On the Role of Market Insurance in a Dynamic Model*

Helge Braun[a] **and Winfried Koeniger**[b,c]

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Abstract

Durables such as cars or houses are a substantial component in the balance sheets of households. These durables are exposed to risk and can be insured in the market. We build a dynamic model in which the agent decides jointly about the size of the durable stock, the stock of a risk-free asset and insurance. This allows us to understand the importance of three different ways to cope with the risk exposure of the durable stock: (i) purchase of market insurance, (ii) buffer-stock saving of the riskless asset or (iii) adjustment of the durable stock. Compared with previous research that did not allow for an endogenous size of the loss, we find that the policy function for market insurance can be non-monotonic in the agent's resources since the covariance between the marginal utility of non-durable consumption and the loss depends both on the stock of risk-free assets and the durable. We calibrate our model to the US economy and find a small role for market insurance even if we add background risk such as volatile permanent labor income.

*a: Northwestern University, Evanston; b: IZA and University of Bonn; c: *Finance and Consumption in the EU* at the European University Institute, Florence. We thank Thomas Hintermaier, Robert Young and participants at various seminars and conferences for very helpful comments. Corresponding author: Winfried Koeniger, IZA, P.O. Box 7240, 53072 Bonn, Germany; koeniger@iza.org

1 Introduction

Durables are very important in the balance sheets of households. In terms of flows, durable consumption accounts for $12 - 28\%$ of total consumption in the US in the period 1990-2002 (depending on whether durable consumption includes housing services; see also Attanasio, 1999). In terms of stocks, the current-cost net stock of private fixed assets has a size of 130% of disposable income where alone the stock of consumer durables amounts to 40% of this income.[1]

The stock of durables is exposed to risk: houses can be broken into or destroyed by natural catastrophes and cars can be stolen. We propose a dynamic model that allows the agent to manage this risk in three different ways through (i) purchase of market insurance, (ii) buffer-stock saving of a riskless asset or (iii) adjustment of the durable stock. The goal of this paper is to analyze the qualitative interplay and quantitative importance of these three ways in which the consumer can manage his risk.

Compared with previous research that did not allow for endogenous adjustment of the durable stock and thus an endogenous size of the loss (Gollier, 2003), we find that the policy function for market insurance can be interestingly non-monotonic in the agent's resources. The reason is that two opposite forces are at work which are endogenous in our model: the size of the loss and self-insurance through buffer-stock saving. An intuitive explanation for the non-monotonicity is that the covariance between the marginal utility of non-durable consumption and the loss is small for a small durable stock (and thus a negligible loss) *as well as* for a large durable stock if the agent holds enough risk-free assets to self-insure. It is in the intermediate range that market insurance plays a role.

Another value added of this paper is that we check whether endogenous interest rates alter the results of the role of market insurance. This is important quantitatively because buffer-stock saving becomes more relevant compared with market insurance if the difference between the discount and

[1] The figures are obtained from the national income and product accounts of the Bureau of Economic Analysis, U.S. Department of Commerce.

interest rate is small. This difference measures the cost of saving for the impatient agent.

We calibrate our model to the US economy and find that market insurance only plays a small role for welfare even if we add additional sources of background risk such as volatile permanent labor income. Although we manage to reproduce statistics of aggregate insurance expenditure, the implied deductibles or retention ratios are an order of magnitude higher than those observed in reality.

Our paper builds on the literature that has explored market insurance mostly in a static framework. For example, Ehrlich and Becker (1972) show that market insurance and actions of agents that decrease the size of the loss are substitutes. Instead, Eeckhoudt et al. (1997) show that market insurance is more desirable if the share of a risky asset in the portfolio increases (if the utility function has the property of decreasing absolute risk aversion). Moreover, Eeckhoudt et al. (1991), Proposition 2, find that market insurance and precautionary savings are substitutes for a particular increase in risk.

Although these static results have been very helpful to understand particular interactions between market insurance and other policies, they cannot be used to address the question whether there is a role for market insurance in the first place within a simple dynamic neoclassical framework. The most important contributions in the literature in this respect are by Gollier (1994, 2003) who investigates dynamic interactions between market insurance and the accumulation of risk-free assets as buffer stock. Whereas Gollier (1994) derives a closed form solution for CRRA utility and no liquidity constraints in a continuous-time model, Gollier (2003) provides a numerical solution to the more general problem. He finds that time diversification through buffer-stock saving reduces the scope for market insurance substantially. As mentioned above, we show in this paper that his assumptions of an exogenous insurable risk and an exogenous interest rate are important and matter for some of the qualitative as well as quantitative results.

The rest of the paper is structured as follows. In Section 2 we present and discuss the model and its optimality conditions. In Section 3 we present the numerical solution for the partial-equilibrium case with an exogenous interest rate. We calibrate our model to match some target statistics in the US and show how the equilibrium changes if we change the most important model parameters. In particular, we show how stochastic permanent labor income affects the equilibrium. Finally, we mention that market insurance is even less important if we allow for an endogenous interest rate in general equilibrium. In Appendix A we provide an analytic approximation of the policy functions for the special case of abundant liquid wealth and in Appendix B we explain our numerical solution procedure in more detail.

2 The model

Agents are risk-averse and have an infinite horizon. They derive utility from a durable good v and a non-durable good c. The instantaneous utility is given by $U(c,v) = u(c) + \phi w(v)$ where $u(.)$ and $w(.)$ are both strictly concave, and ϕ is the weight assigned to utility derived from the durable. We assume that the marginal utility $w'(v)$ is well defined at $v = 0$ so that our model is able to generate agents with no durable stock in at least some states of the world, as is realistic. A possible functional form is $w(v) = (v+\underline{v})^{\tau}$, with $\tau \leq 1$ and $\underline{v} > 0$. The asymmetry in the utility function with respect to non-durable and durable consumption is justified in the sense that durables are less essential than non-durable consumption such as food. Note that we implicitly assume that durables can be transformed into non-durable consumption with a linear technology so that the relative price is unity.

In specifying utility as above we have made a number of simplifying assumptions. We assume v to be a homogenous, divisible good. Moreover, utility is separable over time and at each point in time it is separable between durables and non-durables. Both assumptions are made for tractability given that it is more realistic to assume that durables are a bundle of characteristics and that utility derived from durables depends on non-durable consumption in non-trivial ways. Instead, as in much of the literature, we assume that the service flow derived from durables is proportional to the stock where we have normalized the factor of proportionality to 1 (see Waldman, 2003, for a critical review of these common assumptions).

We specify our model in discrete time so that we have to make assumptions about the timing. At the beginning of each period, financial wealth a_t, the durable stock v_t and its insurance coverage denoted by D_t, the loss l_t and labor income y_t are predetermined. Given these state variables, the agent chooses non-durable consumption c_t, investment into the durable d_t, and insurance coverage for the next period D_{t+1} subject to constraints which we mention below. Finally, the durable stock depreciates at rate δ.

Since v_t is predetermined, utility is derived from the durable stock net of the loss. This implies that the consequences of a loss in terms of utility depend on the speed with which the durable stock can be adjusted. In our numerical solution we calibrate one period as a quarter. Although we think that this is a realistic assumption in terms of adjustment speed, below we also mention how the results change if we choose a higher period frequency.

Before we lay out the dynamic program, let us characterize the insurance contract. A risk neutral insurer offers an actuarially unfair one-period insurance contract with a loading factor $\mu > 1$. Such a contract is closest to the one-period saving technology and thus interesting to analyze in our framework. It is well-known that in this case the optimal insurance contract has a deductible (for example, Raviv, 1979). Since we assume that the loss distribution has only two states (no loss or loss l), the choice of the deductible is equivalent to choosing the retention ratio D. This ratio is defined as the proportion of the loss l which is not insured. Thus, should a loss occur, the agent receives the following payment from the insurance company

$$\max(l - Dl, 0) = l * \max(1 - D, 0) = l * (1 - D) \ ,$$

where the last equality holds since $D \in [0,1]$.

Defining $\mathbf{v}(.)$ as the value function, the dynamic program is

$$\mathbf{v}(a_t, v_t, D_t | l_t, y_t) = \max_{a_{t+1}, v_{t+1}, D_{t+1}} [u(c_t) + \phi w(v_t - l_t) + \beta E_t \mathbf{v}(a_{t+1}, v_{t+1}, D_{t+1} | l_{t+1}, y_{t+1})] \qquad (1)$$

s.t.

$$a_{t+1} = (1+r)a_t + y_t + \overbrace{(1-D_t)l_t}^{\text{insurance claim}} - c_t - d_t - \overbrace{\frac{\mu}{1+r}(1-D_{t+1})El_{t+1}}^{\text{insurance premium}}$$

$$v_{t+1} = (1-\delta)v_t - l_t + d_t$$

$$a_s \geq \underline{a} \ , s \geq t$$

$$v_s \geq 0 \ , s \geq t,$$

$$D_s \in [0;1] \ , s \geq t,$$

where β is the discount factor, r is the interest rate, and δ is the depreciation rate which we assume to be constant for simplicity. The expectation operator is E where a subscript t denotes that expectations are conditional on information available at time t. For the case of deterministic income $y_t = y$ where y is non-random. We now discuss the constraints in some detail.

The first constraint is the budget constraint. The amount of assets tomorrow depends on the amount of assets today plus the interest, the amount of durable and non-durable consumption, and labor income. Moreover, it depends on the amount of insurance demanded in period t, $(\mu/(1+r))(1-D_{t+1})El_{t+1}$, and the money received from the risk-neutral insurance company if damage occurred, $(1-D_t)l_t$. Note that insurance is actuarially unfair, $\mu > 1$, so that risk-averse agents do not necessarily insure fully. Moreover, premiums are discounted to the present since a premium payment today only provides coverage tomorrow. In the meanwhile the insurance company earns the market interest rate r on the paid premium.

