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ABSTRACT

Why is Old Workers' Labor Market More Volatile? Unemployment Fluctuations over the Life-Cycle*

Since the last recession, it is usually argued that older workers are less affected by the economic downturn because their unemployment rate rose less than the one of prime-age workers. This view is a myth: older workers are more sensitive to the business cycle. We document volatilities of worker flows and hourly wage across age groups on CPS data. We find that old worker's job flows are characterized by a higher responsiveness to business cycles than their younger counterparts. In contrast, their wage cyclicality is lower than prime-age workers'. Beyond this empirical contribution, we show that a life-cycle Mortensen & Pissarides (1994) model is well suited to explain these facts: older workers' shorter work-life expectancy endogenously reduces their outside options and leads their wages to be less sensitive to the business cycle. Thus, in a market where wage adjustments are small, quantities vary a lot: this is the case for older workers, whereas the youngest behave like infinitively-lived agents. Our theoretical results point out that Shimer (2005)'s view on the MP model is consistent with prime-age workers' labor market while aging endogenously introduces real wage rigidities, allowing to match what we observe for old workers, without specific assumptions as in Hagendorn & Manovskii (2008).

JEL Classification: E32, J11, J23

Keywords: search, matching, business cycle, life-cycle

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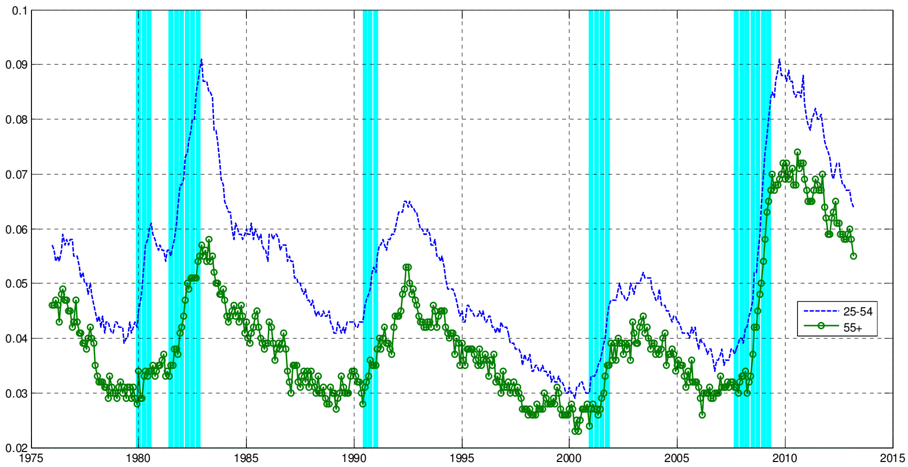
1 Introduction

The last recession has dramatically deteriorated labor market conditions. Figure 1 displays monthly unemployment rate across age groups since 1976. The unemployment ramp up in the aftermath of the financial crisis shows the depth of the so-called Great Recession. Figure 1 shows substantial differences across age groups.

Figure 1: Unemployment rate by age. BLS monthly data. 1976 January - 2013 March



It is often stressed that old workers were less affected by the economic downturn, and more generally by the business cycle. This view is reinforced by the last recession: prime-age workers' unemployment rate has risen by 5.5 percentage points, from 3.5% in 2007 to 9% in 2009. In contrast, the unemployment rate of workers aged 55+ displays an increase of 4 percentage points (from 3% to 7% in the same period). Are these statistic sufficient to establish a stylized fact? In our view, the answer is no. We document the respective roles of job loss and job finding in shaping the change in unemployment across age groups using monthly CPS data. Secondly, we propose a new perspective on the structure of joblessness across age groups along the business cycle. Our findings lead us to challenge the view that the brunt of the recession is not borne by old workers because they experienced a moderate hike in their unemployment rates. We also document, using the same monthly data set, the fact that the highest volatile labor markets are also the ones where the volatility of the hourly wage is the smallest. In this paper, we put forward the view that, when considering several business cycle episodes, old workers' labor market adjustments actually display more volatility than the one prevailing for their younger counterparts. We then argue, using a life cycle Mortensen & Pissarides (1994), that the work-life expectancy plays a key role in accounting for the high responsiveness of old workers' job flows to the business cycle. The basic intuition is the following: because their is no incentive to search close to retirement, a short horizon reduces the outside option value to zero, leading wage to be more rigid at the end of the life cycle. Then, quantities (prices) are more (less) volatile at the end of the life cycle. Numerical experiments support this view. Our key mechanism relies on the distance to retirement. In the model, agents differ only with respect to this dimension. We lay thereby stress on the difference between old workers, close to retirement, and their younger counterparts, prime-age individuals. This then entices us to consider only 2 age groups (older versus prime-age workers). In the last part of the paper, we include a third age group (young workers) as a robustness check. We also include an exogenous increase in the labor productivity in order to match the age-increasing pattern of the hourly wage, and show that the "horizon effect" keeps its dominant role in the explanation of the age-decreasing volatilities.

In this paper, we first investigate the forces behind the age heterogeneity in joblessness. Using Shimer (2012)'s methodology, we measure workers' flows between employment and unemployment using CPS monthly male data between March 1976 and March 2013. Historical averages suggest that older workers (55-61 years old) are characterized by slower employment exit and longer unemployment spell than prime-age workers (25-54 years old). Job finding and separation rates fall with age. These results are consistent with the age-decreasing transitions found in the male population in Choi et al. (2013), Menzio et al. (2012) and Gervais et al. (2012). Beyond these long-run features, we first document business cycle fluctuations in worker flows across age groups in the US. More surprisingly, the volatility of the cyclical component of workers' transition rates display an age-increasing pattern: older workers' are more responsive to the business cycle than their younger counterparts.[1] Data show that older workers' higher relative sensitivity to the business cycle is more striking for the job finding rate than for the job separation rate. We also investigate the business cycle response of real hourly wage, using monthly CPS data, and show that the life-cycle pattern of wage volatility is actually age-decreasing, which is consistent with the large labor market adjustments found for old workers. Our empirical investigation adds to the existing literature along 2 dimensions: first, previous papers (Gomme et al. (2005), Jaimovich & Siu (2009)) have focused only on age-specific cyclicality of worked hours and employment. However, since Mortensen & Pissarides (1994)'s seminal work, it is known that worker flows matter in shaping labor market adjustments. This is the focus of our paper. Elsby et al. (2010) and Elsby et al. (2011) document the age pattern of worker flows but they did not investigate business cycle features. This paper fills this gap. Secondly, we also characterize wage cyclicality across age groups at business cycle frequencies. Jaimovich & Siu (2009) do so using annual data. In our view, since recessions typically last less than a year (with the exception of the 1981 and 2008 episodes), it is paramount to document wage fluctuations using infra-annual data.

We then develop a theoretical model to account for the reason why older workers could be more sensitive (in relative terms) to the business cycle than their younger counterparts. This model must explain three facts: old workers' labor market is characterized by less rotations (levels of job finding

[1] We perform several checks to make sure that this is a robust business cycle features.

and separation rate are lower for old workers) but displays more volatile behavior than prime-age workers. At the same time, the average hourly wages of the older workers are the highest, but its volatilities is the are smallest. We then show that the Mortensen & Pissarides (1994) (hereafter MP) model including aggregate uncertainty and extended to introduce life-cycle feature is well suited for this challenge. Indeed, older workers' shorter horizon prior to retirement endogenously reduces their outside options and then leads their wage to be less sensitive to labor market fluctuations: given that the incentives to search are low for older workers, their threat point during an expansion does not improve in the wage bargaining, therefore preserving the incentive to post new vacancies. As a result, business cycle has large impact on unemployment, and worker flows for old workers. Intuitively, older workers act as static agents ("no future"). They do not smooth fluctuations. As usual, in a market where wage adjustments are small, quantities vary a lot: this the case for older workers, whereas prime-age workers have behaviors closed to the ones of an infinitively-lived agent. Our theoretical results point out that Shimer (2005)'s view on the MP model[2] is consistent with prime-age workers' labor market while aging endogenously introduces real wage rigidities, allowing to match what we observe for old workers, without specific assumptions as in Hagendorn & Manovskii (2008) or in Hall & Milgrom (2008). Our evidence on CPS real hourly wage is consistent with these features. In our view, Shimer (2005)'s criticism on the MP model actually constitutes a powerful opportunity for the model to match age-specific data. Indeed, Shimer (2005) actually describes what is observed on prime-age workers' labor market, with smaller quantity adjustments than the ones prevailing for older workers. But, if the "reservation wage effect" (search opportunity pushes wages upward) is relevant only for prime-age workers because these workers are far away from retirement, older workers, close to retirement (they do not search anymore), agree to moderate wage increases during booms, then older workers' flows are hit harder by the business cycle. The important point is that, unlike Hagendorn & Manovskii (2008), we preserve the Hosios condition and endogenously obtain varying changes in the search value in the wage dynamics through another channel than the one proposed by Hall & Milgrom (2008).

We show that there exists calibration restrictions allowing to match the "steady state" stylized facts. Under these restrictions, theory predicts the age-increasing pattern of the standard deviations of both cyclical job finding and separations rates, as it is observed in the data. These quantitative results underline the impact of work-life expectancy on labor market fluctuations by age.

Finally, we include young workers in this paper. First, this allows us to check the robustness of our statistical results: we find in US data that the volatilities of quantities (prices) estimated for the older worker are larger (lower) than the ones of the younger workers. Previous papers found that

[2]Shimer (2005) underlines that, following an expansion, the increase in outside opportunities puts an upward pressure on wages (wage reservation effect), which reduces firms' incentives to post new vacancies: the MP model generates a large adjustment in prices and small changes in quantities on the labor market, which is not consistent with the aggregate data. Shimer (2005)'s criticism deals with the consistency of the MP model with aggregate data, under the restriction of a representative infinitively-lived agent.

the cyclical volatility of employment and hours worked is U-shaped as a function of age (Gomme et al. (2005), Jaimovich & Siu (2009)). Using CPS data, we actually find an age-increasing volatility profile, using 3 age-groups (16-24, 25-54 and 55-61), thereby providing a robustness check to our stylized facts. If these data confirm that older workers are more sensitive to the business cycle than their younger counterparts, whether prime-age or young. The introduction of the young group adds another challenge to the theory: how can we account for the difference between younger and prime-age workers, as both age-groups are far from retirement (the horizon gap is negligible with respect to the duration of the job)? We then introduce two specificities on the youth labor market. First, given that approximately all men participate, those among them who have decided to opt for the labor market when they were still young are relatively more likely to fit the tasks requested by firms. Second, the cost of a young hiring in the labor market is a lack of information on the match between individual aptitude and that requested by firms (mismatch).[3] In the MP model, the first phenomena can be captured by a higher level of the average of the match-specific productivity for these people [4]. Concerning the mismatch, we introduce an age-specific probability to draw a new opportunity inside the firm: if there is more mismatch in the youth labor market segment, then more transitions from current job to a new job come after an unemployment episode, and less through a new opportunity inside the same firm.[5] Quantitative results at the business cycle frequencies show that these additional features explain a fraction of the gaps between the youth and the prime-age labor markets.

This paper bridges the gap between two strands of the literature: the one aiming at matching *levels* of employment rates across age groups without considering business cycle features (Ljungqvist & Sargent (2008), Cheron et al. (2013), Menzio et al. (2012), Gervais et al. (2012)) and the one looking at *fluctuations* in unemployment rates, without considering age heterogeneity (Fujita & Ramey (2012) among others). Unlike Cheron et al. (2013), we propose a directed search setting. With this assumption, under the Hosios condition, labor market fluctuations are efficient. We restrict our analysis to this efficient allocation in order to focus on the impact of age heterogeneity on unemployment fluctuations. [6] Moreover, this simplifying assumption allows us to solve the model more easily, in the spirit of the solution provided by Menzio & Shi (2010) (block recursive equilibrium).

The paper is organized as follows. Section 2 documents workers' transitions rates by age groups

[3]This is not the case for those who pursue their studies and test their ability to work in different areas.

[4]In the following, we refer to this assumption as the "composition effect" that accounts for the fact that participants are on average closer to employment than the aggregate population. In the quantitative analysis, we also introduce human capital accumulation over the life-cycle. Thus, this "composition effect" does not imply that the young workers' average total productivity is larger than for the other age groups.

[5]This simple assumption, which leads to reduce the labor hoarding value of younger workers, is in the same spirit as the more sophisticated model proposed by Gervais et al. (2012).

[6]Future research will be devoted to reducing the gap between the observed and predicted fluctuations by our first rank allocation. Moreover, given that the undirected search equilibrium is inefficient, the additional trade externality gives some foundations for age-specific labor market policies (See Cheron et al. (2011) and Cheron et al. (2013)).

and age patterns in their responsiveness to business cycles on US data. We consider only old versus prime-age workers. We also investigate wage fluctuations by age. In Section 3, we develop life cycle matching model with directed search, endogenous destruction, search effort and aggregate uncertainty uncertainty. All workers are the same except for their work-life expectancy. We first provide a steady state analysis (section 4) to uncover the conditions under which the model can match the historical averages of finding and separation rates across age groups based on heterogeneity in work-life expectancy only. In section 5, we provide an analytical characterization of the age specific impact of the business cycle on labor market outcomes in the model. Section 6 brings the model to the data. After calibrating the key parameters of the model to match age-specific levels of job finding and separation rates, and real hourly wages, we show that the model matches the age-increasing volatility in transition rates between prime-age and old workers as well as the age-decreasing volatility in real hourly wage. We bring the model closer to the data by adding youth unemployment (16-24) and an exogenous increase in human capital with age. The distance to retirement plays a key role in accounting for old workers' high responsiveness to the business cycle. In addition, in order to match the different volatilities between young and prime-age workers, we show that the "composition" and the "mismatch" effects explains a fraction of the gap between the youth and the prime-age labor market fluctuations.

2 Measuring fluctuations in unemployment, workers' transition rates and hourly wages by age in US data

The life cycle is divided into 2 age groups: 25-54 and 55-61. We chose the same age groups as in Elsby et al. (2010), except that we consider only individuals prior to retirement (as we do not consider retirement choices in the model). In addition, the choice of age groups is consistent with the evidence found in Choi et al. (2013) and Menzio et al. (2012) who assess age profiles of worker flows. They find sharp changes in job finding probabilities and separation for workers aged 55+. Finally, we restrict our attention to men, since female transitions are also linked to fertility and child rearing, that are not modeled in the paper.

We aim at analyzing the cyclical behavior of workers' transition between employment and unemployment by age. We compute the age profile of transitions from employment to unemployment (job separation) and transitions from unemployment to employment (job finding).

2.1 Measuring fluctuations in unemployment and workers' transition rates by age in US data

Using monthly CPS data, between January 1976 and March 2013, we follow all the steps described in Shimer (2012). We compute sample-weighted gross flows between employment states and seasonally adjust the time series using the same ratio-to-moving average technique as in Shimer (2012). We then correct for time aggregation in order to take into account transitions occurring within the month. We get time series for the instantaneous transition rates for each age group and consider quarterly averages to smooth out the noise. We then get quarterly data on workers' instantaneous transition rates (JSR_t and JFR_t) and the corresponding unemployment steady state $\left(u_t = \frac{JSR_t}{JSR_t+JFR_t}\right)$ between 1976Q1 and 2013Q1. The time series are displayed in figures 2 and 3.

