$$

From the first-order condition of the logarithm of Eq. (9), we obtain

$$\tilde{\pi}_{\mathcal{N}-n}(n) - \tilde{\pi}_{\mathcal{N}-n-1}(n) = \frac{1-\phi}{\phi}n\left(\tilde{w}_{\mathcal{N}-n}(n) - \tilde{w}_{\mathcal{N}-n-1}(n)\right)$$

$$\Leftrightarrow n\left(\tilde{w}_{\mathcal{N}-n-1}(n) - \tilde{w}_{\mathcal{N}-n}(n)\right) = \frac{1-\phi}{\phi}n\left(\tilde{w}_{\mathcal{N}-n}(n) - \tilde{w}_{\mathcal{N}-n-1}(n)\right)$$

$$\Leftrightarrow \tilde{w}_{\mathcal{N}-n}(n) = \tilde{w}_{\mathcal{N}-n-1}(n)$$

From induction over the number of unionized ready-to-employ workers, we obtain that $\tilde{w}_{\mathcal{N}-n}(n) = \tilde{w}_{\mathcal{N}-n-1}(n) = \ldots = \tilde{w}_1(n) = \tilde{w}_0(n)$. It is easy to check that the result holds for any number of employees, i.e. $\tilde{w}_{\mathcal{N}-i}(i) = \tilde{w}_0(i)$ for $i \in \{1, \ldots, n\}$. ∎

We give an intuitive interpretation for Proposition 2. Suppose that for a given employment level n, the union and the firm agree upon a wage scheme that negatively depends on the size of the replacement pool. Consider n employees and $\mathcal{N} - n$ unionized ready-to-employ workers. In this case, the firm wants to keep the replacement pool as large as possible in order to reduce the wage bill. However, the union has an incentive to deploy the replacement pool in order to increase the total sum of union members' wages. As a result, the wage scheme cannot be stable. A similar argument, where the incentives of the firm and the union are reversed, holds when the wage scheme depends positively on the replacement pool.

Propositions 1 & 2 allow to answer the question whether the FG underemployment result is robust to a change in the labor organization. The answer is negative since the presence of a replacement pool does not affect the wage-employment equilibrium and the latter coincides with the binding EB equilibrium. Hence, we obtain the following corollary.

Corollary 2. *Due to unionization, the FG underemployment result disappears in a non-binding contract setting with a finite replacement pool.*

We end Section 2 by commenting on the effect of abstaining from the risk-neutrality assumption. Importantly, since the equivalence between our contract and the efficient bargaining contract is driven by the stability requirement, this equivalence would still hold if workers were risk-averse (risk-loving). In other words, risk-neutrality is not needed to obtain Proposition 1. However, as the Pareto-efficient contract lies on a negatively (positively) sloped contract curve, the optimal employment level no longer coincides with neoclassical employment, leading to underemployment (overemployment). As a result, without imposing risk neutrality, Corollaries 1 & 2 would no longer hold.

3 A cooperative game-theoretical characterization

This section provides different cooperative game-theoretical characterizations of our non-cooperative equilibrium wages and profits.

Consider the $(n+1)$-player cooperative game (N,v), where $N = \{0,1,\dots,n\}$ is the set of players in which we index the firm as 0 and the employees as the positive integers 1 to n. The mapping $v : 2^{|N|} \to \mathbb{R}$ represents the characteristic function, assigning to any possible coalition $S \subseteq N$ a real number $v(S)$ called the value of coalition S. The value of the empty coalition equals zero, i.e. $v(\emptyset) = 0$. Any coalition S excluding the firm does not have access to the firm's production process and obtains its outside option, i.e. $v(S) = |S|\,\underline{w}$ when $0 \notin S$. Any coalition S including the firm engages in production, i.e. $v(S) = F(|S|-1)$ when $0 \in S$. The value of the grand coalition equals $v(N) = F(|N|-1) = F(n)$. Stole and Zwiebel (1996a) demonstrate that if the firm's bargaining power equals $1/2$ $\left(\text{i.e. } 1-\phi = \frac{1}{2}\right)$, SZ wages and SZ profits respectively coincide with the Shapley values of the workers and the firm for this $(n+1)$-player cooperative game, i.e. $w_{SZ}(n^*) = Sh_1(N,v) = \dots = Sh_n(N,v)$ and $\pi_{SZ}(n^*) = Sh_0(N,v)$ respectively, where $Sh_i(N,v) = \sum\limits_{S \subseteq N\setminus\{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!}(v(S\cup\{i\}) - v(S))$ for all $i \in N$.