The second constraint is the law of motion of the durable stock. The size of the durable stock tomorrow depends on its size today net of depreciation minus the loss plus the investment today. In order to economize on notation in the equations, we assume that the loss is exogenous in this section. In the numerical part of the paper instead, the loss is endogenous, proportional to the size of the durable stock. Note also that we abstract from adjustment costs. Adjustment costs certainly imply more realistic adjustment dynamics for the durable stock. However, the model structure is already quite rich so that we chose to abstract from adjustment costs as a starting point. For an analysis of durable investment under uncertainty with adjustment costs see for example Bertola et al. (2004) or Lam (1991).

The third constraint is the solvency constraint. It implies that the agent cannot borrow more than $\underline{a}$ which guarantees repayment of the debt. Aiyagari (1994) derives in a model with non-durable consumption that $\underline{a}= -\frac{y_l}{r}$, where y_l is the smallest y attainable on the support of the distribution.[2] For the numerical part of this paper we will assume $\underline{a}= 0$, a tighter borrowing constraint unless $y_l = 0$. This is done to decrease the size of the interval of a and speed up the numerical algorithm. In robustness checks we find that relaxing this constraint does not affect the quantitative results much (see below).

The fourth and fifth constraint imply that the agent cannot go short in the durable or sell insurance. Finally, for both state variables a and v a transversality condition has to be satisfied, respectively.

Ignoring the constraints $a_s \geq \underline{a}$, $v_s \geq 0$ and $D_s \in [0; 1]$, problem (1) yields the following Euler equations for the controls c_t, d_t and D_{t+1}, respectively:

$$u'(c_t) = \beta(1 + r)\ E_t u'(c_{t+1}) \ , \tag{2}$$

$$u'(c_t) = \beta \left[(1 - \delta)E_t u'(c_{t+1}) + \phi E_t w'(\widetilde{v}_{t+1})\right] \tag{3}$$

and

$$u'(c_t)\mu E l_{t+1} = \beta(1 + r)E_t\{u'(c_{t+1})l_{t+1}\} \ , \tag{4}$$

where primes denote first-order derivatives of the functions with respect to the variables in brackets and $\widetilde{v}$ is the durable stock net of the loss. Equations (2) and (3) can be used to solve for the intertemporal behavior of $\widetilde{v}$:

$$E_t w'(\widetilde{v}_{t+1}) = \beta(1 + r)\ E_t w'(\widetilde{v}_{t+2}) \ . \tag{5}$$

[2] Alternatively, one could assume that the durable can be used as a collateral, too, in which case the solvency constraint would vary together with v.

We now provide some intuition for these intertemporal optimality conditions. Equation (2) is standard and relates non-durable consumption intertemporally. Equation (5) is the equivalent for the durable stock. More interestingly, on the left-hand side of equation (3) are the costs for one unit of durable investment d, in terms of utility derived from non-durable consumption. These have to equal the benefits on the right-hand side of equation (3). The benefits are the discounted expected utility afforded by the increase of the durable stock, $\phi E_t w'(\widetilde{v}_{t+1})$, plus the expected utility of non-durable consumption resulting from selling one unit of the durable stock after depreciation in the next period.[3] Note that it is important that the agent derives utility from the durable in our model. Otherwise, the agent would not invest in the durable since it is risky and return-dominated by the risk-free asset.[4]

Equation (4) is the intertemporal optimality condition for insurance demand. Again, marginal costs on the left-hand side of the equation equal marginal benefits which are on the right-hand side. The marginal benefits are that the agent can consume more tomorrow in bad states of the world if he buys insurance today because in expectation more resources are available due to insurance payments. Since $E_t\{u'(c_{t+1})l_{t+1}\} = E_t u'(c_{t+1}) E l_{t+1} + cov(u'(c_{t+1}), l_{t+1})$, the agent wants to buy more insurance if large realizations of l_{t+1} occur in states of the world in which the agent's non-durable consumption is already small. In terms of equation (4), a higher $cov(u'(c_{t+1}), l_{t+1})$ implies a higher $u'(c_t)$: the marginal cost on the left-hand side is relatively higher because agents are willing to forego more non-durable consumption to buy more insurance. In order to understand the policy function for D which we obtain in our numerical solutions, it is important to keep in mind that the attractiveness of insurance crucially depends on $cov(u'(c_{t+1}), l_{t+1})$. As we will see below, the amount of risk-free assets a and the size of the durable stock v are both important for

[3] With an endogenous loss proportional to the durable stock, equation (3) is modified as follows: on the left-hand side of the equation an additional term captures the costs resulting from higher insurance payments because of a higher loss; on the right-hand side the expectation of the future durable stock takes into account that the size of the loss increases if the agent invests.

[4] Compared to standard portfolio-choice models, it is noteworthy that in our model utility functions with constant-relative-risk aversion do not imply that a constant share of wealth is invested in the risky asset.

this covariance since they matter for the variability of $u'(c_{t+1})$ and the size of the loss l_{t+1}.

Before we continue to solve the model numerically, let us mention that in our model precautionary motives arise even if the agent has abundant liquid wealth. As mentioned above this is because utility is derived from the durable stock net of the loss. Thus, the agent cannot smooth immediately the fluctuations in the durable stock. We show this explicitly in Appendix A where we provide an approximation of the model.

3 Numerical solution

In this section we first solve the model numerically for the partial equilibrium case in which the interest rate is exogenous. After we have shown how the solution of the model depends on the most important parameter values, we mention how our quantitative results change if we allow for an endogenous interest rate.

3.1 Partial equilibrium

Before we solve the model numerically we rewrite the maximization problem in terms of resources "cash-in-hand" x_t which is commonly done in the literature to reduce the state-space. Cash-in-hand at the beginning of period t is defined as

$$x_t \equiv (1+r)a_t + y_t + (1-\delta-D_t\eta I_t)v_t \ ,$$

the sum of risk-free assets and their returns, labor income, and the cash value of the durable asset after depreciation and net of the loss which remains after accounting for the retention rate D_t and the insurance payments. Note that now we also let the loss $l_t = \eta I_t v_t$ endogenously depend on the durable stock v_t with a proportionality factor η. Whether a loss occurs or not is summarized by the indicator variable I_t. This variable takes the value 1 if a loss of size ηv_t occurs which happens with probability λ, and the value 0 if no loss occurs. Recalling that $\widetilde{v}_t = (1-\eta I_t)v_t$ is the durable stock net of the loss, we can rewrite the value function as

$$V^{+}(x_t, \widetilde{v}_t) = \max_{a_{t+1}, v_{t+1}, D_{t+1}} \left[u(\underbrace{x_t - a_{t+1} - v_{t+1} - \frac{\mu}{1+r}(1 - D_{t+1})\eta\lambda v_{t+1}}_{c_t}) + \phi w(\widetilde{v}_t) + \beta E_t V^{+}(x_{t+1}, \widetilde{v}_{t+1}) \right]$$

We can further simplify the problem by noting that $\widetilde{v}_t$ is predetermined in period t and that the additive separable term $\phi w(\widetilde{v}_t)$ does not affect the optimal choices of the consumer. Defining

$$V(x_t) \equiv V^{+}(x_t, \widetilde{v}_t) - \phi w(\widetilde{v}_t)$$

the transformed maximization problem is

$$V(x_t) = \max_{a_{t+1}, v_{t+1}, D_{t+1}} \left[u(\underbrace{x_t - a_{t+1} - v_{t+1} - \frac{\mu}{1+r}(1 - D_{t+1})\eta\lambda v_{t+1}}_{c_t}) + \beta\phi E_t w(\widetilde{v}_{t+1}) + \beta E_t V(x_{t+1}) \right] \tag{6}$$

s.t.

$$a_s \geq \underline{a}\ ,\ v_s \geq 0\ ,\ D_s \in [0;1] \text{ with } s \geq t+1\ .$$

It is easy to verify that the problem (6) satisfies Blackwell's sufficient conditions for a contraction mapping. This allows us to solve numerically for the optimal value function V and the corresponding policy functions using Chebychev polynomials to approximate the functions. In our algorithm we first search for an upper bound of cash-in-hand $\overline{x}$ at which the optimal policies $\{a_{t+1}, v_{t+1}, D_{t+1}\}$ imply that the maximal attainable cash-in-hand without a durable loss, $x_{t+1}^{\max}$, is smaller than this upper bound:

$$x_{t+1}^{\max} = (1+r)\, a_{t+1} + y + (1-\delta)\, v_{t+1} < \overline{x}.$$

Using as a lower bound $\underline{x} = y$ gives us a compact state space. More details on the numerical solution of the problem are provided in Appendix B.

Table 1 (at the end of the paper) displays the benchmark parameter values that we use to solve the model numerically. We calibrate one period as one quarter of a year so that the parameter values imply an annual risk-free interest rate of .01 (see Mehra and Prescott, 1985) and a discount factor of .95 or a discount rate of .053 (see Aiyagari, 1994). Thus, agents are quite impatient in the benchmark of our partial equilibrium computations. It is well-known that $\beta < 1/(1+r)$ is necessary in models with incomplete markets for the stock of risk-free assets to be finite. Indeed, if marginal utility is convex, uninsured risk gives rise to a precautionary savings motive and the additional savings lower the interest rate in general equilibrium (see Aiyagari, 1994). Since holding the durable stock is costly, $\delta > 0$, the results in Deaton and Laroque (1992) imply that also cash-in-hand is finite. Thus, in our partial equilibrium computations, we choose values for the interest rate that are consistent with impatience and check in the next subsection which interest rates prevail in general equilibrium.