Figure 2: Job Separation Rate by age group, JSR, CPS monthly data, Men, 1976 Q1 - 2013Q2. Means. Authors' calculations. Recession in shaded area.

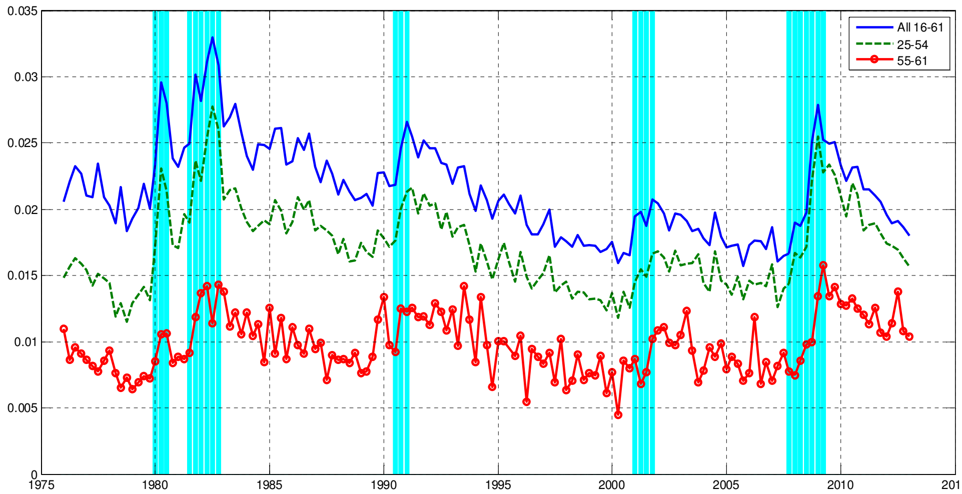
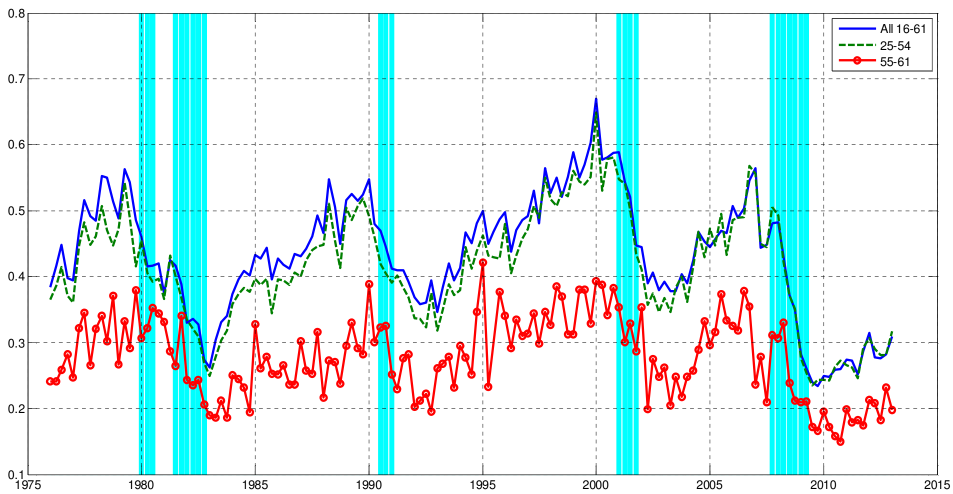


Figure 3: Job Finding Rate by age group, JFR, CPS monthly data, Men, 1976 Q1 - 2013Q2. Means. Authors' calculations. Recession in shaded area.



2.1.1 Mean: Old workers face slower exit from employment and longer unemployment spells

The mean of the time series is reported in Table 1. Like Elsby et al. (2010), we find large differences in levels of separation rates across age groups. Old workers who face low unemployment rates are characterized by low rates of entry into unemployment. The average job tenure when prime-age amounts to 4.9 years (58.8 months, $\frac{1}{1-e^{-0.017163}}$) versus 7.2 years when old (86.7 months, $\frac{1}{1-e^{-0.011608}}$). Differences in job finding rates are less striking but significant: the length of unemployment spell is 2.9 months ($\frac{1}{1-e^{-0.41274}}$) for prime-age workers versus 3.5 months ($\frac{1}{1-e^{-0.3362}}$) for old workers. The levels of inflow and outflow rates of unemployment fall with age. As stressed by Elsby et al. (2011) on UK data, with slower exit from employment and longer unemployment spell, old workers face a less fluid labor market than their younger counterparts. This slower exit from employment and unemployment is missed when one simply looks at unemployment rates across age groups. In addition, the differences in job finding rates would actually predict an age increasing profile for unemployment. As a result, Table 1 suggests that the low level of unemployment rate for old workers is actually driven by their low level of exit from employment. This is consistent with Elsby et al. (2011) and Gervais et al. (2012).

Estimated means are consistent with the decreasing transitions with age found in the male population in Choi et al. (2013), Menzio et al. (2012) and Gervais et al. (2012). The level of our transition rates is larger in our data than in their calculations because we discard employment states that are considered in previous papers (namely inactivity for Choi et al. (2013) and job-to-job transitions for Menzio et al. (2012)). In addition, Menzio et al. (2012) restrict their sample to individuals with a high school degree. [7] Elsby et al. (2010) also compute unemployment outflows and inflows, using employment and unemployment stocks in which separations are captured by the flows of workers who report having been unemployed for less than one month (as in Shimer (2012)). The BLS provides time-series for short-term unemployment by age and sex. This yields higher transition rates than those obtained from disaggregated data. However, changes along the business cycle are consistent with the ones they report.

2.1.2 Business cycle : Old workers face more cyclical job flows

The cyclical behavior transition rates are characterized in recession by a rise in separations (figure 2) and a significant drop in the job finding rate (figure 3). It is striking that, in recession, the fall in the outflow rate seems of the same magnitude across age groups. For instance, in the 2001 recession, the job finding rate fell by 0.2 for all age groups (from 0.6 to 0.4 for prime age and from 0.5 to

[7] Steady state unemployment is consistent with the BLS unemployment rate across age groups (0.051714 for 25-54,0.041289 for 55+ and 0.064696 for 16+. Source: BLS monthly SA data, 1976 Jan-2013 March, Men). In appendix A.9, we also check that steady state unemployment rate by age is consistent with BLS time series

Table 1: Mean. CPS monthly data, Men, quarterly averages. 1976Q1 - 2013Q1. Authors' calculations.

		All: 16-61	Prime-age: 25-54	Old: 55-61
JSR	Job Separation Rate	0.020766	0.017163	0.011608
			1	0.68 [a]
JFR	Job Finding Rate	0.42995	0.41274	0.3362
			1	0.81
u	Unemployment rate	0.048914	0.042691	0.036589
			1	0.86

[a] Old workers' JSR is 0.68% prime-age workers' JSR.

nearly 0.3 for old workers). However, the magnitude of this fall is very different across age groups when scaled it by the average job finding rate of each age group. Since old (prime age) workers are characterized by lower (higher) levels of job finding rates, the fall in unemployment outflows in the 2001 recession is large (small) for this age group. This high (low) responsiveness of old (prime age) workers, relative to their younger (older) counterparts, is the key business cycle fact that we want to investigate.

In order to do so, we identify the cyclical component of each time series using the Hodrick Prescott filter, with smoothing parameter 10^5 on monthly logged data. Detrended data are reported in figures 4 and 5 in Appendix A.1. The figures confirm the contrast in the responsiveness of old and young workers' transition rates to the business cycle. Old workers transition rates display larger fluctuations than the ones observed for prime age individuals, with sharper changes in transition rates in periods of expansion or economic downturn.

Business cycle facts on the de-trended data are reported in Table 2. We indeed find that the volatility of transitions rates increases with age. The increase in volatility in the job finding rate is striking from prime - aged to old individuals (the volatility goes up from 0.17 to 0.22). This is the key business cycle feature that we want to understand using our theoretical model.

2.1.3 Robustness

We show in appendix A that this salient feature is robust when we consider an alternative smoothing parameter for the HP filter, inactivity (Appendix A.3), transition for men and women (Appendix A.4) or Elsby et al. (2010) data (Appendix A.5). In addition, in appendix A.6, we compute rolling 10-year standard deviation of de-trended data and show the evolution of changes in relative volatilities. Old workers' job flows display more cyclical behavior. In Appendix A.7, in the spirit of Jaimovich & Siu (2009), we evaluate the impact of changing age decomposition on our aggregate results. The high sensitivity of old workers' transition rates remains a robust feature. Finally, we check that our

Table 2: Standard deviation. CPS Monthly data, Men, 1976 June - 2012 Sept, HP filter with smoothing parameter 10^5. Authors' calculations.

		All: 16-61	Prime-age: 25-54	Old: 55-61
JSR	Job Separation Rate	0.11025	0.14385	0.20151
			1	1.4 [a]
JFR	Job Finding Rate	0.16434	0.17025	0.22407
			1	1.3161
u	Unemployment rate	0.23997	0.2742	0.32582
			1	1.1883

[a] Old workers' HP filtered-JSR standard deviation is 1.4 times higher than prime-age workers'.

age effect is not a composition due, e.g. to a higher proportion of low education in the population of older workers. We show in Appendix A.8 that the level and the volatilities have the same age profiles within each sub-group, "High school degree and less" and "More than High school". Our age effect is not a skill effect.

2.2 Real hourly wage

Using monthly CPS data[8], between January 1979 and March 2013, we document the business cycle response of male real hourly wage across age groups. [9] Hourly wage w is usual weekly earnings divided by usual weekly hours, for men only. We then deal with outliers, deflate by inflation and technology growth, seasonally adjust then take quarterly averages (see Appendix B for further details).

Table 3 reports the descriptive statistics for male real hourly wage and figure 6 in Appendix A.1 plots the corresponding time series. The levels are age-increasing, which is consistent with the view that experience makes workers more productive. Interestingly, wage cyclical volatility actually falls with age.

In a nutshell, the empirical evidence seems to point at different patterns of cyclicality across age groups : old worker's job flows are characterized by larger volatility than their younger counterparts and lower wage responsiveness to the business cycle. Our findings on the volatilities per age of the

[8]This is still work in progress. As a result, we have only wage from 1995 January onwards. We still need to add wages from January 1979 to January 1995. Data on earnings are available in the Basic CPS from January 1979 onwards.

[9]Jaimovich et al. (2013) use the annual March Supplement to look at wage cyclcality. However, among the 5 recessions that occurred in our sample (1976Q1-2013Q1), only 2 of them (1981 and 2008) lasted more than 1 year. It is then paramount to document fluctuations in wage using infra-annual data. Monthly CPS provide information on earnings from only one-fourth of the CPS total sample of approximately 60,000 households. We check that our levels of wage are consistent with Heathcote et al. (2010) and BLS weekly earnings by age and that the HP filtered volatility is consistent with Jaimovich et al. (2013)'s.

Table 3: Real hourly wage w. CPS monthly data, quarterly averages, male, 1995Q1-2013Q1. Authors' calculations.

	All: 16-61	Prime-age: 25-54	Old: 55-61
Mean(w)	16.3672	17.68812	19.41792
		1	1.1 [a]
Std(w)	0.017	0.019	0.018
		1	0.93

[a] Old workers' w 1.1 times higher than prime-age workers'.

hourly wage can view as consistent to our finding on worker flows and unemployment stock. Indeed, in a market with more rigid prices, the large part of the adjustments are done by the quantities. Nevertheless, on cannot consider real wage rigidity as an acceptable assumption : there is a decline in the wage volatility with the worker age, but the lowest volatility is still larger than zero.

3 A life-cycle matching model with aggregate uncertainty

In this section, we present an extension of the Mortensen & Pissarides (1994) model that introduces life-cycle features and aggregate shocks.[10] Beyond the life-cycle pattern of the job flows and stocks, our objective is to evaluate the impact of age heterogeneity on business cycle elasticities. Thus, in our economy à la MP, there is stochastic aging, as in Castañeda et al. (2003), Ljungqvist & Sargent (2008) and Hairault et al. (2010). Unlike Cheron et al. (2013), we have aggregate shocks, as in e.g. Fujita & Ramey (2012). In addition, unlike Cheron et al. (2013), we have age-directed search as in Menzio et al. (2012). This implies that there is no externality coming form heterogenous workers in the matching function. The decentralized allocation is then efficient if the Hosios condition is satisfied.[11]

In sections 3 to 5, all workers are the same except for their work-life expectancy. In our view, this is the key mechanism that drives the observed differences in job flows cyclicality between old workers and their younger counterparts, prime-age workers. In order to provide a good theory for the different cyclicality between prime-age and young workers, we need additional features that will be presented in section 6.

[10]Since Mortensen & Pissarides (1994) or Mortensen (1994), there have been a lot of papers that provide empirical evaluation of the MP model with respect to its ability to explain the labor market fluctuations. This model has been also tested in the context of Dynamic Stochastic General Equilibrium models: see Merz (1994), Langot (1995) or Andolfatto (1996) with exogenous separations, Denhaan et al. (2000) with endogenous separations. Pissarides (2009)'s paper have revived this literature.

[11]Job-to-job transitions without saving (as in Fujita & Ramey (2012)) or with savings (as in Lise (2013)) are discarded. More generally, the incorporation of the matching frictions within a Bewley-Huggett-Aiyagari type model, as it is done by **?** in the case of a infinitely lived agent, is left for future research.

3.1 Demographic setting, shocks and matching

The Life Cycle. Unlike the large literature following MP, we consider a life-cycle setting characterized by an age at which workers exit the labor market, interpreted as the exogenous retirement age. There are 2 age groups i: prime age and old workers, respectively A and O. All prime-age workers A enter the labor market as unemployed workers. We assume stochastic aging. The probability of remaining a prime age worker in the next period is (π_A. Conversely, the probability of becoming old is $1 - \pi_A$. We divide the periods as old workers into 7 years, one for each year: $O = \{O_i\}_{i=1}^{7}$, with a probability of remaining in age class O_i in the next period is π_{O_i}. With probability $1 - \pi_{O7}$, old workers reach the retirement age T: in order to have a constant population size, we assume that this mass of exiting workers is replaced by an equal mass of prime-age workers.

We need weekly data[12], but only unfrequent transitions are significant with respect to the long run life-cycle features: thus the number of value functions is given by the number of age classes in the labor market, whereas the step of the each value function is the month.

With $\pi_i \neq \pi_j$, for $i, j = 1, ..., 8$ and $i \neq j$, the size of age groups are not equal. The size of each group is deduced from Π, given that the total size of the population is normalized to unity. The population of each group can be divided into two types of agent: the unemployed u_i and the employed n_i, such that $m_i = u_i + n_i$, with $1 = \sum_i m_i$. We thereby discard the participation margin. In our view, this is not a very restrictive assumption because we introduce an age-specific search effort which can converge towards zero at the end of the working life, before the retirement age. These old unemployed workers can thus be considered as non-participants. Their number is endogenously determined at the equilibrium.