3.1 The 2-player cooperative game

Before establishing a cooperative game-theoretical characterization of our collective bargaining non-binding wage-employment contract in the $(n+1)$-player cooperative game, we first consider the 2-player cooperative game $(\{0,1\},v)$, denoting the firm as 0 and the union consisting of n employees as 1. For this 2-player game, it holds that $v(\emptyset) = v(0) = 0$, $v(1) = n\underline{w}$ and $v(0,1) = F(n)$.

We obtain Proposition 3.

Proposition 3. *If the firm's bargaining power equals $1/2$, then workers' wages equal the Shapley value of the union divided by the number of employees and the firm's profit equals its Shapley value.*

Proof. The proof proceeds by contradiction. Suppose that $\tilde{\pi}(n) = \frac{1}{2}(F(n) - n\underline{w})$ and $n\tilde{w}(n) = \frac{1}{2}(F(n) + n\underline{w})$ are not the respective Shapley values of the firm and the union in the cooperative game $(\{0,1\},v)$. Then, following Myerson (1980), at least one of the following two conditions must be violated: (*i*) *Balanced contributions:* $Sh_0(\{0,1\},v) - Sh_0(\{0\},v) = Sh_1(\{0,1\},v) - Sh_1(\{1\},v)$ or (*ii*) *Efficiency:* $Sh_0(\{0,1\},v) + Sh_1(\{0,1\},v) = v(0,1)$. It is straightforward that condition (*ii*) is satisfied. Since $Sh_0(\{0\},v) = 0$ and $Sh_1(\{1\},v) = n\underline{w}$, it follows that condition (*i*) is also satisfied, thereby contradicting that $\tilde{\pi}(n)$ and $n\tilde{w}(n)$ are not the Shapley values of the firm and the union in the cooperative game $(\{0,1\},v)$. $\blacksquare$

Proposition 3 can be generalized for any bargaining power $\phi \in [0,1]$. Define a unanimity game u_S as a game such that $u_S(T) = 1$ if $S \subseteq T$ and $u_S(T) = 0$ otherwise. It is well known that every coalitional game (N,v) can be written as a linear combination of unanimity games in a unique way, i.e. $v = \sum_{S \subseteq N} \lambda_S(v) u_S$. The coefficients $\lambda_S(v)$ are called unanimity coefficients of the game (N,v) and are given by $\lambda_S(v) = \sum_{T \subseteq S}(-1)^{|S|-|T|}v(T)$. The weighted Shapley value for any coalitional game (N,v) and weight vector $(\theta_i)_{i \in N}$ is then given by $Sh_i(N,v,\theta) = \sum_{i \in S} \lambda_S(v)\frac{\theta_i}{\sum_{j \in S}\theta_j}$. We obtain Proposition 4.

Proposition 4. *Workers' wages equal the weighted Shapley value of the union divided by the number of employees and the firm's profit equals its weighted Shapley value for any weight vector $\theta = (\theta_0, \theta_1)$ where $\frac{\theta_0}{\theta_0+\theta_1} = 1-\phi$ and $\frac{\theta_1}{\theta_0+\theta_1} = \phi$.*

Proof. In our setting, $\lambda_{\{0\}}(v) = 0$, $\lambda_{\{1\}}(v) = n\underline{w}$ and $\lambda_{\{0,1\}}(v) = F(n) - n\underline{w}$. When $\theta = (\theta_0, \theta_1)$ where $\frac{\theta_0}{\theta_0 + \theta_1} = 1 - \phi$ and $\frac{\theta_1}{\theta_0 + \theta_1} = \phi$, the reader can check that $Sh_0(N, v, \theta) = (1 - \phi)(F(n) - n\underline{w}) = \tilde{\pi}(n)$ and $Sh_1(N, v, \theta) = n\underline{w} + \phi(F(n) - n\underline{w}) = \phi F(n) + (1 - \phi)n\underline{w} = n\tilde{w}(n)$. ∎