Concerning the other parameter values, the implied annual depreciation rate is 15% which is consistent with micro evidence on cars provided by Alessie et al. (1997). The probability of a loss per quarter, $\lambda = .05$, is an upper bound based on claim rates for property and casualty insurance in the US provided by the Insurance research council (http://www.ircweb.org). The loading factor (mark-up) for insurance is set to $\mu = 1.3$, as in Gollier (2003), which is consistent with evidence on direct written premiums and claims for the US (see the Financial Services Fact Book, 2004, provided free of charge by the joint venture of the Insurance Information Institute and The Financial Services Roundtable under http://www.financialservicesfacts.org).

We normalize labor income y to 1, and parametrize the utility functions

$$u(c) = \frac{c^{1-\sigma_1} - 1}{1 - \sigma_1} \text{ and } w(v) = \frac{(v + \underline{v})^{1-\sigma_2} - 1}{1 - \sigma_2},$$

where as mentioned in the previous section, $\underline{v} > 0$ allows the consumer to hold no durable stock. We set risk aversion for the non-durable and durable good $\sigma_1 = \sigma_2 = 2$ which is well within the range of commonly used values, and assume $\underline{v} = .1$ which is rather small. Indeed, it turns out that this parameter is rather unimportant and can be set to negligibly small values without changing the quantitative results much. This is because the region of v close to zero is not important in our simulations. Since we do not have any information on the remaining two parameters η and ϕ, we set $\eta = .6$ and $\phi = 1.5$ so that we match three target statistics:

i) a durable stock between 40% and 130% of disposable income, depending on whether we consider consumer durables or all private fixed assets (see the data of the Bureau of Economic Analysis mentioned in the introduction). In the calibration we target the average of these values.

ii) a flow of durable consumption between $12-28\%$ of total consumption in the US in the period 1990-2002 (depending on whether durable consumption includes housing services; the source of the data is again the BEA). As above we target the average.

iii) an expenditure on household property and casualty insurance including motor-vehicle insurance, of 2% of disposable income (see the Financial Service Fact Book, 2004).

Figure 1 displays the implied value function as a function of risk-free assets a_t and the durable stock v_t.[5] Not surprisingly the value function is increasing and concave in both a_t and v_t. More interestingly, Figure 2 plots the policies as a function of cash-in-hand for our benchmark case. All variables but the retention ratio are expressed as percent of annual disposable income defined as the sum of quarterly interest and labor income $i \equiv ra + y$ multiplied by 4.

Figures 2b and 2d show that the planned durable stock and non-durable consumption are concave functions of cash-in-hand x. Both functions are quite steep as long as agents do not hold any risk-free assets and the marginal propensity to consume declines substantially as soon as agents hold a buffer stock of risk-free assets (see Figure 2a). This shape of the consumption functions is similar as in the classic models of non-durable consumption with uninsured labor income risk and/or liquidity constraints (see Deaton, 1991, and Carroll and Kimball, 1996 and 2001). Note that the agent only holds a positive amount of risk-free assets in our model because the durable stock is exposed to risk. If that were not the case, an impatient agent would always consume all his resources and never hold a positive amount of risk-free assets.

The main new result in our model is the policy function for the retention ratio D that captures the importance of market insurance (see Figure 2c). The retention ratio is interestingly non-monotonic as a function of cash-in-hand. This is different to the results in Gollier (2003) where

[5] Note that we iterate over $V(x_t)$ and then back out the implied value function over a_t and v_t after the algorithm has converged since this plot is slightly more informative.

the retention ratio (deductible) monotonically increases as a function of cash-in-hand. The reason is that in our model the risk is endogenous and the durable stock is a part of total wealth x. The loss depends on the size of the durable and thus can be influenced by choices of the agent. Market insurance is not desirable for high values of cash-in-hand if a sizeable amount of risk-free assets allows the agent to self insure losses of the durable good. This mechanism implies that the retention ratio increases with cash-in-hand as in Gollier (2003). But in our model, more cash-in-hand also implies a larger amount of the durable stock which increases the exposure but also makes any loss less important in marginal utility terms. This effect reinforces that the retention ratio approaches 1 for large values of cash-in-hand. Low values of cash-in-hand instead imply a small amount of risk-free assets so that losses of the durable cannot be replaced quickly. An important difference to Gollier (2003) is, however, that a small value of cash-in-hand implies a small value of the durable stock and thus also smaller losses. In fact the loss is negligibly small as v approaches 0. In this case market insurance serves no purpose so that the retention ratio also approaches 1 if the cash-in-hand becomes sufficiently small. Moreover, the cost of insurance in terms of marginal utility of forgone non-durable consumption is high for small values of cash-in-hand.

Formally, the desirability of market insurance depends on the covariance of the marginal utility of non-durable consumption and the loss. The optimality condition (4) implies that for an endogenous loss

$$u'(c_t)\mu\eta\lambda v_{t+1} = \beta(1+r)\left[\eta\lambda v_{t+1}E_t u'(c_{t+1}) + cov(u'(c_{t+1}), \eta I_t v_{t+1})\right] \ .$$

Market insurance is more attractive if the covariance is larger since in this case a loss implies a higher marginal utility of non-durable consumption $u'(c_{t+1})$. This can be seen heuristically from the equation above: a larger covariance (a larger right-hand side of the equation) makes agents more willing to spend one more unit in insurance and forgo consumption in period t so that the marginal utility $u'(c_t)$ increases until the equation again holds with equality. As explained above the covariance is small for large values of cash-in-hand if the agent accumulates a substantial amount of risk-free assets (in this case a loss leaves the marginal utility of non-durable consumption essentially unchanged). And the covariance also decreases substantially as v becomes negligibly small. This is illustrated graphically in Figure 2c which plots both the retention ratio and the covariance as a function of cash-in-hand (the scale of the covariance is displayed on the right). The covariance is calculated by setting $D_{t+1} = 1$ but keeping all other policies at their previous optimal values. This gives us the counterfactual covariance between $u'(c_{t+1})$ and $\eta I_t v_{t+1}$ had the agent not chosen to insure himself in the next period.

Finally, in Figures 2a-c we plot the first-order conditions (FOC) for the three choice variables a_{t+1}, v_{t+1} and D_{t+1} as a further check for the accurateness of our computations. Indeed, the Figures show that as soon as interior optima are attained the respective FOC is zero.[6]

We use the policy functions to simulate the model for 1,000 periods. Figure 3 displays the results for the variables of interest for an arbitrary interval of 150 periods within the last 900 periods of the simulation in which the initial conditions are irrelevant. If no shock occurs, the consumer accumulates a substantial amount of the risk-free asset (up to 55% of disposable income in Figure 3b). If a loss occurs these risk-free assets are used to adjust the durable stock close to its old level (see Figure 3a), unless risk-free assets have been exhausted by previous shocks and durable expenditure crowds out non-durable consumption. In this case the durable expenditure is smoothed over a number of periods (see Figure 3e). This implies that durable consumption has spikes in periods after a durable loss occurs whereas non-durable consumption is quite smooth and changes only slightly in the aftermath of a shock as long as risk-free assets are sufficient to smooth out some of the durable loss (see Figure 3d). The fall of the retention ratio in times with a small amount of risk-free assets is not enough to compensate for the lack of financial resources (see Figure 3c). The retention rate fluctuates substantially between .6 in the aftermath of a shock when the agent is liquidity constrained, and 1 in periods in which the stock of risk-free assets is substantial. As a result also insurance expenditure fluctuates between 0 and 5% (see Figure 3f). Rather short time horizons of 15 quarters without a loss are sufficient to accumulate enough liquid assets for

[6] Note that cash-in-hand $x_t \geq 1$ because labor income $y = 1$. Thus, $v_{t+1} > 0$ for all values of cash-in-hand for which we plot the policies so that the corresponding FOC is always zero.

market insurance to become irrelevant. Since time diversification across such periods is likely to matter for real-life consumer decision-making, we can expect our model to match average statistics observed in the US.

Table 2, column 1, displays the averages of the last 900 periods of the simulations for the stocks and flows. Moreover, we compute the relevant measures to be compared with the target statistics where as before we define disposable income as the sum of interest and labor income $i \equiv ra + y$ and convert flows to an annual frequency. In the benchmark case agents hold a durable stock that amounts to 80% of disposable income, durable consumption is 23% of total consumption and the insurance expenditure is 2.4% of disposable income. Obviously, these values are in line with our target statistics since we have chosen η and ϕ accordingly. Moreover, buffer-stock saving is important since agents hold risk-free assets that amount to 14% of disposable income. However, the average retention ratio $D = .81$ is an order of magnitude larger than observed in real world insurance contracts (for example, as mentioned by Gollier, 2003, insurance contracts for cars often have retention ratios of .025: a car with a value of \$20,000 would have insurance contract with a deductible of \$500; see also Drèze, 1981). Quantitatively, the retention ratio in our model is higher than in Gollier (2003) who finds $D = .63$ for similar parameter values. The difference is not surprising. Whereas in the model of Gollier the agent has only the choice between self insurance with risk-free assets and market insurance, in our model the agent also can adjust the durable stock to manage the risk. This decreases the need for market insurance further.