Shocks. A worker-firm match can produce an output level of $z\epsilon$ during month t, where z and ϵ are respectively the aggregate and the match-specific productivity factor. The aggregate productivity component follows the exogenous process:

$$\log(z') = \rho \log(z) + \nu' \tag{1}$$

where ν' is an i.i.d. normal disturbance with mean zero and standard deviation σ_ν.

Firms are small and each has one job. For a common aggregate component of the productivity z, the destruction flows derive from idiosyncratic productivity shocks that hit jobs at random. At the end of each month t, a new productivity level for month $t + 1$ is drawn with probability $\lambda_i \leq 1$ in the distribution $G(\epsilon)$, with $\epsilon \in [0, 1]$. The higher λ_i, the lower the persistence of the current productivity draw. The probability to draw a new match-specific productivity depends on workers' age. This assumption allows to account for the heterogeneous persistence across age groups: when persistence is high, it is more difficult to improve match-specific productivity.

[12]This is needed to capture the high frequencies transitions on the labor market, even if we aggregate these simulated data to compute stylized facts on quarterly averages of monthly data.

Once a shock occurs, the firm either keeps on producing or destroys the job. Each week, employed workers are faced with layoffs when their job becomes unprofitable. The firms decide to close down any job whose productivity is below a productivity threshold (the reservation productivity) denoted $R_i(z)$.

Finally, unlike MP, new jobs are not opened at the highest productivity: their productivity level is also drawn in the distribution $G(\epsilon)$. The age-$i-1$ workers who become age-i workers (with probability $1-\pi_{i-1}$), whereas they have been contacted at the age $i-1$, will be hired if and only if their productivity is larger than the threshold $R_i(z)$, ie. the reservation productivity of an age-i worker, because the productivity value is revealed after the firm has met the worker.

Matching with directed search. We consider an economy where labor market frictions imply that there is a costly delay in the process of filling vacancies. We assume that age is perfectly observed and that a worker who applies to a job not matching its age-characteristic, will have a nil production, and thus a nil surplus. Firms choose how many and what type of vacancies to open, where the type of a vacancy is simply defined by worker's age. Since search is directed, the probability for a worker to meet a firm depends on his age.

Since firms can *ex-ante* age-direct their search, there is one matching function by age. Let $v_i(z)$ be the number of vacancies, $u_i(z)$ the number of unemployed workers, and $e_i(z)$ the endogenous search effort for a worker of age i. The matching function gives the number of contact, $M(v_i(z), e_i(z)u_i(z))$, where M is increasing and concave in both its arguments, and with constant returns-to-scale. From the perspective of a firm, the contact probability is $q(\theta_i(z)) \equiv \frac{M(v_i(z),e_i(z)u_i(z))}{v_i(z)} = M(1, \theta_i^{-1}(z))$ with $\theta_i(z) = \frac{v_i(z)}{e_i(z)u_i(z)}$ the corresponding labor market tightness. The probability for unemployed workers of age i of being employed is then defined by $e_i(z)p(\theta_i(z))[1-G(R_i(z))]$ with $p(\theta_i(z)) \equiv \frac{M(v_i(z),e_i(z)u_i(z))}{e_i(z)u_i(z)} = M(\theta_i(z), 1)$ the contact probability of the effective unemployed worker. Note that the hiring process is then age-differentiated via an age-specific search intensity ($v_i(z)$), an age-specific reservation productivity ($R_i(z)$) and an age-specific search effort ($e_i(z)$).

3.2 Firms' and workers' inter-temporal values

Firms' problem. Any firm is free to open a job vacancy directed to an age-specific labor market and engage in hiring. c denotes the flow cost of hiring a worker and $\beta \in [0, 1]$ the discount factor. Let $V_i(z)$ be the expected value of a vacant position in age-i labor market, given the aggregate state of the economy z at time t, and $J_i(z, \epsilon)$ the value of a job filled by a worker of age i with productivity ϵ in aggregate state z. The firm's search value is given by:

$$V_i(z) = -c + q(\theta_i(z))\beta E_z\left[\pi_i \int_0^1 J_i(z', x)dG(x) + (1-\pi_i)V_i(z')\right] + (1-q(\theta_i(z)))\beta E_z V_i(z')$$

where the operator E_z denote the expectation with respect to the aggregate productivity component z. Given that search is directed, if the worker ages between the meeting and the production processes (with a probability $1-\pi_i$), the job is not filled. We will assume hereafter the standard free-entry condition, ie. $V_i(z)=0$, $\forall i, z$, which leads to:

$$\frac{c}{q(\theta_i(z))} = \beta\pi_i E_z \int_0^1 J_i(z', x)dG(x)$$

Vacancies are determined according to the expected value of a contact with an age-i unemployed worker, which depends on the uncertainty in the hiring process arising from the two components of productivity z and ϵ.

Given a state vector (z,ϵ) and for a bargained wage $w_i(z,\epsilon)$, the expected value $J_i(z,\epsilon)$ of a filled job by a worker of age i, $\forall i \in [Y, ..., O_6]$, is defined by:

$$J_i(z,\epsilon) = \max\left\{ \begin{array}{l} z\epsilon - w_i(z,\epsilon) \\ +\beta\pi_i(1-s_e)\left(\begin{array}{c} \lambda_i E_z \int_0^1 J_i(z',x)dG(x) \\ +(1-\lambda_i)E_z J_i(z',\epsilon) \end{array} \right) \\ +\beta(1-\pi_i)(1-s_e)\left(\begin{array}{c} \lambda_{i+1} E_z \int_0^1 J_{i+1}(z',x)dG(x) \\ +(1-\lambda_{i+1})E_z J_{i+1}(z',\epsilon) \end{array} \right) \end{array} ; 0 \right\}$$

Notice that, for $i = O_7$, aging implies retirement. The value function becomes:

$$J_{O_7}(z,\epsilon) = \max\left\{ \begin{array}{l} z\epsilon - w_{O_7}(z,\epsilon) \\ +\beta\pi_{O_7}(1-s_e)\left(\lambda_{O_7} E_z \int_0^1 J_{O_7}(z',x)dG(x) + (1-\lambda_{O_7})E_z J_{O_7}(z',\epsilon)\right) \end{array} ; 0 \right\}$$

The short horizon reduces the value of a filled job, for a given wage. When the horizon is short, if the wage falls, older workers' value can still be positive such that it can be profitable for firms to hire old workers.

Workers' problem. Values of employed (on a match of productivity ϵ) and unemployed workers of any age $i \neq O_7$, are respectively given by:

$$\mathcal{W}_i(z,\epsilon) = \max\left\{ \begin{array}{l} w_i(z,\epsilon) \\ +\beta\pi_i \left[\begin{array}{l} (1-s_e)\left(\begin{array}{c} \lambda_i E_z \int_0^1 \mathcal{W}_i(z',x)dG(x) \\ +(1-\lambda_i)E_z \mathcal{W}_i(z',\epsilon) \end{array} \right) \\ +s_e E_z \mathcal{U}_i(z') \end{array} \right] \\ +\beta(1-\pi_i) \left[\begin{array}{l} (1-s_e)\left(\begin{array}{c} \lambda_{i+1} E_z \int_0^1 \mathcal{W}_{i+1}(z',x)dG(x) \\ +(1-\lambda_{i+1})E_z \mathcal{W}_{i+1}(z',\epsilon) \end{array} \right) \\ +s_e E_z \mathcal{U}_{i+1}(z') \end{array} \right] \end{array} ; \mathcal{U}_i(z) \right\}$$

$$\mathcal{U}_i(z) = \max_{e_i(z)} \left\{ \begin{array}{l} b - \phi(e_i(z)) \\ +\beta\pi_i \left(\begin{array}{l} e_i(z)p(\theta_i(z))E_z \int_0^1 \mathcal{W}_i(z',x)dG(x) \\ +(1-e_i(z)p(\theta_i(z)))E_z\mathcal{U}_i(z') \end{array} \right) \\ +\beta(1-\pi_i) \left(\begin{array}{l} e_{i+1}(z)p(\theta_{i+1}(z))E_z \int_0^1 \mathcal{W}_{i+1}(z',x)dG(x) \\ +(1-e_{i+1}(z)p(\theta_{i+1}(z)))E_z\mathcal{U}_{i+1}(z') \end{array} \right) \end{array} \right\}$$

with $b \geq 0$ denoting the instantaneous opportunity cost of employment and $\phi(.)$ the convex function capturing the disutility of search effort e_i. For $i = O_7$, these values are simply given by

$$\mathcal{W}_{O_7}(z,\epsilon) = \max \left\{ \begin{array}{l} w_{O_7}(z,\epsilon) \\ +\beta\pi_{O_7} \left[\begin{array}{l} (1-s_e) \left(\begin{array}{l} \lambda_{O_7} E_z \int_0^1 \mathcal{W}_{O_7}(z',x)dG(x) \\ +(1-\lambda_{O_7})E_z\mathcal{W}_{O_7}(z',\epsilon) \end{array} \right) \\ +s_e E_z \mathcal{U}_{O_7}(z') \end{array} \right] \end{array} ; \mathcal{U}_{O_7}(z) \right\}$$

$$\mathcal{U}_{O_7}(z) = \max_{e_{O_7}(z)} \left\{ \begin{array}{l} b - \phi(e_{O_7}(z)) \\ +\beta\pi_{O_7} \left(\begin{array}{l} e_{O_7}(z)p(\theta_{O_7}(z))E_z \int_0^1 \mathcal{W}_{O_7}(z',x)dG(x) \\ +(1-e_{O_7}(z)p(\theta_{O_7}(z)))E_z\mathcal{U}_{O_7}(z') \end{array} \right) \end{array} \right\}$$

The optimal search effort decision of the worker then satisfies the following condition:

$$\phi'(e_i(z)) = \beta\pi_i p(\theta_i(z))E_z \left[\int_0^1 \mathcal{W}_i(z',x)dG(x) - \mathcal{U}_i(z') \right]$$

The marginal cost of search effort at age i is equalized to its expected marginal return.

3.3 Job surplus, Nash sharing rule and reservation productivity

The surplus $S_i(z,\epsilon)$ generated by a job of productivity $z\epsilon$ is the sum of the worker's and the firm's surplus: $S_i(z,\epsilon) \equiv \mathcal{W}_i(z,\epsilon) - \mathcal{U}_i(z) + J_i(z,\epsilon)$, given that $V_i(z) = 0$ at the equilibrium. Thus, using the definitions of $J_i(z,\epsilon)$, $\mathcal{W}_i(z,\epsilon)$, and $\mathcal{U}_i(z)$, the surplus is given by:

$$S_i(z,\epsilon) = \max \left\{ \begin{array}{l} z\epsilon - b + \phi(e_i(z)) \\ +\beta\pi_i(1-s_e) \left(\begin{array}{l} \left[\lambda_i - \frac{\gamma e_i(z)p(\theta_i(z))}{1-s_e}\right] E_z \int_0^1 S_i(z',x)dG(x) \\ +(1-\lambda_i)E_z S_i(z',\epsilon) \end{array} \right) \\ +\beta(1-\pi_i)(1-s_e) \left(\begin{array}{l} \left[\lambda_{i+1} - \frac{\gamma e_{i+1}(z)p(\theta_{i+1}(z))}{1-s_e}\right] E_z \int_0^1 S_{i+1}(z',x)dG(x) \\ +(1-\lambda_{i+1})E_z S_{i+1}(z',\epsilon) \end{array} \right) \end{array} ; 0 \right\}$$

The reservation productivity $R_i(z)$ can then be defined by the condition $S_i(z, R_i(z)) = 0$. As in MP, a crucial implication of this rule is that the job destruction is mutually optimal, for the firm and the worker. $S_i(z, R_i(z)) = 0$ indeed entails $J_i(z, R_i(z)) = 0$ and $\mathcal{W}_i(z, R_i(z)) = \mathcal{U}_i(z)$. Note that the lower bound of any integral over $S_i(z,\epsilon)$ is actually the reservation productivity, as no productivity levels below the reservation productivity yield a positive job surplus. Given $S_i(z,\epsilon)$, the Nash bargaining leads to

$$\mathcal{W}_i(z,\epsilon) - \mathcal{U}_i(z) = \gamma S_i(z,\epsilon) \quad \text{and} \quad J_i(z,\epsilon) = (1-\gamma) S_i(z,\epsilon)$$

Using this sharing and the definitions of the value functions, the wage rule is

$$w_i(z,\epsilon) = \gamma \left(z\epsilon + c e_i(z)\theta_i(z) + \frac{1-\pi_i}{\pi_{i+1}} c e_{i+1}(z)\theta_{i+1}(z) \right) + (1-\gamma)\left(b - \phi(e_i(z))\right) \tag{2}$$

Because workers age, the returns on search activity is an average between age i and age $i+1$. In addition, when agents are identical except for their work-life expectancy, prime-age worker's wage is higher than old workers' because the search efforts, $\theta_i(z)$ and $e_i(z)$, are age-decreasing. This is counterfactual. To solve this problem, we introduce in section 6 an exogenous increase in human capital with age. More precisely, we assume that $z_i = zh_i$ and $b_i = bh_i$, where h_i is the exogenous human capital of the agent at the age i. We then show that distance to retirement still plays a key role in shaping age-specific cyclicality even when the model takes into account this age-increasing human capital.

3.4 Equilibrium

Definition 1. *The labor market equilibrium with directed search in a finite-horizon environment is defined by the search efforts of vacant jobs and unemployed workers, respectively $\theta_i(z)$ and $e_i(z)$, and the separation rule (the reservation productivity), $R_i(z)$:*

$$\frac{c}{q(\theta_i(z))} = (1-\gamma)\beta\pi_i E_z \overline{S}_i(z') \tag{3}$$

$$\phi'(e_i(z)) = \gamma p(\theta_i(z))\beta\pi_i E_z \overline{S}_i(z') \tag{4}$$

$$\begin{aligned} zR_i(z) = {} & b - \phi(e_i(z)) \\ & -\beta\pi_i(1-s_e)\begin{pmatrix} \left[\lambda_i - \frac{\gamma e_i(z)p(\theta_i(z))}{1-s_e}\right] E_z\overline{S}_i(z') \\ +(1-\lambda_i)E_z S_i(z', R_i(z)) \end{pmatrix} \\ & -\beta(1-\pi_i)(1-s_e)\begin{pmatrix} \left[\lambda_{i+1} - \frac{\gamma e_{i+1}(z)p(\theta_{i+1}(z))}{1-s_e}\right] E_z\overline{S}_{i+1}(z') \\ +(1-\lambda_{i+1})E_z S_{i+1}(z', R_i(z)) \end{pmatrix} \end{aligned} \tag{5}$$

given the average and individual surpluses :

$$\overline{S}_i(z') \equiv \int_{R_i(z')}^{1} S_i(z',x)dG(x) \tag{6}$$

$$S_i(z,\epsilon) = \max\left\{ \begin{matrix} z(\epsilon - R_i(z)) + \beta\pi_i(1-\lambda)(1-s_e)E_z[S_i(z',\epsilon) - S_i(z',R_i(z))] \\ +\beta(1-\pi_i)(1-\lambda)(1-s_e)E_z[S_{i+1}(z',\epsilon) - S_{i+1}(z',R_i(z))] \end{matrix} ; 0 \right\} \tag{7}$$

The stock-flow dynamics on the labor market are given in Appendix C.1 by equations (25), (26), (27) and (28), whereas the dynamics of the aggregate shock is given by (1).