3.2 The $(n+1)$-player cooperative game

The previous section allows for a characterization of equilibrium wages and profits in terms of (weighted) Shapley values. Returning to the $(n+1)$-player cooperative game (N, v), we obtain an alternative characterization in terms of modified Shapley values, known as Owen values (Owen, 1977), that takes into account possible coalitional structures that may form between players. The standard textbook interpretation of the Shapley value is that of a queue of players, where each player is entering a room and is obtaining her marginal contribution to the coalition of players already present in the room. In case of the Shapley value, all queues are formed with equal probability and the Shapley value is precisely the expected marginal contribution to coalitions with respect to this random order of players. In contrast, the Owen value restricts the possible formation of queues according to the coalitional structure. We formally define a coalitional structure $B = \{S_1, \ldots, S_m\}$ which partitions N into m disjoint subsets. Let ω be a permutation on N and let Ω be the set of all permutations on N. Define $\Omega(B)$ as the subset of Ω, which includes only the orders in which players of the same component of B appear successively; i.e. $\Omega(B) = \{\omega \in \Omega : \text{if } i, j \in S_k \text{ and } \omega(i) < \omega(l) < \omega(j), \text{ then } l \in S_k\}$. Then, the Owen value assigns to each player her expected marginal contribution to the coalition of preceding players with respect to a uniform distribution over the set of orders in $\Omega(B)$; i.e. $O_i(B, v) = \frac{1}{|\Omega(B)|} \sum_{\omega \in \Omega(B)} (v(P_i^\omega \cup i) - v(P_i^\omega))$ for all $i \in N$, where $P_i^\omega = \{j \in N, \omega(j) < \omega(i)\}$.

In our framework, $\bar{B} = \{S_1, S_2\}$ where $S_1 = \{0\}$ is containing the firm, and $S_2 = \{1, \ldots, n\}$ is containing the workers.

We obtain Proposition 5.

Proposition 5. *If the firm's bargaining power equals $1/2$, then workers' wages and the firm's profit equal their Owen values in the cooperative game with coalitional structure $\bar{B}$.*

Proof. First, consider the firm. Note that, given the coalitional structure $\bar{B}$, the firm enters either first or last in the order of players, implying that $|\Omega(\bar{B})| = 2n!$. The marginal contribution of the firm entering first equals 0, the marginal contribution of the firm entering last equals $F(n) - n\underline{w}$. Hence, $O_0(\bar{B}, v) = \frac{n!}{2n!}(F(n) - n\underline{w}) = \frac{1}{2}(F(n) - n\underline{w}) = \tilde{\pi}(n)$. The result for the workers' wages follows by noting that (i) the cooperative game among workers when the firm is absent is inessential, implying that $O_i(\bar{B}, v) = O_j(\bar{B}, v)$ for all $i, j \in S_2$ and that (ii) Owen values satisfy efficiency with respect to the grand coalition, implying that $\sum_{i \in N} O_i(\bar{B}, v) = v(N)$. Hence, we obtain, for all $i \in S_2$, $O_i(\bar{B}, v) = \frac{v(N) - O_0(\bar{B}, v)}{n} = \frac{F(n) - \frac{1}{2}(F(n) - n\underline{w})}{n} = \frac{1}{2n}F(n) - \frac{1}{2}\underline{w} = \tilde{w}(n)$. ∎

Proposition 5 can be generalized for any bargaining power $\phi \in [0, 1]$ in terms of "weighted" Owen values in the cooperative game with coalitional structure $\bar{B}$. However, to the best of our knowledge, the latter solution concept is not yet defined in the literature (for any coalitional structure B) and doing so goes beyond the scope of this paper. Nevertheless, for our specific coalitional structure $\bar{B}$,

an elegant interpretation can be given which resembles Owen (1968)'s original interpretation of the weights of the weighted Shapley value as a measure of players' delay to reach the grand coalition. Owen showed that the introduction of weights amounts to distorting the equal probabilities with which queues form in the following way: the higher the weight of a player, the higher the probability of the queues in which this player arrives the last. In our setting with coalitional structure $\bar{B}$ and the firm's bargaining power equal to $1/2$, the firm ends up at either end of the order with equal probability, yielding the Owen value of $1/2$ times the firm's marginal contribution to the grand coalition (remember that the marginal contribution of the firm entering first equals zero). Generalizing, with coalitional structure $\bar{B}$ and the firm's bargaining power equal to $(1 - \phi)$, the firm's bargaining power exactly reflects the probability that the firm enters the last in the order of players, yielding the "weighted" Owen value of $(1 - \phi)$ times the firm's marginal contribution to the grand coalition.