We assess the welfare gains of granting the consumer access to market insurance in addition to the risk-free asset. We compute the value function for the restricted problem $V^r(.)$ without market insurance and compare it with the unrestricted value function $V(.)$ at the steady-state cash-in-hand of the restricted problem, x^r. We then perform the following *Gedankenexperiment*: how much additional cash-in-hand x^* does the consumer need to receive in the restricted case, in order to be compensated for not having access to market insurance? Formally this can be computed as

$$V^r(x^r + x^*) = V(x^r) \ .$$

Note that the unrestricted value function $V(.)$ is also evaluated at the steady state of the restricted problem because the welfare effect of market insurance includes the transition period to the new steady state. We find that market insurance is worth an additional 2.3% in the stock of cash-in-hand or, expressed in certainty-equivalent flows, an additional 0.12% flow of non-durable consumption. Given the rather high retention ratio in the unrestricted equilibrium (which is even higher in our model than in Gollier, 2003, as mentioned above), this welfare effect of market insurance is substantially lower than the 0.66% increase of non-durable consumption in Gollier (2003), Table 1. Thus, allowing for an endogenous risk not only matters for the qualitative shape of the policy function for the retention ratio but also for the quantitative importance of market insurance.

3.1.1 Sensitivity analysis

We now provide a sensitivity analysis for different parameter values in order to investigate whether the result of a high retention ratio is robust. We change one parameter at a time, displaying the new changed parameter value at the top of each column in Table 2.

In column 2 we decrease the discount factor (to an annual value of .935). This increases the spread between the discount rate and the interest rate and makes buffer-stock saving more costly so that the stock of risk-free assets falls to 4.5% of disposable income. Market insurance instead becomes more attractive and the retention ratio falls to $D = .69$. More impatience also makes durable consumption less attractive since utility can only be derived from it next period. However, the durable stock falls only slightly so that insurance expenditure increases to 4.1% of disposable income. Qualitatively similar is the effect of lowering the cost of insurance in column 3. An unrealistically low loading factor of $\mu = 1.1$ is necessary however, in order to lower the retention ratio to more plausible values, $D = .27$. Moreover, in this case, the insurance expenditure of 8.6% of disposable income is much above its empirically observed value.

In columns 4 and 5 we lower the probability of the loss and increase the size of the loss, respectively. Both make market insurance more attractive. A small probability of the loss implies that insurance premiums become relatively cheaper (although market insurance is actuarially unfair this matters less for premiums if the probability of the loss is small). Letting the probability of a loss fall to $\lambda = .01$ implies a retention ratio $D = .59$, and increases the durable stock and thus the risk exposure. Although market insurance becomes more attractive so that the agent no longer holds risk-free assets, the much lower probability of the loss decreases insurance premiums so that insurance expenditure falls to 1.3% of disposable income. Instead, increasing the size of the loss from 60% of the durable stock to 80%, implies a lower retention rate $D = .69$, increases insurance premiums and thus insurance expenditure to 5%, and the stock of risk-free assets to 18% of disposable income. Both non-durable consumption and the durable stock are smaller.

In column 6 we lower the depreciation rate to an annual value of 10% (from previously 15%). This makes it less costly to accumulate the durable so that the durable stock increases. Since this also increases the exposure, the buffer stock of risk-free assets slightly increases whereas the retention ratio falls only slightly from $D = .81$ to $D = .80$.

We also experimented with a monthly instead of quarterly period frequency (results are not reported in Table 2). A shorter frequency decreases the consequences of a durable loss in marginal-utility terms since agents can readjust their durable stock more quickly. This increases the retention rate even further to $D = .88$.[7] Furthermore, relaxing the liquidity constraint so that agents can borrow makes little difference since the stock of cash-in-hand adjusts downward and the wealth effect resulting from interest payments is quantitatively small.

We now investigate in columns 7-11 whether changes in the parameters of the utility function can achieve more realistic values for the retention ratio. In columns 7 and 8 we change the marginal utility the agent derives from durables. A smaller $\phi = 1.3$ implies a lower marginal utility for the durable, ceteris paribus, so that the durable stock and the exposure is smaller, the retention rate slightly increases and the insurance expenditure falls. Analogously, a smaller $\underline{v}= .001$, increases the marginal utility which has the opposite effect. Note that setting $\underline{v}$ to a negligibly small value is innocuous quantitatively. This parameter does not play a big role in our numerical results.

[7] A shorter frequency also changes the intertemporal elasticity of substitution for $\sigma_i \neq 1$ and thus the optimal ratio of non-durable to durable consumption.

Finally, we experiment with higher values of risk aversion. We first symmetrically set $\sigma_1 = \sigma_2 = 9$ in column 9, before we change one parameter at a time in columns 10 and 11, respectively. Not surprisingly higher risk aversion decreases the retention ratio and the durable stock. Market insurance becomes relatively more attractive as the buffer stock of risk-free assets falls. Of course, substitution away from the durable stock towards non-durable consumption would even occur in the certainty case. To provide a heuristic argument, let us assume for simplicity that $\delta = 1$, $\underline{v} = 0$ and an exogenous loss in which case the first-order condition (3) implies

$$\frac{c}{v} = \left(\frac{1}{\beta\phi}\right)^{\frac{1}{\sigma}}.$$

Since $\beta\phi > 1$ for the used parameters, a larger $\sigma = \sigma_1 = \sigma_2$ implies that c/v increases. Moreover, a higher σ also implies a lower intertemporal elasticity of substitution (which we cannot disentangle from risk-aversion using a CRRA utility function). Thus, one could expect that a smaller planned durable stock is chosen to lower intertemporal fluctuations (because the loss increases in the size of the durable stock). However, the simulations reveal that the realized durable stock is adjusted relatively more quickly to its previous level in the aftermath of a shock so that its average value increases, ceteris paribus. Comparing column 9 in which $\sigma_1 = \sigma_2 = 9$ and column 11 in which $\sigma_1 = 9$ and $\sigma_2 = 2$, with our benchmark of $\sigma_1 = \sigma_2 = 2$ suggests that quantitatively the intertemporal elasticity of substitution of the durable alone is not crucial for a sizeable drop of the average durable stock.

More interestingly, our computations show that both σ_1 and σ_2 need to increase for the retention rate to fall substantially to $D = .38$ in column 9. Nonetheless, quantitatively even a sizeable increase in risk aversion to $\sigma_1 = \sigma_2 = 9$ is not enough to generate retention ratios that are consistent with the low levels observed in real-world insurance contracts.

For commonly used values of risk aversion a simple dynamic neoclassical model does not manage to replicate the low retention ratios observed in real-world data. This "insurance" puzzle is reminiscent of the well-known equity premium puzzle (see Mehra and Prescott, 1985, and Gollier, 2003). In reality, agents are "puzzlingly" unwilling to invest in risky assets (notwithstanding their much higher expected returns) similarly as they are unwilling to bear more exposure of the durable stock. However, there are some important differences in our model since agents derive utility from the risky durable stock (which makes them hold the durable in the first place) and can insure the risk of the durable in the market.

One reason why market insurance is not very important in our model might be that we assume that shocks are i.i.d. Permanent shocks are likely to make buffer-stock saving a less perfect substitute for market insurance. However, i.i.d. shocks seem a realistic assumption for most property and liability damages associated with durables. Thus, we explore how other sources of risk with more permanent consequences affect the agent's insurance decision.

3.1.2 Permanent income risk

We assume that labor income risk and the durable risk are independently distributed. Labor income now follows the process

$$y_{t+1} = y_t \varepsilon_t \ ,$$

where ε_t has the support $\{\varepsilon_b, 1, \varepsilon_g\}$ with the corresponding probabilities $\{p_b, 1 - p_g - p_b, p_g\}$. In order to insure comparability with the previous results we do not allow for a change in the mean so that the expected value of income is 1. Moreover, we assume that the standard deviation of permanent income shocks is .1% per year based on estimations for the PSID in the US (see Carroll, 1997). The two implicit restrictions on the first and second moment allow us to retrieve p_g and p_b once we have chosen $\varepsilon_g = 1.1$ and $\varepsilon_b = .75$: a permanent good shock increases labor income by 10% whereas a bad shock decreases it by 25%. These are quite sizeable shocks which help us to find out whether empirically plausible permanent income risk can make market insurance more important.

All other parameters are as in the benchmark case but for $\underline{v}= 0$. In this case we can exploit that the value function is homogenous of degree $1 - \sigma$, $\sigma = \sigma_1 = \sigma_2$, so that permanent income shocks do not add another state variable to the value function (see, for example, Haliassos and

Michaelides, 2001). The independence of the two risks allows us to write

$$EV(x_t, \widetilde{v}_t, y_t) = E_l E_{y|l} V(x_t, \widetilde{v}_t, y_t)$$

and homogeneity then implies

$$E_{y|l} V(x_t, \widetilde{v}_t, y_t) = E_{y|l} \left[\varepsilon_t^{1-\sigma} \widetilde{V}(x_t, \widetilde{v}_t) \right] \ ,$$

which simplifies the computations substantially.

Before we turn to the results let us mention that labor income risk will induce buffer-stock saving even without any durable risk. This has been shown in the seminal papers on precautionary savings in models with non-durable consumption and liquidity constraints (see the partial-equilibrium model of Deaton, 1991, and the general-equilibrium model with heterogenous agents of Aiyagari, 1994). Diaz and Luengo-Prado (2003) and Gruber and Martin (2003) have extended this work adding durable goods with adjustment costs to the model. They find that it is ambiguous in general whether the importance of the precautionary motive increases in dynamic models with durable and non-durable consumption. In these models, durables let current utility depend on past expenditure so that the precautionary savings motive is smaller ceteris paribus. An important difference in our model is that the risk associated with the durable stock strengthens the precautionary motive for the durable good (see also the analytic approximation of the policy functions in Appendix A, for the special case in which liquid wealth is abundant).