Proposition 1. *The directed search assumption implies that the problem is block-recursive.*

Proof. As in Menzio & Shi (2010), if we find a fix point for $S_{O_7}(z, \epsilon)$ which is a function of choices at the age O_7 (the terminal age) only, we then obtain $\{S_{O_7}(z, \epsilon), \theta_{O_7}(z), R_{O_7}(z), e_{O_7}(z)\}$, $\forall z, \epsilon$, using the equations (3), (5) and (4). Given these solutions for the labor market of the age-O_7 workers, we can solve for the age-O_6 workers using the system given in definition 1 until age $i = A$. □

The algorithm for this first step is the same as the one described in Fujita & Ramey (2012) for the solution of the MP model in the infinite horizon case. It is extended to account for the endogenous search effort from unemployed workers. See Appendix C.5 for a description of the algorithm.

Proposition 2. *If the Hosios condition is satisfied, the equilibrium is efficient.*

Proof. By backward induction, it is trivial to show that, if $\gamma = \eta$, then the labor market allocation for older workers is efficient. Given this result, the same conclusion applies on prime-age workers' labor market. □

In this paper, we assume that this restriction is satisfied. This simplifying assumption, in the spirit of Menzio & Shi (2010), is also used in Menzio et al. (2012). It then allows us to measure the gap between optimal fluctuations and the observed business cycle. Cheron et al. (2011) discuss the impact of a non-directed search equilibrium per age. They show that non-directed search generates additional trade externality giving some foundations to age-specific labor market policies: because older workers create a negative externality in the hiring process, it could be optimal to retain them inside the firms or to subsidy their non-participation.

4 Steady state analysis: Accounting for the age-decreasing levels of transition rates

In this section, we show that the model replicates the age-decreasing levels of transition rates found on the data (Table 1). We then consider the steady state of our economy, and stress the key mechanisms that allow to match the pattern of labor reallocation of prime-age and old workers. This steady state analysis is a first step for the validation of our theory: the model is required to match the transition rates measured as the average over all the sample. This will also simplify the exposition of economic mechanisms when we consider the stochastic economy (section 5).

Transition rates fall with age in the data, meaning that, at the steady state, for age i, the model must generate an age-pattern of transition rates such that

$$JSR_i \approx s_e + (1 - s_e)\lambda_i G(R_i) > JSR_{i+1} \quad (8)$$

$$JFR_i \approx e_i p(\theta_i)[1 - G(R_i)] > JFR_{i+1} \quad (9)$$

If we only focus on endogenous variables[13], the job separation rate is only a function of the reservation productivity (R_i), whereas the job finding rate is a function of search efforts ($\{e_i; \theta_i\}$ and the reservation productivity. This observed pattern is puzzling since, in the model, the value of a match is determined by a single variable, its surplus. Using this single variable, the model must produce an equilibrium outcome in which, it is optimal to have old workers kept inside the firm while, at the same time, firms are less willing to hire them.

4.1 Steady state equilibrium by age

Since Mortensen & Pissarides (1994)'s seminal paper, the information in the surplus (equation (7) in definition 1) can be decomposed in two parts: *(i)* the expected average value of the surplus gives the incentive to search ($\overline{S}_i$ in equations (3) and (4)), *(ii)* whereas its marginal value provides information on the separation decision (equation (5)). Given that $e_i(z)$ can be easily deduced from $\theta_i(z)$ (equations (4) and (3)), we only focus on the labor market tightness and reservation productivity. At the conditional steady state, ie. a steady state indexed by a permanent level of z, we have, omitting z to simplify the notations:

$$\frac{c}{q(\theta_i)} = (1-\gamma)\beta\pi_i\overline{S}_i \tag{10}$$

$$R_i = b + \Sigma_i - \Lambda_i - \Gamma_i(R_i) \tag{11}$$

where Σ_i, Λ_i and $\Gamma_i(R_i)$ denote respectively the value of search opportunities $\Sigma_i = \pi_i\gamma e_i p(\theta_i)\beta\overline{S}_i + (1-\pi_i)\gamma e_{i+1}p(\theta_{i+1})\beta\overline{S}_{i+1} - \phi(e_i)$, labor hoarding $\Lambda_i = \pi_i(1-s_e)\lambda_i\beta\overline{S}_i + (1-\pi_i)(1-s_e)\lambda_{i+1}\beta\overline{S}_{i+1}$, and the continuation value $\Gamma_i(R_i) = (1-s_e)(1-\lambda_i)(1-\pi_i)\beta S_{i+1}(R_i)$. Given that $\overline{S}_i$ is a function of R_i and Σ_i a function of θ_i, equation (10) represents the job creation curve (JC), whereas (11) is the job destruction curve (JD). At each age i, and thus for a given set of variable representing the equilibrium at age $i+1$, JC defines a negative relationship between θ_i and R_i: the firm's investments in search are large when a wide set of jobs is profitable (low value of R_i). JD defines a positive relationship between θ_i and R_i: the set of profitable jobs is reduced when the search process allows to be more selective (high value of θ_i).

We will show in the following sections that there exists an equilibrium such that reservation productivity falls with age (section 4.2) as well as labor market tightness hence search effort (section 4.3). Both elements play a role in generating an age-decreasing profile of transition rates.

4.2 The steady-state life-cycle pattern of reservation productivity

Equation (8) suggests that the age-decreasing job separation rate can be obtained as soon as the reservation productivity falls with age. The reservation productivity (equation (11)) is the sum of

[13] The job separation rate is also a function of the age-specific probability to draw an new match-specific productivity (λ_i).

unemployed worker's current surplus (unemployment benefit and home production, b) and the net value of new opportunities. The latter consists of the difference between the return on opportunities outside the firm (Σ_i) and within the firm at the current productivity level ($\Gamma_i(R_i)$) or after a change in productivity (Λ_i). The reservation productivity differs across age groups because workers differ with respect to their expected time on the labor market. For workers who are close to retirement, only current surplus matters. The reservation productivity converges to the unemployed worker's current surplus b as the worker ages. In contrast, prime-age workers have a long work-life expectancy on the labor market. We have basically two cases. In the first, the value of the labor hoarding Λ_i is larger than the search returns Σ_i. In this case, $R_i < b$ and the reservation productivity can only be age-increasing, which is not in accordance with the data of the job separation rates. We then exclude this case. In the other case, we have $\Sigma_i > \Lambda_i$, leading to $R_i > b$: a age-decreasing pattern of the job separation is then possible.

Thus, reservation productivity for prime age workers is larger than that of their older counterparts ($R_i > R_{i+1}$) as long as the expected return on their search activity is larger than the expected gain from opportunities within the firm ($\Sigma_i > \Lambda_i + \Gamma_i(R_i)$). If we consider the marginal job, we have $\Gamma_i(R_i) \to 0$. Thus, a sufficient condition for $R_i > R_{i+1}$ is $\Sigma_i > \Lambda_i$ or $\gamma e_i p(\theta_i) > (1 - s_e)\lambda_i$, $\forall i$. [14] This gap between $\gamma e_i p(\theta_i)$ and λ_i simply measures the efficiency of unemployment search relative to labor hoarding. This condition underlines that the value of search can be manipulated by agents through their choices on $\{e_i, \theta_i\}$ whereas the labor hoarding value is driven by an exogenous process λ_i: the younger they are, the larger the incentive to invest in the labor market because a longer horizon allows them to recoup more easily their search costs. This age-dynamic of reservation productivity is consistent with the evidence found in the US data reported in Table 1. [15]

4.3 Hirings and search efforts along the life-cycle

According to Equation (9), the decline in the reservation productivity tends to increase the job finding rate along the life cycle, since more jobs become acceptable. This is not consistent with the data (Table 1). For the job finding rate to decline with age, the model must predict a large decline in search efforts ($\{e_i; \theta_i\}$) and thus a large decline in the meeting rate (through the contact probability of the unemployed worker ($e_i p(\theta_i)$)) at the end of the working life. The fall in e_i and θ_i

[14] The intuition beyond this restriction is the following: at the end of the life-cycle, we have $R_S \to b$. If $\Sigma_A > \Lambda_A \Rightarrow \Sigma_A - \Lambda_A > 0$, then $R_A > R_S$. In order to have $R_Y > R_A$, we also need $\Sigma_Y - \Lambda_Y > \Sigma_A - \Lambda_A > 0$. This can be an equilibrium result if the horizon effect is large enough and/or if the labor hoarding value is low for younger workers. It is indeed the case because λ_Y is small: younger workers' opportunities to change productivity inside the firm is lower then the one prevailing for their older counterparts. See the discussions below concerning these two points.

[15] In Cheron et al. (2013), all the other cases are analyzed: the age-increasing reservation productivity case, implying $\Sigma_i < \Lambda_i \ \forall i$, and the U-shape pattern of the reservation productivity, implying $\Sigma_i > \Lambda_i$, for $i < \widehat{i}$ and $\Sigma_i < \Lambda_i$, for $i \geq \widehat{i}$. Given that the US data are not in accordance with these two last cases, we restrict our analysis to case where the steady state of the model match the long run value of the JSR.

must compensate the decline in the reservation productivity. Recall that, for age group i, $\{e_i, \theta_i\}$ are positively linked to the expected surplus $\overline{S}_i$, which is given by equation (6). We determine below the age profile of the expected surplus from a match.

If the reservation productivity is age-decreasing, implying an age-decreasing separation rates as in the data, then we have $S_i(\epsilon) = \Omega_i(\epsilon - R_i)$ where Ω_i is a polynomial which accounts for actualization, leading to $\Omega_i > \Omega_{i+1}$. In this case, we have $\overline{S}_i = \int_{R_i}^1 S_i(x)dG(x) = \Omega_i \int_{R_i}^1 (x - R_i)dG(x) = \Omega_i \int_{R_i}^1 [1 - G(x)]dx$. Thus we have [16]:

$$\overline{S}_i - \overline{S}_{i+1} = (\Omega_i - \Omega_{i+1}) \int_{R_i}^1 [1 - G(x)]dx - \Omega_{i+1} \int_{R_{i+1}}^{R_i} [1 - G(x)]dx$$

where *(i)* the first term accounts for the "horizon effect", leading the expected surplus to be age-decreasing ($\overline{S}_i > \overline{S}_{i+1}$) because, for a given draw of match-specific productivity ϵ, a shorter horizon prior to retirement leads each match-specific surplus to decline with the worker's age ($S_i(\epsilon) > S_{i+1}(\epsilon) \Leftrightarrow \Omega_i - \Omega_{i+1} > 0$), and *(ii)* the second term captures a "selection effect" leading surplus to be age-increasing ($\overline{S}_i < \overline{S}_{i+1}$) because, when $R_i > R_{i+1}$, older workers are less selective than younger workers when new opportunities are available for them. If the number of new jobs that become profitable after aging, relative to the number of jobs that are already profitable at younger ages, is small enough, the *expected* surplus can be age-decreasing. In this case, where "the horizon effect" dominates the "selection effect", the expected surplus declines with age, leading search efforts (e_i and θ_i) to be age-decreasing. [17]

4.4 The wage age-profile

In our MP-life-cycle model, there is heterogeneity within each age group because each job is characterized by a match-specific productivity. Thus, the model generates a wage distribution per age. If we want to compare the model predictions with the stylized facts, it is then necessary to compute an *Average wage* per age, which is:

$$\mathcal{W}_i(z) \quad = \quad \gamma z \mathcal{G}(R_i(z)) + (1 - \gamma)\left(b + \Sigma_i(z)\right) \tag{12}$$

where $\mathcal{G}(R_i(z)) = \frac{1}{n_i(z)} \int_{R_i(z)}^1 x dn_i(z, x)$ denotes the average productivity of age-i workers, whereas $dn_i(z, x)$ is the mass of age-i employees on a x-productivity job. This distribution is endogenous and

[16] see Appendix C.2

[17] This restriction seems to be realistic simply because new jobs becoming profitable with aging are actually in a *small* interval ($[R_i, R_{i+1}]$), whereas the other jobs are in a *large* interval (at least $[R_i(z), 1]$). Empirically, we need the selection effect to be small enough. In which case, the age-dynamics of the expected surplus is driven by the horizon effect. The model can then generate an age-decreasing JFR. Notice that beyond its impact on the expected surplus, the selection effect has also a direct impact on the JFR though $1 - G(R_i)$: after a meeting, it is easier to be productive for old workers. Thus, at the end, we need a large enough horizon effect in order to dominate the other two channels (the direct channel through e_i and θ_i and the indirect effect through R_i) through which the selection effect generates an age-increasing profile for the JFR.

depends on the worker movements in the job distribution (entries, exits and productivity changes). Another difference with respect to the exogenous distribution $G(\epsilon)$ is that $dn_i(z,\epsilon)$ change with the business cycle. [18]

If we assume that search efforts ($\theta_i(z)$ and $e_(z)$) and reservation productivity are age-decreasing [19], how is the age-dynamics of the average wage? When a worker ages, the decline in $\Sigma_i(z)$ pushes $\mathcal{W}_i(z)$ downward: the horizon effect reduces the outside options and thus the average wages. But, at the same time, when workers age, they become less selective: the reservation productivity $R_i(z)$ falls. This produces two opposite effects: firstly, the set of accepted wage increases, leading to push-up $\mathcal{W}_i(z)$, but secondly, this decline in the selective process raises the employment rate $n_i(z)$ and concentrates the wage at the bottom of the distribution because $\frac{d}{dR_i(z)}dn_i(z,x) > 0$.[20] Thus, steady-state average wage can be age-decreasing. Since it is not the case in the data, we introduce human capital accumulation in the quantitative section.[21]

5 The age-specific impact of the business cycle in the model

Does the restrictions allowing to match the steady state per age of the worker flows compatible with the stylized facts by which older worker flows' are more sensitive to the business cycle than their younger counterparts? Indeed, in the previous section, we have shown that the sizes of valuations of future options depend on workers' age. This necessarily affects business cycle elasticities which become age-dependent. This section is devoted to explain why a model that takes into account the life cycle gives more chance to the Diamond-Mortensen-Pissarides (DMP) setting to reproduce the large volatility of labor market outcomes. Following Shimer (2005) or Nagypal & Mortensen (2007), we derive comparative statics results that describe how the equilibrium market tightness implied by the model changes with aggregate productivity across steady states. For this type of analysis, it is sufficient to focus on conditional steady states (see Nagypal & Mortensen (2007)), ie. the equilibrium contingent to a permanent realization of the aggregate productivity z, and to approximate the impact of shock on z as a permanent change. In the same spirit, we present analytical results under the approximation that transitions between ages do not matter for the fluctuations within an age class: the high persistence in an age class leads to neglect the low probability of aging at business cycle frequencies[22].