4 Conclusion

To represent a widespread, institutional characteristic of contemporary labor markets, this paper introduces collective bargaining in a non-binding contract setting where the firm and its risk-neutral employees bargain over wages alone. We show that the wage-employment equilibrium coincides with the outcome of the binding efficient bargaining framework. The driving force behind this result is that, due to the stability requirement in a non-binding contract setting, the firm is not able to determine employment unilaterally even if bargaining is over wages alone. The availability and the size of a finite replacement pool leaves the wage-employment equilibrium unchanged. These findings allow to conclude that due to unionization, the Stole and Zwiebel (1996a, 1996b) overemployment result and the de Fontenay and Gans (2003) underemployment result disappear. Furthermore, the firm and the workers receive their expected marginal contribution of the corresponding cooperative game with the coalitional structure that follows from all workers being represented by a single union.

Within our static collective non-binding bargaining framework, an evident continuation is to explore applications regarding hiring decisions, technological choice and organizational design. Furthermore, if data on labor contract specificities were available, our equivalence result would provide the foundation of an original test of Pareto efficiency in the empirical collective bargaining literature. An interesting extension of our framework, following Horn and Wolinsky (1988), is to introduce worker heterogeneity and to study the formation of multi-union patterns, possibly exploited by the firm to its advantage.

Recently, within a dynamic framework, a number of studies have introduced individual wage bargaining in a search and matching economy (e.g. Cahuc and Wasmer, 2001; Cahuc et al.; 2008; Helpman et al., 2008; Mortensen, 2009). Bauer and Lingens (2010) are the first to analyze and compare the wage-employment equilibrium under collective and individual wage bargaining in a large firm search model in an attempt to answer the question whether collective wage bargaining can restore efficiency in the labor market or not. Under the assumption that collective wage bargaining takes the form of all employees delegating the wage negotiation to a representative worker and deciding jointly whether to work or not, they show that both collective and individual wage bargaining regimes deliver inefficient allocations. Whether these conclusions also hold when allowing employees in a unionized firm to make individual employment decisions still remains an open

question.

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Appendix : A non-cooperative representation

In this Appendix we present an extensive-form bargaining game whose unique subgame perfect equilibrium corresponds with the equilibrium wage-employment contract described in the main text. Consider a firm with n unionized employees. Bargaining proceeds as a finite sequence of pairwise bargaining sessions over wages between the union and the firm.

In Figure A.1, each bargaining session is depicted by a box, representing the number of employees on which behalf the union is negotiating with the firm. In the first bargaining session, the union represents n employees. In each bargaining session, either the union and the firm reach an agreement (A), or negotiations break down (B). Whenever an agreement is reached, the game ends. Whenever a bargaining session ends in a breakdown, one randomly chosen employee exits the game forever, after which bargaining starts again between the firm and the union representing the remaining employees. At most n bargaining sessions can occur before the game terminates in which case all employees have dropped out following failed bargaining sessions.
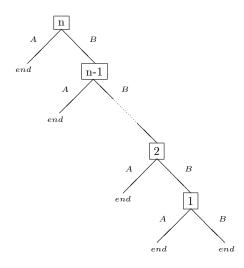

Figure A.1: Game representation in extensive-form.

Within each bargaining session, the union and the firm play the alternating-offer bargaining game of Binmore et al. (1986) in which there is an exogenous probability of breakdown following each rejected offer. Breakdown probabilities differ following a rejection by the firm or the union. The game is described as follows. Starting with the firm, the firm and the union alternate wage proposals. If a proposal is accepted, negotiations terminate. If a proposal is rejected, negotiations break down with probability p_f if a rejection is made by the firm and with probability p_u if a rejection is made by the union. When a breakdown does not occur, the rejecting party makes a counterproposal. Proposals are made until one is accepted or a breakdown occurs. There is no discounting. It is straightforward to demonstrate that every bargaining power $\phi \in [0,1]$ is consistent for some pair of

probabilities (p_f, p_u) in the following way:

$$\phi = \frac{p_f(1 - p_u)}{p_f(1 - p_u) + p_u}$$

We look for the limiting outcome as breakdown probabilities approach zero. Binmore et al. (1986) show that for such a bargaining session the generalized Nash bargaining solution emerges.