The results are summarized in Table 3. In the first column we display the equilibrium values for our benchmark case without permanent labor income risk but with $\underline{v}= 0$. The quantitative results are nearly identical to the results reported for $\underline{v}= 0.001$ in column 8 of Table 2. In Table 3, column 2 we add permanent labor income risk with a standard deviation of .1%. This leaves the quantitative results nearly unchanged. Finally, we increase the permanent income risk to 10% in column 3. This increases the buffer stock of risk-free assets to 20% of disposable income and slightly reduces non-durable consumption. The higher level of risk-free assets makes market insurance even less important so that the retention ratio increases to 84%. The durable stock increases slightly but insurance expenditure falls due to the lower retention ratio. Thus, in the new steady state with a larger stock of durables and risk-free assets, market insurance is even less important although ceteris paribus insurance payments are more valuable if the bad income shock is accompanied by a loss of the durable.

So far we have shown that a dynamic neoclassical model of market insurance has difficulties to match realistic values of retention ratios even if we add additional uninsurable permanent labor income risk to the model. We now briefly mention that this conclusion does not change if we allow for an endogenous interest rate in a general equilibrium setup.

3.2 General equilibrium considerations

In order to analyze dynamic market insurance in general equilibrium we have to adapt our maximization problem slightly. In our model insurance companies make profits since they charge a mark-up and in general equilibrium these profits are part of the consumer's budget constraint. The "new" budget constraint for the average agent reads

$$a_{t+1} = (1+r)a_t + y_t + \overbrace{(1-D_t)El_t}^{\text{insurance claim}} - c_t - d_t - \overbrace{\frac{\mu}{1+r}(1-D_{t+1})El_{t+1}}^{\text{insurance premium}} + \overbrace{\pi_t}^{\text{insurance profits}}$$

where

$$\pi_t = (1+r)\frac{\mu}{1+r}(1-D_{t-1})El_t - (1-D_{t-1})El_t.$$

Note that we use the law of large numbers so that the average loss over a large cross-section of individuals is equal to the expected value.

Using this expression for π_t in the budget constraint and defining

$$b_t \equiv \frac{\mu}{1+r}(1-D_{t-1})El_t$$

we get

$$a_{t+1} + b_{t+1} = (1+r)(a_t + b_t) + y_t - c_t - d_t \ .$$

Aggregate assets in the economy, $a_t + b_t$, are risk-free assets a and the insurance premiums which the insurers invest at interest rate r. With this slight modification, we can solve the model numerically as before for a given interest rate r. This will give us the aggregate "supply" of capital.

In order to find out which interest rate is market clearing, we specify a standard Cobb-Douglas production function with constant returns and a capital-labor ratio k. In intensive form this function is given by

$$f(k) = Ak^{\alpha}$$

with $\alpha = .36$. We normalize with the endogenously computed wage income and exploit the homogeneity of the value function as before. Assuming that factors are paid their marginal product then implies a "demand" for capital[8]

$$k = \frac{\alpha}{(1-\alpha)(r+\xi)}$$

where ξ is the physical depreciation of capital and we assume an annual depreciation rate of $\xi = .08$. For each "supply" of assets computed for given r, we can use the equation for the "demand" for capital to update the interest rate and restart the algorithm until convergence. This method is essentially as in Aiyagari (1994). It turns out that the interest rate r which would clear the market for our parameter values is so close to the discount rate that there is no role for market insurance at all. For example, at an annual interest rate of $r = .048$ (which is close to the implied annual discount rate of 5.25%) the supply for capital in our model is only one quarter of the demand. Since we know from Aiyagari (1994), that the assets held by the agent (capital "supply") approach infinity as the interest gets ever closer to the discount rate from below, the market clearing interest rate is even higher. We find that already at the interest rate of $r = .048$, the retention rate is 99.61 so that market insurance is negligible. Changing the parameters η and ϕ (which were chosen above to match target statistics for $r = .01$) is not likely to make market insurance more important in general equilibrium. Increasing the size of the loss to .8 in Table 2, column 5 does not increase the stock of risk-free assets enough to change the quantitative result; and choosing a larger ϕ to increase

[8] The first-order conditions are $A\alpha k^{\alpha-1} = r + \xi$ and $A(1-\alpha)k^{\alpha} = w$. Dividing both conditions and setting $w = 1$ results in the expression in the text.

the stock of risk-free assets and to reduce the retention ratio would imply unrealistic spending patterns of durable compared with non-durable consumption (see Table 2, column 7). Finally, for the empirically observed risk-free rate of $r = .01$ to clear the market in general equilibrium, we would need to decrease the impatience of consumers. This, however, decreases the relevance of market insurance. The small quantitative role of market insurance remained robust for many alternative parameter combinations that we have tried.

4 Conclusion

We set up and solve numerically a dynamic neoclassical model of market insurance in which agents derive utility from a durable and non-durable good. The durable good is exposed to risk and can be insured. We analyze the role of market insurance if agents also have access to a risk-free asset. We provide intuition for our results using first-order conditions, and a second-order approximation for the special case of abundant liquid wealth.

We find that the retention ratio (the fraction of the durable that is not insured) is a non-monotonic function of cash-in-hand. This differs from previous results in the literature and can be explained with the endogenous loss associated with the durable stock. Calibrating our model to the US economy, we find that the average retention ratio is an order of magnitude larger than observed in reality. This result remains unchanged if we add permanent labor income risk to the model or allow for endogenous interest rates. This "insurance" puzzle is reminiscent of the equity-premium puzzle although our model is different in a number of respects.

Future research could relax some of the simplifying assumptions in our model such as the perfect divisibility of durable goods or the separability of the utility function in order to be more confident about the results. One could also allow for more institutional detail on insurance contracts. Regulation on insurance contracts might restrict agents in their choice of retention ratios although in reality mandatory insurance is hard to enforce (see Smith and Wright, 1992). Finally, more complex models with life-cycle patterns of durable and non-durable consumption might help to increase the importance of market insurance. If agents accumulate durables early in life to relax collateral constraints (see Krueger and Villaverde, 2002), this makes a loss of the durable especially costly so that market insurance might become more desirable.

Appendices

Appendix A: Approximation of the policy functions for abundant liquid wealth

The model presented in the text does not have a closed-form solution in general. We present the results of a second-order approximation of the policy functions and laws of motion to develop some intuition. The approximation is done for the model with two sources of risk in labor income and the durable stock, keeping the interest rate constant. The main point of this exercise is to show that in our model precautionary motives are important even if agents own abundant liquid wealth because durables are a state variable and directly enter the utility function.

To approximate the solution we employ the perturbation method which is explained in detail in Schmitt-Grohé and Uribe (2001) using the Euler equations (2)~(4) and the two laws of motion for a and v. We first provide the solution of the approximation and some intuition before we get to the derivation. Because we do allow the state variables to be at interior optima only, for the purpose of the approximation, the results simplify considerably and can be used to develop intuition on the mechanics of the model. That is, we assume that the agent's stock of risk-free assets is sufficient to buffer income shocks and allow the durable stock to return to the steady state after one period through the appropriate durable investment if a loss occurs. The policy functions can then be approximated[9] by

[9] We derived the approximation for the more general case in which households do not have abundant liquid wealth. However, expressions become very messy so that they do not help much to develop intuition on the mechanics of the model. Hence, we focus on a special, but important case.

$c = c_{ss}$

$d = d_{ss} - (1-\delta)(v - v_{ss}) + \frac{1}{2}\gamma\sigma_l^2$

$D = D_{ss}$

$a = a_{ss} + (1+r)(a - a_{ss}) - (1-\delta)(v - v_{ss}) - \frac{1}{2}\gamma\sigma_l^2$

$v = v_{ss} + \frac{1}{2}\gamma\sigma_l^2$,

where

$$\gamma \equiv -\frac{w'''(v_{ss})}{w''(v_{ss})} \ .$$

The steady state is obtained solving the model without uncertainty. Hence, no market insurance is demanded, i.e., $D_{ss} = 1$. Note that the steady state of risk-free assets a_{ss} is such that the steady state non-durable consumption c_{ss} and durable investment $d_{ss} = \delta v_{ss}$ are feasible. We now discuss the solution in detail.

First-order deviations The first-order deviations of a or v from the steady state do not affect non-durable consumption or the durable stock. Shocks occurring to the durable stock are offset after one period (net of the depreciation rate) by durable investment which is fully financed by risk-free assets. Shocks occurring to risk-free assets do not result in any reaction of the controls but only change risk-free assets by the same amount and the rate of return.

Second-order deviations and variances Neither the second-order deviations of risk-free assets from the steady state nor the one of the durable stock do matter if agents have abundant liquid wealth. Instead, the variance of the durable shock turns out to be important whereas the variance of income does not affect the solution. The asymmetric effect of the variances is resulting from the model's structure: since v enters the utility function, the fluctuations of v directly result in variation of utility whereas this is not the case for fluctuations of a.

The importance of the variance of the loss depends on γ, a measure of prudence with respect to the durable stock, $-w'''(v_{ss})/w''(v_{ss})$, defined according to Kimball (1990). Since agents derive utility from the durable stock, they invest more into durables if durables are more exposed to risk. Thus, the durable stock rises and liquid wealth falls. This is in contrast to investment behavior for risky assets from which agents do not directly derive utility but only indirectly since more assets afford more units of non-durable consumption. We now present the derivation of the approximation in some detail.