[18] See the section 5.4 for more details on the impact of the business cycle on the employment distribution.

[19] These restrictions are simply a sufficient condition to match the age-decreasing pattern or JSR and JFR observed in the data.

[20] See appendix C.3.

[21] More specifically, we introduce age-specific labor efficiency ($z_i = zh_i$) and age-specific home production efficiency ($b_i = bh_i$)

[22] This assumption is also consistent with the measures of the worker flows in the data where we consider that the transitions within an age class are not influenced by the fraction of workers who age in a period (the month). This approximation is acceptable if $\frac{1-\pi_i}{\pi_{i+1}} \to 0$. In the calibrated model, we have $\frac{1-\pi_i}{\pi_{i+1}} \in [7e-4; 2e-2]$ showing that this

For expositional simplicity, we also assume in what follows that the disutility of search effort is such that $\phi(e_i) = \frac{e_i^{1+\phi}}{1+\phi}$ and that the matching function is $M(v_i, e_i u_i) = v_i^{1-\eta}(e_i u_i)^{\eta}$, with $\phi > 0$ and $\eta \in (0; 1)$.

Volatilities can be inferred from a log-linear approximation of equations (8) and (9)

$$\widehat{JSR}_i = \frac{JSR_e}{JSR} \varepsilon_{G|R_i} \widehat{R}_i \tag{13}$$

$$\widehat{JFR}_i = \widehat{e}_i + (1-\eta)\widehat{\theta}_i - \frac{G(R_i)}{1-G(R_i)} \varepsilon_{G|R_i} \widehat{R}_i \tag{14}$$

where $\varepsilon_{G|R_i}$ denotes the elasticity of the function G with respect to R_i. The job separation rate are driven by counter-cyclical movements in $\widehat{R}_i$ while the counter-cyclical behavior of the reservation productivity reinforces the pro-cyclical behavior of the job finding rate. To determine the age profile of volatility in JSR and JFR, we will examine below the responsiveness of reservation productivity (section 5.1) and search efforts (section 5.2). Both variables, reservation productivity and labor market tightness, are the solution of a system of equations involving the free entry condition and zero surplus.

More specifically, in order to decompose the macroeconomic impact of the aggregate productivity shock, we use the following system:[23]

$$(JC) \quad \begin{cases} \frac{c}{q(\theta_i(z))} = \beta \pi_i J_i(z) \\ J_i(z) = ze(R_i(z)) - w_i(z) + (1-\gamma)[(1-G(R_i(z)))\Lambda_i(z) + \Gamma_i(z)] \\ w_i(z) = \gamma z e(R_i(z)) + (1-\gamma)\left(b + \Sigma_i(z)\right)\left(1 - G(R_i(z))\right) \end{cases}$$

$$(JD) \quad \begin{cases} zR_i(z) = w_i(z, R_i(z)) - (1-\gamma)[\Lambda_i(z) + \Gamma_i(z, R_i(z))] \\ w_i(z, R_i(z)) = \gamma z R_i(z) + (1-\gamma)\left(b + \Sigma_i(z)\right) \end{cases}$$

where $J_i(z) = \int_{R_i(z)}^{1} J_i(z,x)dG(x)$, $w_i(z) = \int_{R_i(z)}^{1} w_i(z,x)dG(x)$, $e(R_i(z)) = \int_{R_i(z)}^{1} x dG(x)$ and $\Gamma_i(z) = \int_{R_i(z)}^{1} \Gamma_i(z,x)dG(x)$. These variables denote respectively the expected value of a filled vacancy for the firm, the *ex-ante* average wage cost, the average productivity and the average of the continuation value.

The Log-linear approximation of the previous system leads to the following dynamic of the (JC)

approximation is acceptable.

[23]Given that all the variables are evaluated conditionally on a particular level of z, we have:

$$\begin{aligned} \Sigma_i(z) &= -\phi(e_i(z)) + \beta\gamma\left[\pi_i e_i(z) p(\theta_i(z)) \overline{S}_i(z) + (1-\pi_i) e_{i+1}(z) p(\theta_{i+1}(z)) \overline{S}_{i+1}(z)\right] \\ \Lambda_i(z) &= \beta(1-s_e)\left[\pi_i \lambda_i \overline{S}_i(z) + (1-\pi_i)\lambda_{i+1}\overline{S}_{i+1}(z)\right] \\ \Gamma_i(z,\epsilon) &= \beta(1-s_e)(1-\pi_i)(1-\lambda_{i+1}) S_{i+1}(z,\epsilon) \end{aligned}$$

curve: [24]

$$(JC) \quad \begin{cases} \widehat{\theta}_i &= \frac{1}{1-\eta}\widehat{J}_i \\ \widehat{J}_i &= \frac{ze(R_i)}{ze(R_i(z))-w_i(z)}\widehat{z} - \frac{w_i}{ze(R_i(z))-w_i(z)}\widehat{w}_i - R_iG'(R_i)\frac{zR_i+(1-\gamma)\Lambda_i}{ze(R_i(z))-w_i(z)}\widehat{R}_i \\ \widehat{w}_i &= \gamma\frac{ze(R_i)}{w_i}\widehat{z} + (1-\gamma)\frac{\Sigma_i(1-G(R_i))}{w_i}\widehat{\Sigma}_i - R_iG'(R_i)\frac{w(R_i)}{w_i}\widehat{R}_i \end{cases}$$

Fluctuations in the search effort of the firm, $\widehat{\theta}_i$ are driven by the dynamic of the job value $\widehat{J}_i$. The wage equation then shows that changes in (JC) are dampened if the search value is highly pro-cyclical: high wages in boom reduce the increase in the firm value and thus moderates its incentive to hire more workers. Another crucial point is the fact that, when we combine the $\widehat{J}_i$ and the $\widehat{w}_i$ equations, the term on $\widehat{R}_i$ disappears: indeed, the ex-ante productivity draw leads firms to compare expected productivity to expected labor costs, both take into account the dynamics of the selective process accounting by R_i. Hence, this model with endogenous destructions shares the same properties with respect to the JC curve dynamics as the simple model with exogenous destructions. This (JC) system then provides the intuition behind Shimer (2005)'s criticism of the DMP model: ex-ante wage variations dampen the firm surplus volatility, thereby reducing the incentive to invest in vacancy in economic booms. This leads the DMP model to predict small changes in labor market quantities.

The Log-linear approximation of the (JD) system are given by:

$$(JD) \quad \begin{cases} \widehat{R}_i &= -\widehat{z} + \frac{w_i(R_i)}{b+\Sigma_i}\widehat{w}_i^r - \frac{(1-\gamma)\Lambda_i}{b+\Sigma_i}\widehat{\Lambda}_i - \frac{(1-\gamma)\Gamma_i(R_i)}{b+\Sigma_i}\widehat{\Gamma}_i^r \\ \widehat{w}_i^r &= \gamma\frac{zR_i}{w_i(R_i)}(\widehat{z}+\widehat{R}_i) + (1-\gamma)\frac{\Sigma_i}{w_i(R_i)}\widehat{\Sigma}_i \end{cases}$$

The system shows that movements in (JD) curve are also dampened if the wage is highly pro-cyclical.

The size of the age-specific ex-ante wage response to the business cycle is then at the heart of the age-specific volatilities of worker flows. In the two wage equations, inter-temporal behaviors, summarized by fluctuations in the search value $\widehat{\Sigma}_i$, magnify the wage sensitivity to the business cycle. The longer the worker's opportunity to find a job, the larger the ex-ante wage sensitivity to changes in the value of the labor market prospects. Thus, others things being equal, if, for older workers, Σ_i is small, the pro-cyclical behavior of ex-ante wage is dampened. Given that our steady state analysis leads to the restrictions $\Sigma_i > \Sigma_{i+1}$ (the horizon effect), the elasticity of ex-ante wages (the average wage for the (JC) curve, or the reservation wage for the (JD) curve) to the search option (Σ_i) declines with the worker's age. Hence, ex-ante wages becomes more rigid when workers age. Other thinks being equal, this generate more volatility in the quantities (worker inflows and outflows) in the labor market for older workers.

[24]Using the approximation $\frac{1-\pi_i}{\pi_{i+1}} \to 0$, we obtain $\Lambda_i(z) = \beta(1-s_e)\pi_i\lambda_i\frac{J_i(z)}{1-\gamma}$. This leads to

$$J_i(z) = \frac{ze(R_i(z)) - w_i(z)}{1-\beta(1-s_e)\pi_i\lambda_i\frac{1-(1-\lambda_i)G(R_i(z))}{1-\lambda_i}} \quad \text{and} \quad \widehat{\Lambda}_i = \widehat{J}_i$$

5.1 Job separation rate volatility: elasticity of reservation productivity by age

Given that the job separation rate is a monotonic function of the reservation productivity, the analysis of this variable is sufficient to characterize the age pattern of separation fluctuations. Combining the wage reservation equation and the reservation productivity of the (JD)-curve, using the definition of the surplus and its approximation[25], we have $\widehat{\Gamma}_i^r(z) \approx \widehat{R}_i$, leading to

$$\widehat{R}_i = -\frac{b+\Sigma_i}{b+\Sigma_i+\Gamma_i(R_i)}\widehat{z} + \frac{\Sigma_i}{b+\Sigma_i+\Gamma_i(R_i)}\widehat{\Sigma}_i - \frac{\Lambda_i}{b+\Sigma_i+\Gamma_i(R_i)}\widehat{\Lambda}_i$$

Given that the Log-linear approximations of the free entry condition and the FOC w.r.t e lead to $\widehat{\Sigma}_i \approx \frac{1+\phi}{\phi}\widehat{\theta}_i$ and $\widehat{\Lambda}_i \approx \eta\widehat{\theta}_i$ respectively, we deduce that $\widehat{\Sigma}_i > \widehat{\Lambda}_i$. Thus, assuming that Σ_i is age-decreasing (our assumption in order to match the observed first order moments characterizing the steady state), the variations in search values ($\widehat{\Sigma}_i$) dominate the volatility in labor hoarding values ($\widehat{\Lambda}_i$). The expression of $\widehat{R}_i$ then shows that fluctuations in inter-temporal values, dominated by changes in $\widehat{\Sigma}_i$, dampen fluctuations in $R_i(z)$. In recession, prime-age workers are highly sensitive to the decrease in labor market opportunities because they have a future. The change in their reservation wage dampens the impact of the business cycle. For old workers, the same effect is at work, even if the future is very short. However, at the end of the life cycle, we have $\Sigma_i \to 0$ and thus $\Lambda_i \to 0$ because $\Sigma_i > \Lambda_i$, leading R_i to be highly sensitive to current aggregate shocks. In other words, $\widehat{R}_i = -\widehat{z}$ for old workers: old workers' reservation productivity respond to the aggregate shock on a one-for-one basis. In contrast, for prime-age workers, the reservation productivity responsiveness is less than one, in absolute value.

5.2 Job finding rate volatility: elasticity of search efforts by age

Let us first analyze the volatility of labor market tightness that depends on the expected value of a filled vacancy.[26] By introducing the log-linear approximation of the ex-ante wage in the log-linear approximation of the expected firm surplus, we obtain:

$$\widehat{J}_i = \frac{ze(R_i)}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))}\widehat{z} - \frac{\Sigma_i(1-G(R_i))}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))}\widehat{\Sigma}_i \tag{15}$$

Recall that Σ_i is an increasing function of γ (a large bargaining power leads to a high search value). Using equation (15), we can recover all the cases discussed in the literature.

In Shimer (2005), without age-heterogeneity, equation (15) boils down to :

$$\widehat{J} = \frac{z}{z-(b+\Sigma(\gamma))}\widehat{z} - \frac{\Sigma(\gamma)}{z-(b+\Sigma(\gamma))}\widehat{\Sigma}(\gamma) \tag{16}$$

[25] See Appendix C.4.

[26] Equation (14) suggests that in order to understand the volatility age profile of JFR, we need to characterize the elasticity of search efforts for firms and workers. Labor market tightness and search effort ($\{\theta_i, e_i\}$) depend on the expected value of a filled vacancy J_i. The Log-linear approximations of the free entry condition and the FOC w.r.t the search effort of the unemployed workers leads to $\widehat{e}_i = \frac{1}{\phi}\widehat{\theta}_i$. Thus, we focus in this section on θ_i. The volatility of e_i behaves in a similar way.

In Shimer (2005), b the value of leisure is restricted to measure the replacement rate in the United-States: thus, b is small and denoted by $b = b^-$. Concerning the bargaining power, Shimer (2005) assumes that the Hosios condition is satisfied and thus sets γ to be equal to η. For the calibration, Shimer (2005) uses an information on η and sets it at the upper end of the range of estimates reported in Petrongolo & Pissarides (2001). Given that $\eta = \gamma$, this implies that the bargaining power is large, $\gamma = \gamma^+$, and thus the value of the search is also large ($\Sigma(\gamma^+) >> 0$). With Shimer (2005)'s calibration [27], the direct impact of the productivity shock is small ($\frac{z}{z-(b^-+\Sigma(\gamma^+))}$) because the direct impact of b is larger than the indirect impact of γ on this multiplier, whereas the impact of fluctuation in search opportunities are not negligible ($\Sigma(\gamma^+) \neq 0$). This calibration leads to minimize the predicted impact of an aggregate shock on hiring decision in the DMP model.

In Hagendorn & Manovskii (2008), in equation (16), a new calibration of this model is proposed. First, the value of leisure includes unemployment benefits as well as home production, leading to a high value for b, denoted b^+ and such that $b^+ > b^- + \Sigma(\gamma^+)$. Concerning the bargaining power, Hagendorn & Manovskii (2008) do not assume that the Hosios condition is satisfied and set a low value for γ such that $\gamma \approx 0 \Rightarrow \Sigma(\gamma^-) \to 0$. With this calibration, the impact of search opportunities is negligible ($\Sigma \approx 0$) and the direct impact of $\widehat{z}$ is maximized $\frac{z}{z-b^+}$. With this calibration strategy [28], Hagendorn & Manovskii (2008) show that the DMP model can generate the observed fluctuation of worker flows. Nevertheless, their solution is obtained in a economy where the unemployment rates are lower than their optimal counterparts, because $\gamma < \eta$, suggesting that something is missing in the model.

An alternative solution, suggested in Hall (2005), is provided in Hall & Milgrom (2008): this setup provides a game where the weight of the search value Σ in the wage equation can be arbitrary very small. [29].

In our case, prime-age workers lie between Shimer (2005)'s and Hagendorn & Manovskii (2008)'s workers, and look like Hall & Milgrom (2008)'s workers:

$$\widehat{J}_A = \frac{ze(R_A(z))}{ze(R_A(z)) - (b+\Sigma_A)(1-G(R_A(z)))}\widehat{z} - \frac{\Sigma_A(1-G(R_A(z)))}{ze(R_A(z)) - (b+\Sigma_A)(1-G(R_A(z)))}\widehat{\Sigma}_A \quad (17)$$

In order to reproduce the average volatility of workers flows, it is necessary to calibrate the value of home production at a larger value than the one proposed by Shimer (2005), which is consistent with Hall (2005). The important point is that, unlike Hagendorn & Manovskii (2008), we preserve

[27] The values in Shimer (2005) are $\gamma = 0.72$ and $b = 0.4$.