Derivation Consistent with the notation used in Schmitt-Grohé and Uribe (2001) we define the matrix F as

$$F \equiv \begin{bmatrix} u'(c_t) - \beta(1+r)\ E_t u'(c_{t+1}) \\ u'(c_t) - \beta\left[(1-\delta)E_t u'(c_{t+1}) + \phi E_t w'(\widetilde{v}_{t+1})\right] \\ \beta(1+r)E_t\{u'(c_{t+1})l_{t+1}\} - u'(c_t)\mu E l_{t+1} \\ a_{t+1} - (1+r)a_t + c_t + d_t + \frac{\mu}{1+r}(1-D_{t+1})El_{t+1} - (1-D_t)l_t - y_t \\ v_{t+1} - (1-\delta)v_t - d_t + l_t \end{bmatrix},$$

where $F = 0$. We define the controls as $\zeta = (c, d, D)'$ and the state variables as $x = (a, v)'$. The shocks can be rewritten as

$$l = m_l + \sigma_l \varepsilon_l$$

and

$$y = m_y + \sigma_y \varepsilon_y\ ,$$

where $\varepsilon_i \sim N(0,1)$, $i = l, y$ and $m_y = Ey$, $m_l = El$. The shocks are assumed to be i.i.d.

We know that the solution will take the form $\zeta_t = g(x_t', \sigma')$ and $x_{t+1} = h(x_t', \sigma') + \eta\varepsilon_t$, where $\varepsilon_t = (\varepsilon_{lt}, \varepsilon_{yt})'$. The 2x2 matrix η and $\sigma = (\sigma_y, \sigma_l)'$ are known. In our model

$$\eta = \begin{bmatrix} (1-D)\sigma_l & \sigma_y \\ -\sigma_l & 0 \end{bmatrix}.$$

Note that y_t and l_t are i.i.d. distributed shocks. To perform a second-order approximation, first and second derivatives of the functions $g(.)$ and $h(.)$ need to be determined. As explained in more detail in Schmitt-Grohé and Uribe (2001) this is done by taking first and second derivatives of F with respect to x and exploiting the fact that these derivatives are 0.

We find that

$$F_x = \begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1})\left[g_a^c h_a^a + g_v^c h_a^v\right] + u''(c_t) g_a^c \\ -\beta(1-\delta)E_t u''(c_{t+1})\left[g_a^c h_a^a + g_v^c h_a^v\right] + u''(c_t) g_a^c - \beta\phi E_t w''(\widetilde{v}_{t+1}) h_a^v \\ \frac{\partial F(3,1)}{\partial c_{t+1}}\left[g_a^c h_a^a + g_v^c h_a^v\right] - \frac{\mu}{1+r} E l_{t+1}\ u''(c_t) g_a^c + \frac{\partial F(3,1)}{\partial D_{t+1}} g_a^D \\ g_a^c + g_a^d - \frac{\mu}{1+r} E l_{t+1} g_a^D + h_a^a - (1+r) \\ -g_a^d + h_a^v \\ -\beta(1+r)E_t u''(c_{t+1})\left[g_a^c h_v^a + g_v^c h_v^v\right] + u''(c_t) g_v^c \\ -\beta(1-\delta)E_t u''(c_{t+1})\left[g_a^c h_v^a + g_v^c h_v^v\right] + u''(c_t) g_v^c - \beta\phi E_t w''(\widetilde{v}_{t+1}) h_v^v \\ \frac{\partial F(3,1)}{\partial D_{t+1}}\left[g_a^c h_v^a + g_v^c h_v^v\right] - \frac{\mu}{1+r} E l_{t+1}\ u''(c_t) g_v^c + \frac{\partial F(3,1)}{\partial D_{t+1}} g_v^D \\ g_v^c + g_v^d - \frac{\mu}{1+r} E l_{t+1} g_v^D + h_v^a \\ -g_v^d + h_v^v - (1-\delta) \end{bmatrix},$$

where derivatives of F with respect to a are in rows 1-5 and derivatives with respect to v are in rows 6-10. The notation g_a^c denotes the derivative of non-durable consumption c with respect to a and $F(3,1)$ is the element in the third row and first column of F. Expressions for $\frac{\partial F(3,1)}{\partial c_{t+1}}$ and $\frac{\partial F(3,1)}{\partial D_{t+1}}$ can be derived using the fact that $Eab = EaEb + cov(a,b)$, but are lengthy so that we use shorthand notation. F_x is a system of 10 equations in 10 unknowns g_j^i, h_j^k with $i = c, d, D$; $j = a, v$ and $k = a, v$.

It turns out that one solution of the system of equations is

$$\begin{aligned} g_a^c &= g_v^c = g_a^d = g_a^D = g_v^D = h_a^v = h_v^v = 0, \\ h_a^a &= 1 + r, \\ g_v^d &= -(1-\delta), \\ h_v^a &= 1 - \delta. \end{aligned}$$

This is the case of abundant liquid wealth when $D = D_{ss}$ and $c = c_{ss}$. The solution for the general case is messy and does not add to the intuition.

We calculate F_{xx} which gives us 20 equations to determine 20 second-order derivatives. Using that some of the first-order derivatives are zero, we get that

$$F_{xa} = \begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1}) g^c_{aa} \left(h^a_a\right)^2 + u''(c_t) g^c_{aa} = 0 \\ -\beta(1-\delta)E_t u''(c_{t+1}) g^c_{aa} \left(h^a_a\right)^2 + u''(c_t) g^c_{aa} - \beta\phi E_t w''(\widetilde{v}_{t+1}) h^v_{aa} = 0 \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g^c_{aa} \left(h^a_a\right)^2 - \frac{\mu}{1+r} El_{t+1} u''(c_t) g^c_{aa} + \frac{\partial F(3,1)}{\partial D_{t+1}} g^D_{aa} = 0 \\ g^c_{aa} + g^d_{aa} - \frac{\mu}{1+r} El_{t+1} g^D_{aa} + h^a_{aa} = 0 \\ -g^d_{aa} + h^v_{aa} = 0 \\ -\beta(1+r)E_t u''(c_{t+1}) g^c_{aa} h^a_a h^a_v + u''(c_t) g^c_{va} = 0 \\ -\beta(1-\delta)E_t u''(c_{t+1}) g^c_{aa} h^a_a h^a_v + u''(c_t) g^c_{va} - \beta\phi E_t w''(\widetilde{v}_{t+1}) h^v_{va} = 0 \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g^c_{aa} h^a_a h^a_v - \frac{\mu}{1+r} El_{t+1} u''(c_t) g^c_{va} + \frac{\partial F(3,1)}{\partial D_t} g^D_{va} = 0 \\ g^c_{va} + g^d_{va} - \frac{\mu}{1+r} El_{t+1} g^D_{va} + h^a_{va} = 0 \\ -g^d_{va} + h^v_{va} = 0 \end{bmatrix}$$

for the derivatives with respect to aa in rows 1-5 and derivatives with respect to va in rows 6-10 and

$$F_{xv} = \begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1}) g^c_{av} \left(h^a_v\right)^2 + u''(c_t) g^c_{vv} \\ -\beta(1-\delta)E_t u''(c_{t+1}) g^c_{av} \left(h^a_v\right)^2 + u''(c_t) g^c_{vv} - \beta\phi E_t w''(\widetilde{v}_{t+1}) h^v_{vv} \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g^c_{av} \left(h^a_v\right)^2 - \frac{\mu}{1+r} El_{t+1} u''(c_t) g^c_{vv} + \frac{\partial F(3,1)}{\partial D_{t+1}} g^D_{vv} \\ g^c_{vv} + g^d_{vv} - \frac{\mu}{1+r} El_{t+1} g^D_{vv} + h^a_{vv} \\ -g^d_{vv} + h^v_{vv} \\ -\beta(1+r)E_t u''(c_{t+1}) g^c_{av} h^a_v h^a_a + u''(c_t) g^c_{av} \\ -\beta(1-\delta)E_t u''(c_{t+1}) g^c_{av} h^a_v h^a_a + u''(c_t) g^c_{av} - \beta\phi E_t w''(\widetilde{v}_{t+1}) h^v_{av} \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g^c_{av} h^a_v h^a_a - \frac{\mu}{1+r} El_{t+1} u''(c_t) g^c_{av} + \frac{\partial F(3,1)}{\partial D_{t+1}} g^D_{av} \\ g^c_{av} + g^d_{av} - \frac{\mu}{1+r} El_{t+1} g^D_{av} + h^a_{av} \\ -g^d_{av} + h^v_{av} \end{bmatrix}$$

for the derivatives with respect to vv in rows 1-5 and derivatives with respect to av in rows 6-10. It follows from $F_{xa} = F_{xv} = 0$ that all second-order derivatives equal zero.