[28] The values in Hagendorn & Manovskii (2008) are $\gamma = 0.061$ and $b = 0.943$.

[29] The values in Hall & Milgrom (2008) are $\gamma = 0.5445$ and $b = 0.71$. Nevertheless, the solution of the wage equation depends on additional parameters. Indeed, in their "strategic bargaining", if for simplicity it is assumed that *(i)* the worker receives payoff b and the employer incurs no cost while bargaining continues and *(ii)* they renegotiate the division of the match product $z\epsilon$ whenever it changes, then the outcome of a symmetric ($\gamma = 0.5$) alternating-offers game is $w = b + 0.5(z\epsilon - b)$. This shows that it exists a calibration of the costs incurred during the bargaining process leading the wage to be independent of the search opportunity value Σ.

the Hosios condition and we endogenously obtain varying changes in the search value in the wage dynamics though another channel than the one proposed by Hall & Milgrom (2008). We have for old workers

$$\widehat{J}_O = \frac{ze(R_O(z))}{ze(R_O(z)) - b(1 - G(R_O(z)))}\widehat{z} \tag{18}$$

With aging, the returns on search is nil for older workers, leading $\Sigma_O \to 0$, and reducing the elasticity of wages w.r.t. $\widehat{\Sigma}_i$ when workers age. Then, with a calibration for the value of leisure such that $b > b^-$, the aging endogenously leads average flows to look like Hall & Milgrom (2008)'s workers (even if $\gamma = \eta > 0$). In contrast, prime-age workers highly value search opportunities ($\Sigma_A > 0$), the very element that moderates the elasticity of the job value w.r.t. aggregate shocks in equation (17). This then explains the lower elasticity of search efforts to the business cycle for prime-age workers than for old workers.

5.3 Worker flow volatility: the equilibrium analysis

In the two previous sections, we have presented the results on the impact of the business cycle using partial equilibrium analysis in order to decompose in an intuitive way the economic forces at work in the model. In this section, using the same set of simplifying assumptions (*(i)* conditional steady states and *(ii)* negligible impact of age transitions on fluctuations within an age class, ie. $\frac{1-\pi_i}{\pi_{i+1}} \to 0$), we show that the results are robust to the equilibrium analysis. Indeed, we obtain:

$$\widehat{R}_i \approx -\frac{b + \Sigma_i\left(1 - \frac{1+\phi}{\phi}\frac{1}{\eta}\right)}{b + \Sigma_i\left(1 + \frac{1+\phi}{\phi}\frac{1}{\eta}\varepsilon_{I|R}\right) - \Lambda_i(1 + \varepsilon_{I|R})}\widehat{z} \tag{19}$$

$$\widehat{\theta}_i \approx \frac{1}{\eta}\left[1 + \varepsilon_{I|R}\frac{b + \Sigma_i\left(1 - \frac{1+\phi}{\phi}\frac{1}{\eta}\right)}{b + \Sigma_i\left(1 + \frac{1+\phi}{\phi}\frac{1}{\eta}\varepsilon_{I|R}\right) - \Lambda_i(1 + \varepsilon_{I|R})}\right]\widehat{z} \tag{20}$$

$$\widehat{e}_i \approx \frac{1}{\phi}\frac{1}{\eta}\left[1 + \varepsilon_{I|R}\frac{b + \Sigma_i\left(1 - \frac{1+\phi}{\phi}\frac{1}{\eta}\right)}{b + \Sigma_i\left(1 + \frac{1+\phi}{\phi}\frac{1}{\eta}\varepsilon_{I|R}\right) - \Lambda_i(1 + \varepsilon_{I|R})}\right]\widehat{z} \tag{21}$$

where $\varepsilon_{I|R} = \left|\frac{I'R}{I}\right|$, with $I(R_i(z)) = \int_{R_i(z)}^{1}(1 - G(x))dx$.

Proposition 3. *When JFR_i and JSR_i are age-decreasing, older workers are more sensitive to the business cycle than prime-age workers, because old workers have no future.*

Proof. Using (19), (20) and (21), if we assume that the values of Σ_i and Λ_i are at their lowest level for older workers $\Sigma_O = \Lambda_O = 0$, and at their highest level for prime-age workers $\overline{\Sigma}_A = \frac{b}{\frac{1+\phi}{\phi}\frac{1}{\eta} - 1}$, we deduce that

$$\begin{aligned}
\widehat{R}_O &\approx -\widehat{z} & \widehat{R}_A &\approx 0\\
\widehat{\theta}_O &\approx \tfrac{1}{\eta}\left[1 + \varepsilon_{I|R}\right]\widehat{z} & \widehat{\theta}_A &\approx \tfrac{1}{\eta}\widehat{z}\\
\widehat{e}_O &\approx \tfrac{1}{\phi}\tfrac{1}{\eta}\left[1 + \varepsilon_{I|R}\right]\widehat{z} & \widehat{e}_A &\approx \tfrac{1}{\phi}\tfrac{1}{\eta}\widehat{z}
\end{aligned}$$

therefore $|\widehat{R}_O| > |\widehat{R}_A|$, $|\widehat{\theta}_O| > |\widehat{\theta}_A|$ and $|\widehat{e}_O| > |\widehat{e}_A|$. □

Proposition 3 shows that, if the model can reproduce the shape of worker per age at the steady state, then this set of restrictions is sufficient to be in accordance with fluctuations of these data around this steady state.

5.4 Wage elasticity by age

When there is no heterogeneity among matches within a age group ($\epsilon = 1$), the wage equation (2) can be used to test the predictions of the model with respect of wage volatilities. Indeed, in a representative-agent model, without heterogeneity, there is no difference between the wage bargained at the individual level, the aggregate wage (an average over identical workers at a given time) and the ex-ante wage (or the mean wage over homogenous jobs) computed by the firm to predict its expected job value before at match. Thus, in this simple model, one can easily deduce that if the real wage (measured by the aggregate wage) is rigid in the data, then it is the case for the ex-ante wage: the DMP model with a rigid wage can then solve the Shimer volatility puzzle. Nevertheless, the statistics reported in the table 3 show that volatilities are not negligible, which does not provide empirical support for the rigid wage assumption.

In our MP life-cycle model, the heterogeneous match-productivity leads the average wage and the ex-ante wage to be different, even if both take into account the individual wage levels (equation (2)). The average over all workers in t contains an information on the composition of the labor force (the endogenous distribution of the jobs, $dn_i(z, \epsilon)$), whereas the ex-ante wage in t contains information on the draw of the productivity after the match (the exogenous distribution of the productivity, $dG(\epsilon)$). If only ex-ante wages matter for individual decisions, the descriptive statistics can be only based on average wages. Thus, one can have a rigid ex-ante wage, in the sense discussed in the section 5, and a volatile average wage, as in table 3. If it is the case, then the DMP model can predict both workers flow volatilities *and* wage volatility. In this section, we propose to explain the main force at work in the average wage dynamics.

The log-linear approximation of equation (12) leads to

$$\widehat{\mathcal{W}_i} = \gamma \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i} \left(\widehat{z} + \widehat{\mathcal{G}}_i\right) + (1-\gamma)\frac{\Sigma_i}{\mathcal{W}_i}\widehat{\Sigma}_i$$

where the term $\widehat{\mathcal{G}}_i$ gives the impact the changes in the job composition on wage fluctuations. In the appendix C.4, it is shown that on can obtain

$$\widehat{\mathcal{G}}_i = \Gamma_i^z \widehat{z} + \Gamma_i^r \widehat{R}_i + \Gamma_i^s \widehat{\Sigma}_i$$

where Γ_i^z, Γ_i^r and $\Gamma_i^s \equiv \gamma_i^s \Sigma_i$ are the elasticities of the average productivity with respect to z, R_i and Σ_i. [30] This approximation underlines the channels through which the average productivity

[30] See the appendix C.4 for more details on the computation.

$\mathcal{G}_i$ depends on the business cycle. The signs of these elasticity Γ_i^x, for $x = z, r, s$, are ambiguous. The aggregate productivity (z) has a positive impact on aggregate employment, and thus lowers the average productivity, but also raises employment at each level of the productivity distribution through its impact on search efforts ($e_i(z)$ and $\theta_i(z)$). In boom, the fall in R_i increases the set of job (the integral has a larger span) but lowers its quality (more jobs are concentrated at the bottom), Moreover, when R_i declines, employment increases, thus average productivity falls. Finally, a rise in the search value Σ_i reduces the incentive to post new vacancies. This has a negative impact on the two dimensions of employment: by reducing its aggregate level, this raises average productivity, whereas its negative effect on each point lowers average productivity. Finally, we deduce that the average wage dynamics is given by:

$$\widehat{\mathcal{W}_i} = \gamma \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i}\left((1+\Gamma_i^z)\widehat{z} + \Gamma_i^r \widehat{R}_i + \Gamma_i^s \widehat{\Sigma}_i\right) + (1-\gamma)\frac{\Sigma_i}{\mathcal{W}_i}\widehat{\Sigma}_i$$

Thus, for older workers, ie. when $\Sigma_O \to 0$, the average wage can be proxied by

$$\widehat{\mathcal{W}_O} = \gamma \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i}\left((1+\Gamma_O^z)\widehat{z} + \Gamma_O^r \widehat{R}_O\right)$$

This shows that this average wage can be highly pro-cyclical if $\Gamma_i^r > 0$ and large enough. Moreover, given that the volatility of the JSR is age-increasing, implying $\widehat{R}_i < \widehat{R}_{i+1}$, the impact of $\widehat{R}_O$ on $\widehat{\mathcal{W}_O}$ can be reinforced by the large volatility in the reservation productivity.

In the data, the volatility of the hourly wage is age-decreasing. Thus, an additional test of the model, which is different than the rigidity of the ex-ante wage in the sense defined in section 5, is its ability to reproduce the age-pattern of wage cyclical properties. This implies that the calibration must be such that Γ_i^r must be lower enough to ensure that older workers' wage volatility is the lowest.

6 Quantitative assessment

In this section, we bring the model to the data. This allows to check that the model's behavior is consistent with our analytical results when we give up the simplifying assumptions made in the previous sections. First of all, we introduce a deterministic human capital accumulation allowing to the model to match the observed age-increasing pattern of real hourly wages: even if there horizon is shorter, the older workers can compensate via there human capital accumulations. Under these steady state restrictions, our goal is to explain the volatility by age, while respecting those observed at the aggregate level, which is usually computed on the workers aged from 16 years old and more. This therefore requires us to retain the more general version of our model, ie. that integrating youth labor market (16-24 years old). This allows us to generate aggregated data that correspond to macroeconomic time series usually discussed. The "horizon effect" is not discriminating to explain the differences between the young and the prime-age workers. We then introduce two specificities on the youth labor market: a potentially higher "mismatch effect" and a "composition effect" which account for a difference in the quality of the labor force (section 6.1). It is noticeable that, with the introduction of youth labor market, who are also far way from retirement, job flows volatilites found in US data are consistent with our distance-to-retirement effect. Using CPS data, youth job flows (hourly wage) are found to be less (more) responsive to the business cycle than prime-age workers'. The data seems to support our view that aging endogenously generates wage sluggishness and large labor market adjustments.

Secondly, in section 6.2, we gauge the model's ability to replicate the age volatility profile when we calibrate the model to fit historical averages. In doing so, we first uncover structural parameters that allow the model to match the first moments found in the data. Under this calibration, we assess the model's ability to generate second order moments consistent with the data. The challenge is quantitative: will the model's predicted volatilities be close to the ones observed in the data? Our result indeed show that the parameter restrictions imposed by the first order moments are sufficient to generate age-increasing elasticities.

Finally, in section 6.3, a sensitivity analysis is performed.

6.1 Adding the youth labor market : Model's analytical predictions

Steady state reservation productivity for the youth. If horizon matters for the gap between older and prime-age workers, it seems intuitive that it is not sufficiently persistent to explain the large observed differences between young ($i = Y$) and prime-age workers. What matters for this gap could be the specificities of the match-specific productivity. If for young workers, the persistence of this specific draw of productivity is more persistent than for the other workers, then a bad draw represents a mismatch and a high probability to be fired before aging. At the opposite, a good and persistent draw insures the worker to have new opportunities inside the firm. Hence, the high persistence of the match-specific productivity can be viewed as a reduced form of a selection process where less informed workers on the specificities of the job can experiment more jobs via an unemployment transition. This interpretation leads to a restriction: λ_i is smaller for younger workers than for older workers, leading younger workers to be more fragile w.r.t. microeconomic disturbances.[31]

Steady state hiring and search effort for the youth. To account for the differences between youth and prime-age workers, we introduce a "composition" effect: on average, the younger workers who participate have a comparative advantage at work than those who choose to learn at school.[32]

[31]Even if $e_A p(\theta_A) \approx e_Y p(\theta_Y)$, the restriction $\lambda_Y < \lambda_A$ ensures that the gap $\gamma e_i p(\theta_i) - (1 - s_e)\lambda_i > 0$ is larger for younger workers than for prime-age workers, leading to $R_Y > R_A$ as in the data.

[32]Remark that this difference among the population within an age group disappears when worker age because all the males participate after 25 years old until the retirement age.

This composition effect is modeled as an age-specific distribution $G_i(\epsilon)$ where the average of ϵ is larger in the youth labor market leading to large expected gains for posted vacant job.[33]

Sensitivity to the business cycle for the youth. How do these structural specificities affect the business cycle properties of the model?

Proposition 4. *Even if the horizon effect does not matter at the beginning of the life-cycle, a lower* λ_i *for young workers than for the other workers reduces their sensitivity to the business cycle.*

Proof. Using notations from section 5.3, if the horizon effect does not matter, we have $\Sigma_Y \approx \Sigma_A$ and $\Lambda_Y \approx \delta\Lambda_A$ with $\delta = \lambda_A/\lambda_Y < 1$. Using (19), (20) and (21), we deduce that $\widehat{R}_Y < \widehat{R}_A$, $\widehat{\theta}_Y < \widehat{\theta}_A$ and $\widehat{e}_Y < \widehat{e}_A$ if $\delta < 1$, for $\varepsilon_{I|R_Y} \approx \varepsilon_{I|R_A}$. □

A high persistence in the match-specific productivity (low value of λ) reduces the sensitivity of young worker flows to the business cycle. Indeed, young workers' low ability to move up on the productivity distribution leads their current surplus to be highly dependent on current productivity and the search outside option. Since this last component dampens fluctuations in the job value, young worker flows are less sensitive to the business cycle.