It remains to derive F_{σ_i}, $i = l, y$. Using the results for the first derivatives g^i_j, h^k_j with $i = c, d, D$; $j = a, v$ and $k = a, v$, and $E_t\varepsilon_l = E_t\varepsilon_y = 0$, we find

$$F_{\sigma_l}=\begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1})g^c_{\sigma_l}+u''(c_t)g^c_{\sigma_l} \\ -\beta(1-\delta)E_t u''(c_{t+1})g^c_{\sigma_l}+u''(c_t)g^c_{\sigma_l}-\beta\phi E_t w''(\widetilde{v}_{t+1})h^v_{\sigma_l} \\ \frac{\partial F(3,1)}{\partial c_{t+1}}g^c_{\sigma_l}-\frac{\mu}{1+r}El_{t+1}\ u''(c_t)g^c_{\sigma_l}+\frac{\partial F(3,1)}{\partial D_{t+1}}g^D_{\sigma_l} \\ g^c_{\sigma_l}+g^d_{\sigma_l}-\frac{\mu}{1+r}El_{t+1}g^D_{\sigma_l}+h^a_{\sigma_l}-(1-D_t)E_t\varepsilon_l \\ -g^d_{\sigma_l}+h^v_{\sigma_l}+E_t\varepsilon_l \end{bmatrix}$$

and

$$F_{\sigma_y}=\begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1})g^c_{\sigma_y}+u''(c_t)g^c_{\sigma_y} \\ -\beta(1-\delta)E_t u''(c_{t+1})g^c_{\sigma_y}+u''(c_t)g^c_{\sigma_y}-\beta\phi E_t w''(\widetilde{v}_{t+1})h^v_{\sigma_y} \\ \frac{\partial F(3,1)}{\partial c_{t+1}}g^c_{\sigma_y}-\frac{\mu}{1+r}El_{t+1}\ u''(c_t)g^c_{\sigma_y}+\frac{\partial F(3,1)}{\partial D_{t+1}}g^D_{\sigma_y} \\ g^c_{\sigma_y}+g^d_{\sigma_y}-\frac{\mu}{1+r}El_{t+1}g^D_{\sigma_y}+h^a_{\sigma_y}-E_t\varepsilon_y \\ -g^d_{\sigma_y}+h^v_{\sigma_y} \end{bmatrix}.$$

Since F_{σ_l} and F_{σ_y} are linear and homogenous in $g^j_{\sigma_l}$ and $h^k_{\sigma_l}$, $F_{\sigma_l}=F_{\sigma_y}=0$ implies that $h^k_{\sigma_l}=0$ and $g^j_{\sigma_l}=0$ with $j=c,d,D$; $i=l,y$ and $k=a,v$.

For the second-order derivatives we get

$$F_{\sigma_l\sigma_l}=\begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1})g^c_{\sigma_l\sigma_l}+u''(c_t)g^c_{\sigma_l\sigma_l} \\ u''(c_t)g^c_{\sigma_l\sigma_l}-\beta\left[\phi E_t w'''(\widetilde{v}_{t+1})E_t\left(\varepsilon_l^2\right)+(1-\delta)E_t u''(c_{t+1})g^c_{\sigma_l\sigma_l}+\phi E_t w''(\widetilde{v}_{t+1})h^v_{\sigma_l\sigma_l}\right] \\ \frac{\partial F(3,1)}{\partial c_{t+1}}g^c_{\sigma_l\sigma_l}-\frac{\mu}{1+r}El_{t+1}\ u''(c_t)g^c_{\sigma_l\sigma_l}+\frac{\partial F(3,1)}{\partial D_{t+1}}g^D_{\sigma_l\sigma_l} \\ g^c_{\sigma_l\sigma_l}+g^d_{\sigma_l\sigma_l}-\frac{\mu}{1+r}El_{t+1}g^D_{\sigma_l\sigma_l}+h^a_{\sigma_l\sigma_l} \\ -g^d_{\sigma_l\sigma_l}+h^v_{\sigma_l\sigma_l} \end{bmatrix}$$

and

$$F_{\sigma_y\sigma_y}=\begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1})g^c_{\sigma_y\sigma_y}+u''(c_t)g^c_{\sigma_y\sigma_y} \\ u''(c_t)g^c_{\sigma_y\sigma_y}-\beta\left[(1-\delta)E_t u''(c_{t+1})g^c_{\sigma_y\sigma_y}+\phi E_t w''(\widetilde{v}_{t+1})h^v_{\sigma_y\sigma_y}\right] \\ \frac{\partial F(3,1)}{\partial c_{t+1}}g^c_{\sigma_y\sigma_y}-\frac{\mu}{1+r}El_{t+1}\ u''(c_t)g^c_{\sigma_y\sigma_y}+\frac{\partial F(3,1)}{\partial D_{t+1}}g^D_{\sigma_y\sigma_y} \\ g^c_{\sigma_y\sigma_y}+g^d_{\sigma_y\sigma_y}-\frac{\mu}{1+r}El_{t+1}g^D_{\sigma_y\sigma_y}+h^a_{\sigma_y\sigma_y} \\ -g^d_{\sigma_y\sigma_y}+h^v_{\sigma_y\sigma_y} \end{bmatrix}.$$

$F_{\sigma_y\sigma_y}$ is linear and homogenous in $g^j_{\sigma_y\sigma_y}$ and $h^k_{\sigma_y\sigma_y}$, $g^j_{\sigma_y\sigma_y}=h^k_{\sigma_y\sigma_y}=0$. The same holds for $F_{\sigma_y\sigma_l}$ because

$E_t\varepsilon_l\varepsilon_y = 0$; and for $F_{x\sigma_i}$, $i = l, y$. Instead, given that $E_t\varepsilon_l^2 = 1$, $F_{\sigma_l\sigma_l} = 0$ implies

$$h^a_{\sigma_l\sigma_l} = -h^v_{\sigma_l\sigma_l} = -g^d_{\sigma_l\sigma_l} = \frac{w'''(v_{ss})}{w''(v_{ss})} \ ,$$

where the other second-order derivatives are found to be zero.

Appendix B: Description of numerical algorithm

We solve the problem using value function iteration. First, we guess upper bounds $\overline{v}$ and $\overline{a}$ for the durable stock and risk-free assets, respectively, which imply an upper bound for cash-in-hand $\overline{x} = (1+r)\,\overline{a} + (1-\delta)\overline{v} + y$. The additional constraints $a_s \leq \overline{a}$ and $v_s \leq \overline{v}$ help us to generate a compact state space. We then solve the problem given these constraints as described below. If either constraint is binding in the solution of the constrained problem, we increase $\overline{v}$ and $\overline{a}$ and iterate again.

We approximate the value function with a high-order Chebychev polynomial. A high-order polynomial is necessary because the value function becomes steep in the region of the state space where agents do not hold any risk-free assets, i.e. for small x. We check the precision on a fine grid x between the evaluation nodes. We compare $V^{cheb}(x)$ to $V(x)$ with

$$V(x_t) = max_{D_{t+1}, v_{t+1}, a_{t+1}} \left\{ u(c_t) + \beta E_t \left(V^{cheb}(x_{t+1}) + \phi w(\widetilde{v}_{t+1}) \right) \right\}.$$

For the benchmark parameter values, we started with a polynomial of order 29 with 30 evaluation nodes which resulted in $max \left| V^{cheb}(x) - V(x) \right| < .0001$. Building on this precise approximation, we were able to reduce the order of the polynomial to 10 and the number of nodes to 15 for computations with other parameter values, achieving the same precision. We also experimented with cubic splines, but found that computational time increased for similar levels of accuracy.

We solve the maximization problem calling the Matlab routine fminsearch, which uses the Nelder-Mead simplex method. To ensure that we find the global maximum, we evaluate

$$\Lambda \equiv u(c_t) + \beta E_t \left(V^{cheb}(x_{t+1}) + \phi w(\widetilde{v}_{t+1}) \right)$$

on a fine grid of values of $k = [D_{t+1}, v_{t+1}, a_{t+1}]$ and pick the five values of $\left\{k_j^{\text{ini}}\right\}_{j=1:5}$ that yield the highest values of Λ. We use these values to initialize the Nelder-Mead simplex method and retrieve the solution $\left\{k_j^{\text{sim}}\right\}_{j=1:5}$. We then choose the value of $\left\{k_j^{\text{sim}}\right\}_{j=1:5}$ that maximizes the value function. If any of the elements of $\left\{k_j^{\text{sim}}\right\}_{j=1:5}$ are within a specified (small) distance of the respective constraints, we additionally check whether setting the respective element equal to the constraint and reoptimizing the other elements yields an improvement.

For each iteration step i, we calculate the maximum iteration errors $\varepsilon_V \equiv \left|V^i - V^{i-1}\right|$, $\varepsilon_a \equiv \left|a_{t+1}^i - a_{t+1}^{i-1}\right|$, $\varepsilon_v \equiv \left|v_{t+1}^i - v_{t+1}^{i-1}\right|$ and $\varepsilon_D \equiv \left|D_{t+1}^i - D_{t+1}^{i-1}\right|$ on the grid of the evaluation nodes. The iteration procedure is terminated when all errors are less than .001. We further check the accuracy of the solution by calculating the analytical first-order conditions (FOC). As one can observe in Figures 2a-c, the FOC are satisfied up to a tolerance of .001 when the constraints are not binding.

To calculate the expected values of a,v, D, d, c, and insurance expenditure, we simulate the model for $1,000$ periods, discard the first 100 observations and take averages. The results are insensitive to the initial conditions used in the simulation. Except for the retention ratio, all values reported in the table and figures are in percentages of annual average income.