6.2 Matching the age-increasing volatility in transition rates

6.2.1 Benchmark Calibration

The vector of the model's parameter is $\Phi = \{\Phi_1, \Phi_2\}$ with $dim(\Phi) = 48$. All the usual parameters provided in the literature are in

$$\Phi_1 = \{\beta, \{\pi_i\}_{i=Y}^{O_7}, \gamma, \eta, c, s_e, \rho, \sigma_\nu\} \quad dim(\Phi_1) = 16$$

For these parameters, we follow Fujita & Ramey (2012) and Shimer (2005). The parameters $\{\beta, \{\pi_i\}_{i=1}^{O_7}, \gamma = \eta, c, \rho, \sigma_\nu\}$, which are calibrated to match a monthly discount factor consistent with an annual interest rate of 4%, whereas the elasticity parameter of the matching function η and the bargaining weight of workers γ are both set to 0.7. These two values are close to the values of η and γ (0.72) used in Shimer (2005). The calibration of the vacancy posting cost c uses the results of Barron et al. (1997) and Barron & Bishop (1985). These authors suggest that an amount to 17 percent of a 40-hour workweek (nine applicants for each vacancy filled, with two hours of work time required to process each application). This leads to $c = 0.17$. The parameters for the aggregate productivity process ρ and σ_ν are set to the values proposed by Shimer (2005). We set π_i such that

[33] Even in the extreme case where $\Omega_Y \approx \Omega_A$ and $R_Y \approx R_A$, we have $\overline{S}_Y - \overline{S}_A = \Omega_Y \left(\int_{R_Y}^1 [1 - G_Y(x)]dx - \int_{R_Y}^1 [1 - G_A(x)]dx \right) > 0$ if $E_A[\epsilon] > E_Y[\epsilon]$.

an age class correspond to the age groups in the data: $i = Y$ are the $16-24$ years old workers, $i = A$ are the $25 - 54$ years old workers and $i = O_j$ for $j = 1, ..., 7$ are the $55, ..., 61$ years old workers.

For the other parameters, with the normalization $h_Y(1 + \mu_Y) = 1$, we have:

$$\Phi_2 = \{H, \chi, \phi, \sigma_\epsilon, b, \{h_i, \mu_i, \lambda_i\}_{i=Y}^{O_7}\} \quad dim(\Phi_2) = 31$$

where H and χ are the scale parameters, respectively, of the matching function $M(v_i, e_i u_i) = H v_i^{1-\eta}(e_i u_i)^\eta$, and search cost $\phi(e_i) = \chi \frac{e_i^{1+\phi}}{1+\phi}$. The parameters h_i account for the increase in labor efficiency when workers age. The average level of the match-specific productivity ϵ is denoted μ_i.

We need some restrictions in order to identify these parameters using our first order moments. We assume that:

- Older workers share the same level of human capital, leading to $\{h_i\}_{i=Y}^{O_7} = \{h_Y, h_A, h_O\}$,
- Older workers share the same λ_i, leading to $\{\lambda_i\}_{i=Y}^{O_7} = \{\lambda_Y, \lambda_A, \lambda_O\}$,
- Only young workers can be affected by a "composition" effect: $\{\mu_i\}_{i=Y}^{O_7} = \{\mu_Y, 0\}$.

The calibration procedure finds parameter values Φ_2 that minimize the distance between theoretical and observed moments $\Psi^{theo}(\Phi_2) - \Psi$. The numerical solution for $\Psi^{theo}(\cdot)$ is provided by the algorithm described in Appendix C.5. The 11 free parameters are

$$\Phi_2 = \{H, \chi, \phi, h_A, h_O, \mu_Y, b, \lambda_Y, \lambda_A, \lambda_O, \sigma_\epsilon\} \quad dim(\Phi_2) = 11$$

whereas the 11 first-order moments provided by the data are:

$$\Psi = \{\overline{JFR}, \overline{JSR}, \overline{w}, \{JFR_i\}_{i=Y}^{O}, \{JSR_i\}_{i=Y}^{O}, \{w_i\}_{i=A}^{O}\} \quad dim(\Phi_2) = 11$$

where $X_O = \frac{1}{7}\sum_{i=O_1}^{O_7} X_i$ for $i = O_1, ..., O_7$, $\forall X = JFR, JSR, w$. w_Y is normalized to unity. These targeted moments are reported in the table 4.

Table 4: First order moments: Ψ

JFR_Y	JFR_A	JFR_O	JSR_Y	JSR_A	JSR_O
0.49	0.41	0.33	0.048	0.017	0.011
$\overline{JFR}$	$\overline{JSR}$	$\overline{w}$	w_A	w_O	
0.43	0.021	1.55	1.68	1.84	

Table 5 summarizes the calibration. Two important results must be stressed.

First, regarding our evaluation of the process of match-specific productivity, given by $\{\lambda_Y, \lambda_A, \lambda_O, \sigma_\epsilon\}$, our calibration for the probability of having access to a new draw of match-specific productivity encompasses the one used in Fujita & Ramey (2012): $\lambda_Y < \lambda_{FR} < \lambda_A < \lambda_O$. Our calibrated values for λ_i imply a mean waiting time of one year and five months between switches of the match-specific

Table 5: Benchmark calibration

External information Φ_1						
	β	$\eta = \gamma$	c	s_e	h_Y	
	$r = 5\%$	0.7	0.15	0.001	0.875	
	ρ	σ_ν	π_Y	π_A	$\{\pi_i\}_{i=Y}^{O_7}$	
	0.9895	0.0034	16-24	25-54	55-56...60-61	
Calibration Φ_2						
	H	b	σ_ϵ	χ	ϕ	
HLS	0.0975	0.92	0.317	0.29	0.7	
FR	0.061	0.934	0.124	-	-	
	μ_Y	λ_Y	λ_A	λ_O	h_A	h_O
HLS	0.125	0.0495	0.1975	0.225	1.82	1.976
FR	-	-	0.085	-	-	-

HLS = our weekly calibration

FR = Fujita and Ramey (2011)'s weekly calibration

productivity for younger workers, whereas, for other workers, this mean waiting time is only one months and one week (one month) for the prime-age (older) workers.[34] Our results show that young workers are less likely to benefit from changes in productivity during their first nine years of work experience. If we reduce the interpretation of λ to a job-to-job mobility opportunity, our calibration seems to contradict the idea that job-to-job mobility is more important at the early stage of a career (e.g. see the data discussed by Menzio et al. (2012)). However, as the model has only this source of heterogeneity between jobs at each stage of the life-cycle, it may reflect different phenomena for each age: a less restrictive interpretation is needed. Thus, the phenomenon of mismatch can be very relevant in the youth labor market and gradually decrease with experience in the labor market. A small λ for younger workers can indicate an inability to move up within the firm and improve match-specific productivity on the job (mismatch looks like a permanent shock). The best option for these worker is then to look for another job, thereby reducing the distance between the required skill and the worker's skills. We deduce that our small value for λ_Y reflects that mismatch is dominant at the early stage of the life-cycle, whereas, for other workers, the high value of λ may reflect their ability to adapt to new tasks in the firms. This interpretation of $\{\lambda_Y, \lambda_A, \lambda_O\}$ is consistent with Menzio et al. (2012)'s results on the estimations of the production parameters of a match. Indeed, they find that the quality of a match changes once every 8 and a half years.[35] Concerning the uncertainty about the match-specific distribution, our calibrated value of σ_ϵ is larger than Fujita & Ramey (2012)'s. Nevertheless, this result is unsurprising when we compare our results to the ones reported in Menzio et al. (2012), where the productivity of a match in the 90th percentile of the distribution is approximately 3 times larger than the productivity of a match in the 10th percentile

[34] For Fujita & Ramey (2012) this mean waiting time for the representative workers is equal to 3 months.

[35] They also show that the quality of the match is rapidly observed: 70 percent of firms and workers learn the quality of their match within the first four months, whereas in our model, this quality of the match is perfectly observed instantaneously.

of the distribution.

Table 6: Implied values of the outside option

$\sum_{i=Y}^{O_7} m_i \left[\frac{b_i}{h_i} - \phi(e_i^{ss})\right]$	$\frac{b_Y}{h_Y} - \phi(e_Y^{ss})$	$\frac{b_A}{h_A} - \phi(e_A^{ss})$	$\frac{b_O}{h_O} - \phi(e_O^{ss})$
0.73	0.56	0.76	0.78

The second comment deals with the value of b. Results reported in table 5 show that our calibrated value for b is lower than the one used by Fujita & Ramey (2012) in their calibration à la Hagendorn & Manovskii (2008). Our value is also larger than the more realistic one proposed by Hall & Milgrom (2008), which is $b = 0.7$.[36] Nevertheless, in our model with endogenous search effort, the instantaneous value of leisure is actually $b_i - \phi(e_i(z))$, not b. The net value of home production is reported in table 6. On average, the value of leisure is close to 0.7. This result shows that our calibration is closed to Hall & Milgrom (2008)'s estimated value for leisure. If, on average, we are not in the extreme case proposed by Hagendorn & Manovskii (2008), it is not the case for particular workers. Hence, for older workers, $e_{O_7} \to 0$, the value of leisure endogenously converges to b, a value close to Hagendorn & Manovskii (2008)'s calibration. In contrast, for young workers, the net value of home production is 0.56, which is closer to Shimer (2005)'s calibration ($b = 0.4$). Even if $e_i(z)$ is an endogenous variable which adjusts along the business cycle, these calibration results suggest that the model can generate more volatility in the labor market of older workers than for younger workers.

6.2.2 Quantitative results

Table 7 reports the 2nd order moments predicted by the model. Firstly, we comment the model implications with respect to the aggregate data (column "All: 16-61"). This allows to compare easily the performance of our model with the existing evaluations of the DMP model with representative agent. In the data, the standard deviation of unemployment of 23%.[37] This value is under-estimated by the model simulations (16%). This result is largely explained by our underestimation of the job finding rate volatility (11 % in the model vs. 16 % in the data[38]) while our model well predicts the job separation rate volatility (10 % in the model vs. 11 % in the data). Nevertheless, these results provide a first support to the DMP model: using an information on age heterogeneity in order to calibrate the steady state, the theoretical aggregate data on the labor market, stock and worker flows, display a volatility of the same order as the one observed in the data. There exists a discrepancy between theory and data but largely smaller than the one underlined by Shimer (2005)

[36] This value is also used by Menzio et al. (2012).

[37] Our data on unemployment displays a larger volatility than the ones reported in the literature: 9.6% in Fujita & Ramey (2012) and 19% in Hagendorn & Manovskii (2008). These two papers do not take into account the last recession. Moreover, the HP filter used in Fujita & Ramey (2012) extract less cyclical components.

[38] Notice that, in Hagendorn & Manovskii (2008), the standard deviation of the job finding rate is 11.8 %, a lower estimate than the one obtained in our data.

or Fujita & Ramey (2012) in their benchmark calibration. This result comes from our calibration of the outside options, which are on average over the age classes between Shimer (2005)'s and Hagendorn & Manovskii (2008)'s. Obviously, one can increase the value of the outside option in order to match the second order moments of the aggregate data. But, the cost of this strategy is to create a gap between the model and the first order moments, leading the model to depart from observed averages on labor market adjustments.

Table 7: Second order moments: data versus theory

		All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR:	US Data	0.11	0.089	0.14	0.20
	Model	0.10	0.07	0.17	0.19
JFR:	US Data	0.16	0.16	0.17	0.22
	Model	0.11	0.04	0.12	0.15
u:	US Data	0.23	0.18	0.27	0.32
	Model	0.16	0.05	0.25	0.27
w:	US Data	0.015	0.026	0.019	0.018
	Model	0.0123	0.0127	0.0123	0.0122

US data: Standard deviation, CPS Monthly data, Men, 1976Q1-2013Q1
HP filter with smoothing parameter 10^5. Authors' calculations.

Table 7 also reports the second order moments, per age class, implied by the model and those computed using our data set. The main result is that the model predicts that the volatilities of the labor market are age-increasing (stock and flows), as in the data. This result shows that the parameter restrictions imposed by the first order moments are consistent with age-increasing elasticities. Thus, changes in search value dominate all the other components in the expected job surplus, including the exogenous age-increasing labor productivity introduced to match life-cycle pattern of wages. This leads young and prime-age workers to smooth the business cycle because their outside options are also affected by aggregate shocks. In contrast, older workers, without any future in the labor market, do not have any outside option and thus are very responsive to the business cycle.

The second result is that the fit of the model is very good with respect to the prime-age and the older workers. Indeed, for prime-age and older workers, the model is able to approximatively match the volatilities per age of unemployment rates and job separations rates, but it slightly underestimates the volatilities of the job finding rates for these two age groups. These good results come from the endogenous increase in the outside options with worker's age: the horizon effect reduces the search value and thus leads older workers to mimic Hagendorn & Manovskii (2008)'s infinitely lived agents. One can notice that these results are obtained with a model that predicts a volatility of the real wage of the same order of magnitude as the observed one, which cannot be the case with a model with rigid real wage. Moreover, as in the data, the theory predicts a slight decrease in wage volatility at the end of the life-cycle. This suggests that the impact of changes in the job composition ($\widehat{\mathcal{G}}_i$ in section 5.4) over the business cycle are small enough to be negligible in our understanding of real wage fluctuations.

Thirdly, business cycle implications with respect to the youth labor market are not satisfying. Indeed, the predicted volatilities of unemployment and job finding rate are 4 times lower than their observed counterparts. Only the dynamics of the job separation rate is approximatively matched. Thus, our assumptions concerning the specificities of this labor market allows us to match the level of the workers flow and the unemployment rate, but not the elasticity to the business cycle.[39] This mainly comes from the large discrepancy between the first order moments of young and prime-age workers, which leads to strong differences in age-specific parameter values. These calibration restrictions are not compatible with business cycle properties for the young workers. Something is missing in the DMP model to explain flucutations in this labor market, for example participation decisions.

The simulations results are encouraging and support the view that adding a horizon effect in the DMP model is relevant to explain a large part of the differences between prime-age and older workers along the business cycle, even if our structural specificities of the youth labor market are not completely convincing. But the main result is that the MP is able to explain why older workers are more sensitive to business cycle than their younger counterparts.

6.3 Sensitivity analysis

In this section, we simulate the model without any exogenous heterogeneity, except worker's age in order to evaluate the robustness of the thesis of the paper: the horizon effect accounts for the age-increasing pattern of volatilities. Indeed, in the theoretical part of the paper, only horizon heterogeneity matters: we then discuss the case of a "pure" horizon effect. Nevertheless, in the quantitative analysis, the objective to match the average level of the data per age leads us to introduce two exogenous components in our model: firstly, an age-increasing human capital in order to match the age-increasing pattern of the real wage and secondly composition and mismatch effects specific to young workers. Hence, in a first experiment, we simulate the model under the assumption that workers' labor productivity are homogenous across ages: this implies $h_i = h$ and $\mu_i = 0$, $\forall i$. This experience allows to measure the impact of the changes of the human capital on the elasticities per age of the business cycle. Remark that this model is exactly the one presented and discussed in the theoretical part of the paper: only the sufficient sources of heterogeneity are introduced if we want to explain the age-decreasing volatilities of labor market quantities, without any consideration on the model implications with respect of the steady state properties on wages. In a second experience, we simulate the model without any exogenous age heterogeneity ($h_i = h$,

[39]We have tried an alternative identifying assumption, without any success. See appendix C.6.