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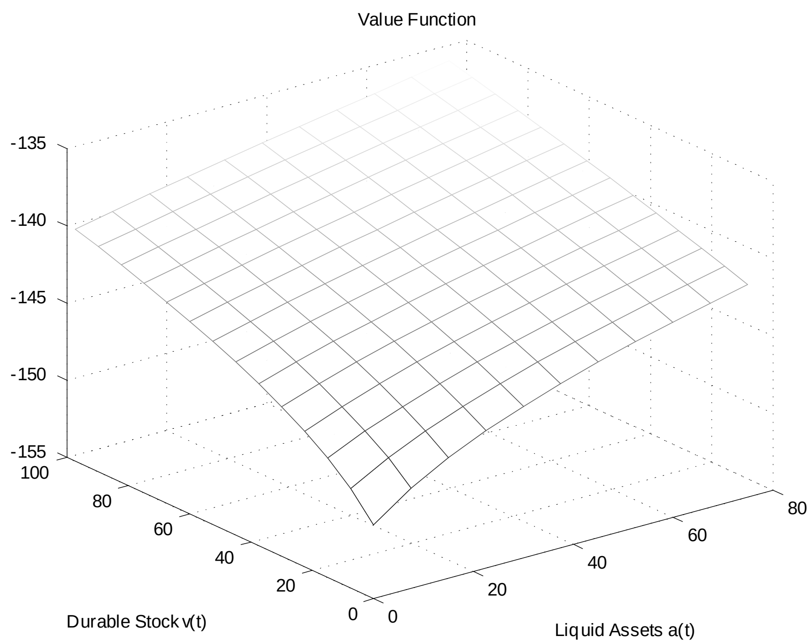

Figure 1: Value function for the benchmark case. Note: the units of the durable stock and liquid assets are % of disposable income.

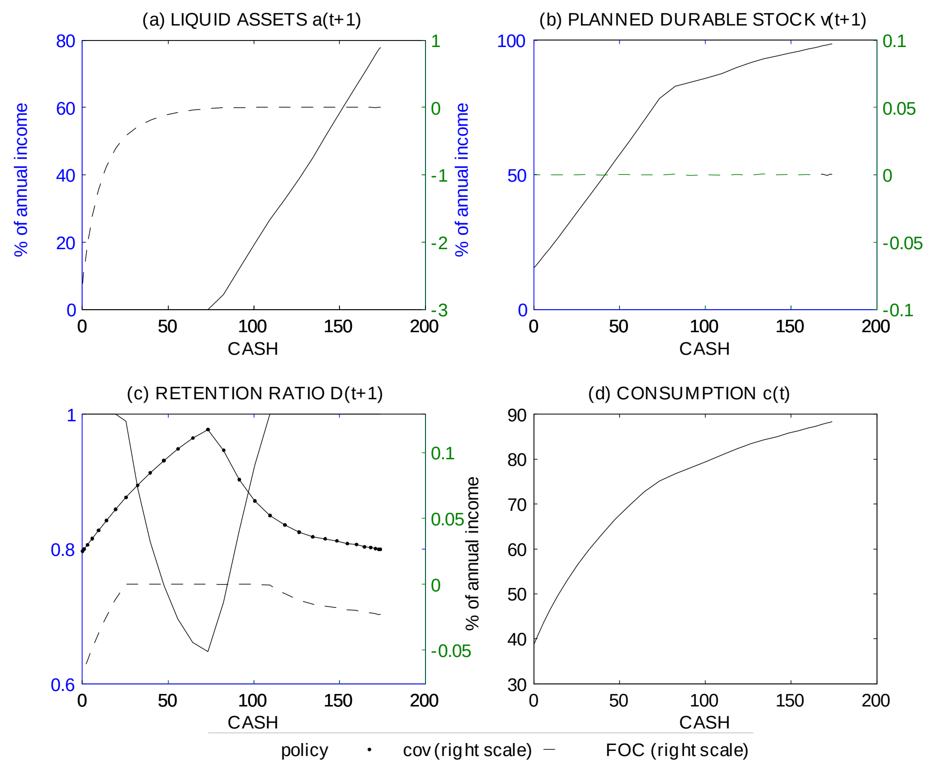

Figure 2: Policies as a function of cash-in-hand x_t for the benchmark parameters. Note: All variables but the retention ratio are in % of disposable income.

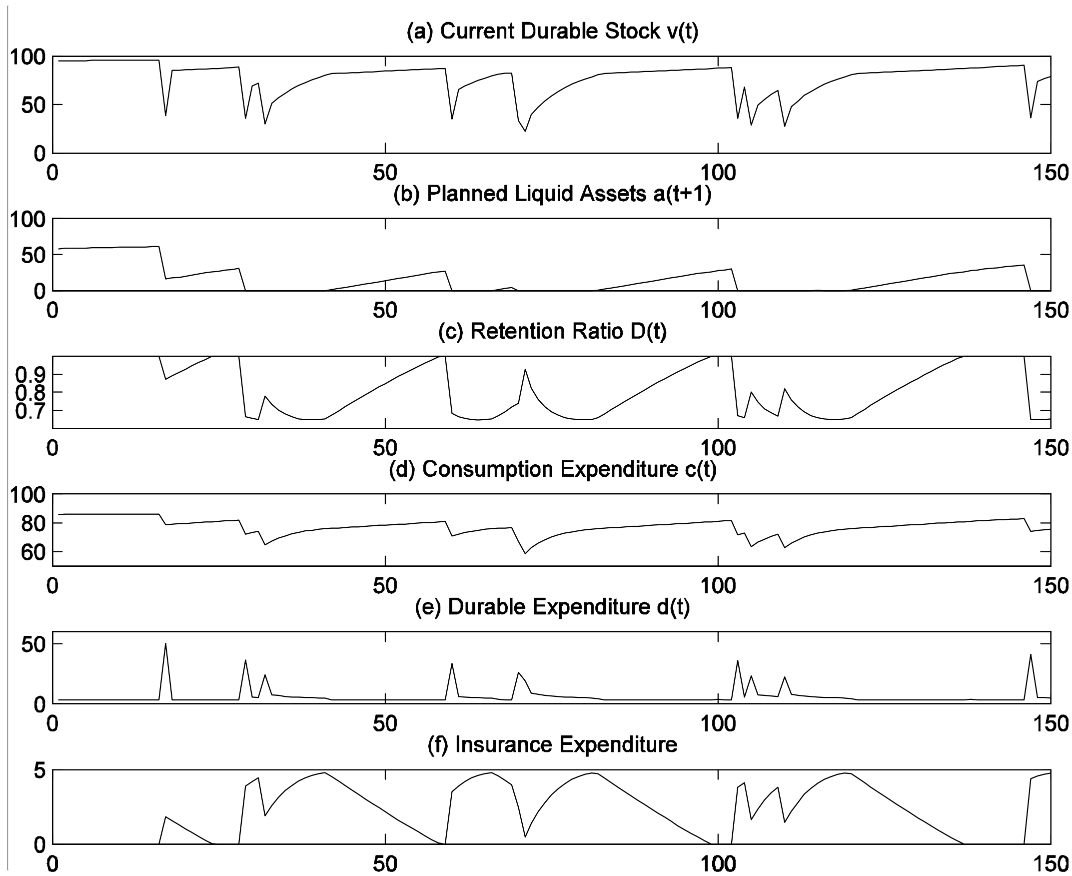

Figure 3: Simulation of the main variables of interest for 150 quarters. Note: Simulations have been performed for 1,000 quarters of which the first 100 quarters have been discarded so that initial conditions do not matter. All variables but the retention ratio are in % of disposable income.

Table 1: Benchmark parameters

$r = .0025$	$\beta = .9873$	$\delta = .0355$	$\lambda = .0543$
$\eta = .6$	$\mu = 1.3$	$\phi = 1.5$	$\underline{v} = .1$
$\sigma_1 = 2$	$\sigma_2 = 2$	$y = 1$	

Table 2: Average equilibrium values

Variables	*Benchmark*	$\beta = .983$	$\mu = 1.1$	$\lambda = .01$	$\eta = .8$	$\delta = .024$
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
liquid assets a	13.9	4.5	0.0	0.0	18.6	14.3
durable stock v	80.7	78.8	81.4	104.4	75.1	88.8
durabl. cons. d	23.0	22.3	23.2	18.0	26.0	21.1
non-d. cons. c	77.1	77.2	76.8	81.9	74.7	78.8
retention ratio D	81.5	68.8	26.5	58.7	69.2	80.2
durabl./total cons.	23.0	22.4	23.2	18.0	25.8	21.1
insurance exp.	2.4	4.1	8.6	1.3	5.0	2.8
	$\phi = 1.3$	$\underline{v} = .001$	$\sigma_1 = \sigma_2 = 9$	$\sigma_2 = 9$	$\sigma_1 = 9$	
	(7)	**(8)**	**(9)**	**(10)**	**(11)**	
liquid assets a	13.2	14.3	5.9	12.6	6.8	
durable stock v	76.3	82.9	42.7	48.1	50.4	
durabl. cons. d	21.7	23.6	12.2	13.7	14.4	
non-d. cons. c	78.4	76.4	87.1	86.2	85.4	
retention ratio D	82.7	79.6	38.4	70.2	67.3	
durabl./total cons.	21.7	23.6	12.3	13.7	14.4	
insurance exp.	2.1	2.7	4.4	2.4	2.7	

Table 3: Permanent labor income risk

Variables	*Benchmark,* $\underline{v} = 0$, $sd = 0$	$sd = .1\%$	$sd = 10\%$
	(1)	**(2)**	**(3)**
liquid assets a	14.39	14.44	19.77
durable stock v	82.87	82.91	83.68
durabl. cons. d	23.60	23.61	23.85
non-d. cons. c	76.43	76.42	76.39
retention ratio D	79.67	79.72	84.39
durabl./total cons.	23.59	23.60	23.79
insurance exp.	2.69	2.67	2.03

Note: sd: standard deviation of permanent income