$\mu_i = 0$ and $\lambda_i = \lambda$, $\forall i$): only the horizon effect induces a endogenous age heterogeneity. This model can only measure the magnitude of the "pure" horizon effect in the explanation of the gaps between prime-age and older workers. Finally, we propose a calibration where the calibration of the home production is lower than the one estimated by Hall & Milgrom (2008). This allows us to test the persistence of the horizon effect even in a economy where the magnitude of labor market fluctuations is largely underestimated.

Simulation results are reported in table 8. Firstly, all these simulations display a age-increasing

Table 8: Second order moments: Sensitivity analysis

		All: 16-61	Y: 16-24	A: 25-54	O: 55-61
JSR:	Benchmark	0.10	0.07	0.17	0.19
	h & μ	0.15	0.14	0.22	0.26
	h & μ & λ	0.23	0.22	0.22	0.24
	Low b	0.006	0.002	0.018	0.024
JFR:	Benchmark	0.11	0.04	0.12	0.15
	h & μ	0.11	0.05	0.12	0.15
	h & μ & λ	0.13	0.12	0.12	0.14
	Low b	0.04	0.03	0.06	0.07
u:	Benchmark	0.16	0.05	0.25	0.27
	h & μ	0.18	0.104	0.28	0.32
	h & μ & λ	0.27	0.22	0.27	0.29
	Low b	0.04	0.025	0.078	0.081

h & μ: Model with homogenous h & μ: $h_i = h_A$ and $\mu_Y = 0$

h & μ & λ: Model with homogenous h & μ & λ: $h_i = h_A$, $\mu_Y = 0$ and $\lambda_i = \lambda_A$

Model with low b: $b_i = rh_i$ such that $r - \sum_i m_i \phi(e_i^{ss}) \approx 0.5$

pattern of the volatilities. Since this effect persists even if no additional heterogeneity is introduced, our theoretical analysis focus then on the main force at work in the data.

Secondly, if we focus only on the gap between prime-age and older workers, it appears that the homogeneity of human capital among these two age groups allows to increase the volatilities generated by the model, in particular the cyclicalities of unemployment and of job separation rate. Indeed, the reservation productivity is perfectly indexed on the age-specific human capital, whereas its counterparts, in particular the search returns Σ, are under-indexed on the age-specific human capital. Thus, when the human capital is homogenous among ages, the countercyclical weight of the outside option is relatively less valuated than the current profit. This leads to an increase in the job separation rate volatility and then in unemployment rate.[40] The experience where human capital

[40]This under-indexation phenomena of the "reservation wage" is less important for the job finding rate dynamics because the weight of the outside options is lower than for the productivity reservation dynamics, see the log-linearized

and λ are homogenous shows that the decline in λ at the end of the life cycle leads to a reduction in the gap between the data and simulated volatilities. Indeed, this calibration moderates the weight of labor hoarding at the end of the life-cycle, this component that dampens the smoothing effect from the reservation wage effect.

Thirdly, as for the youth labor market, our additional features, composition and mismatch effects, go in the right direction: when these age-specific components are omitted, the model largely overestimates the cyclicality of this labor market which is significantly lower than on the other labor markets. Nevertheless, the estimated weight of these two age-specific features, induced by the fit of the first order moments, is larger than their optimal weight with respect to the 2nd order moments. A gap between the steady state and the cyclical properties must be introduced, though e.g. a more complex diffusion process of learning on young workers.

Finally, and as usual, the decrease in the value of the home production leads to a large decline of the predicted volatilities. Nevertheless, our model can predict the age-increase in the cyclicality, underlying that the horizon effect is robust to the level of variances.

7 Conclusion

In this paper, we contribute to the existing literature along two dimensions. First, we document business cycle fluctuations in worker flows across age groups in the US. While previous papers focused on differences in *average* transition rates across age groups, we extend the current literature by looking at the age profile of *volatilities* in workers' transition rates for young, prime-age and old workers on CPS data. We find that old worker's are characterized by a higher responsiveness to business cycles than their younger counterparts. We perform several checks to ensure that this is a robust stylized fact. Thus, we show that the brunt of the recession is borne by older workers. The analysis of the hourly wage dynamics, using monthly data, shows that the cyclicality is age-decreasing. Then, the data presents a consistent picture of labor market adjustments: the market with the most rigid price adjusts largely through quantities. This is the case for older workers.

Secondly, we propose a life-cycle Mortensen & Pissarides (1994) model with age-directed search. First, we show that old workers' short horizon can quantitatively explain the observed gap between their job finding and separations rates and the ones for the prime-age workers. Older workers's shorter horizon endogenously reduces their outside options, thereby leading their wages to be less sensitive to the business cycle. Thus, in a market where wage adjustments are small, quantities vary a lot: this the case for older workers, whereas the youngest behave like infinitively-lived agents. Older workers do not smooth the impact of aggregate shocks through an expected low returns on search, because they have no future (a short horizon reduces to zero the option value of search). We point out that older workers look like Hagendorn & Manovskii (2008)'s workers, but, contrary

systems (JC) and (JD)

to the proposition of these authors, we show that it is not necessary to calibrate the value of home production at a larger value than the one proposed by Hagendorn & Manovskii (2008) in order to match volatilities in workers flows. The important point is that, unlike Hagendorn & Manovskii (2008), we preserve the Hosios condition and we endogenously obtain varying changes in the search value in the wage dynamics through another channel than the one proposed by Hall & Milgrom (2008). Indeed, with aging, the returns on search is nil for older workers, which reduces the elasticity of wages and raises the responsiveness of worker flows. Aging then endogenously leads average flows to look like Hagendorn & Manovskii (2008)'s workers. Our results support Pissarides (2009)'s view that it is not necessary to introduce exogenous wage rigidity in order to reconcile the DMP model with the data. We show that this mechanism allows us to explain why older workers are more sensitive to the business cycle than the other workers.

At the margin of this main objective of the paper, we also try to explain why young workers are less sensitive to the business cycle than the other workers. We show that their short experience reduces their opportunities to improve their match-specific productivity (mismatch effect) whereas a composition effect leads to select the closest to the work (composition effect), are needed to explain the observed gap between their job finding and separations rates and the ones for the prime-age workers.

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A Stylized facts on worker flows

A.1 HP-filtering with $\lambda = 10^5$

This appendix displays Figures 4, 5 and 6 mentioned in section 2.1.

Figure 4: Job Separation Rate by age group, JSR, HP-filtered, Men, 1976Q1 - 2013Q1. Authors' calculations. Recession in shaded area.

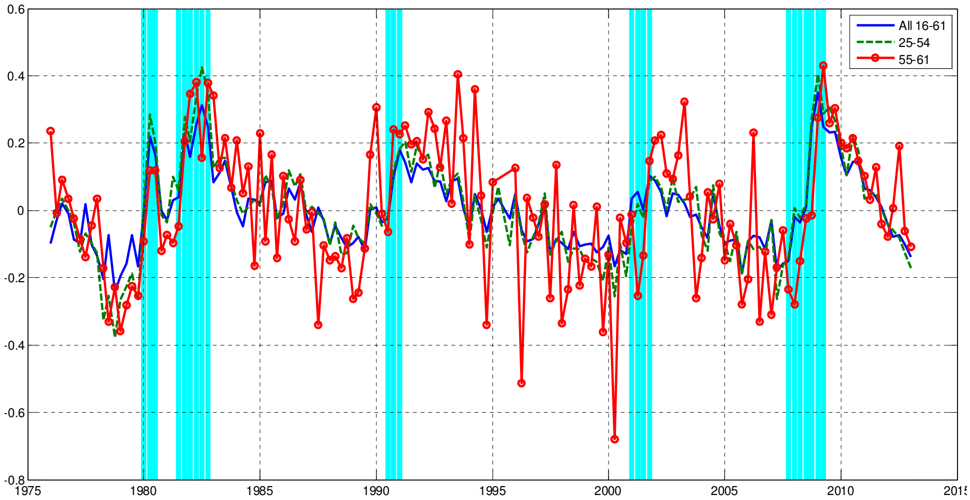
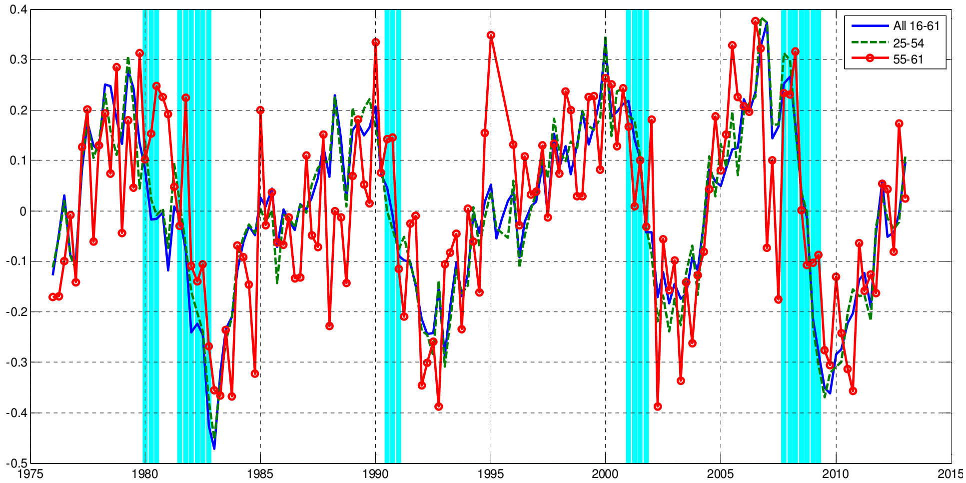


Figure 5: Job Finding Rate by age group, JFR, HP-filtered, Men, 1976Q1 - 2013Q1. Authors' calculations. Recession in shaded area.



A.2 HP-filtering with $\lambda = 1600$

Table 9 reports business cycle facts when the smoothing parameter is 1600 rather than 10^5. The age-increasing pattern in volatility is robust.

Figure 6: w, Monthly CPS, Men, Quarterly averages, 1995Q1-2013Q1. Authors' calculations.

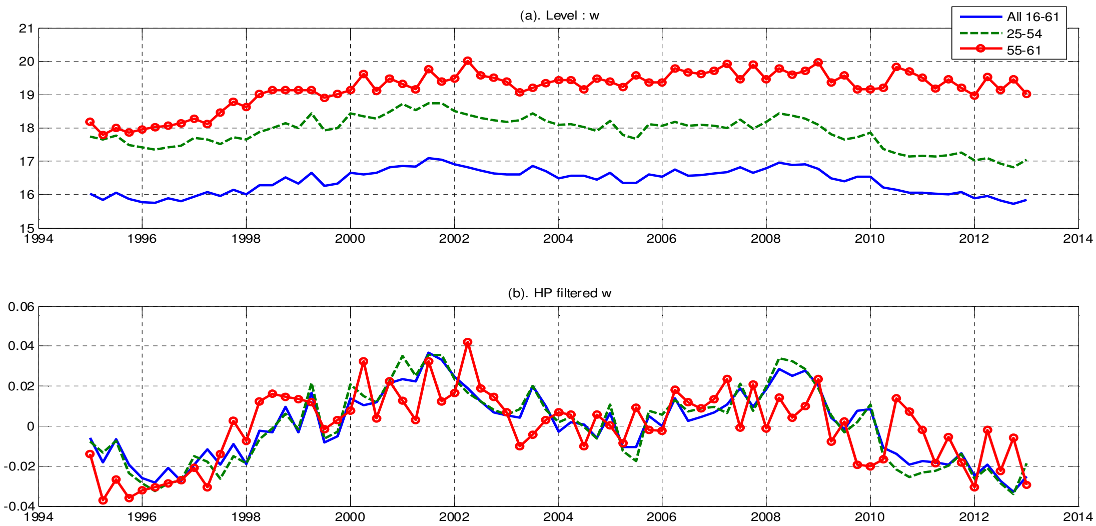


Table 9: Standard deviation. CPS data, quarterly averages of monthly instantaneous transition rates,1976Q1-2013Q1, Men, HP filter with smoothing parameter 1600. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.083638	0.077361	0.10856	0.17571
		0.7126	1	1.6186
JFR	0.11848	0.1219	0.12439	0.18647
		0.98004	1	1.4991
u	0.16876	0.13661	0.19427	0.25264
		0.7032	1	1.3004

A.3 Employment states are employment, unemployment and inactivity

Using Shimer (2012)'s methodology for 3 employment states (Employment, Unemployment and Inactivity), on CPS data for Men, we get results reported in Tables 10 and 11. With 3 employment states, the steady state unemployment includes all transitions rates, including those involving inactivity. Tables 10 and 11 suggest that our business cycle facts across age group remain robust when separations and findings are purged from transition to and from inactivity. The level of exit from employment as well as the job finding rate fall with age with their volatility increases with age.

When we decompose unemployment fluctuations using β computations as in Shimer (2012), based on hypothetical unemployment rates. We find that the transitions between unemployment and unemployment account for 75% of unemployment fluctuations between 1976Q1 and 2013Q1. [41]

[41]We compute counterfactual steady states predicted by time varying finding and separation rates, while other

Table 10: Mean. Monthly CPS data, 3 states (Employment, Unemployment, Inactivity), 1976 June - 2012 Sept, Men. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.021973	0.0498	0.01774	0.012265
		2.8073	1	0.69139
JFR	0.38674	0.41812	0.38077	0.30457
		1.0981	1	0.79988
u	0.061169	0.12544	0.048787	0.044766
		2.5712	1	0.91759
EI	0.020969	0.060632	0.00908	0.017287
UI	0.24631	0.38836	0.17323	0.2194
IE	0.050005	0.1067	0.089367	0.039833
IU	0.045497	0.12421	0.097117	0.026462

Table 11: Standard deviation. Monthly CPS data, 3 states (Employment, Unemployment, Inactivity), 1976 June - 2012 Sept, Men. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.10656	0.095019	0.14095	0.19785
		0.67413	1	1.4037
JFR	0.15555	0.15902	0.1637	0.21629
		0.97144	1	1.3213
u	0.21213	0.15902	0.25506	0.29212
		0.62344	1	1.1453
EI	0.055755	0.073621	0.074905	0.10924
UI	0.13217	0.098655	0.15932	0.23929
IE	0.07725	0.11697	0.10092	0.12961
IU	0.094497	0.09714	0.12326	0.23116

A.4 Employment and Unemployment for all workers

Using Shimer (2012)'s methodology for 2 employment states (Employment, Unemployment), on CPS data for Men and women, we get results reported in Tables 12 and 13. The main stylized facts remain relevant : the mean transition rates fall with age while their volatility increases with age.

Table 12: Mean. Monthly CPS data, Employment and Unemployment, 1976Q1 - 2013Q1, Men and Women. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.018822	0.04208	0.015499	0.010614
		2.715	1	0.68479
JFR	0.42647	0.49789	0.40141	0.33929
		1.2404	1	0.84524
u	0.044507	0.080907	0.039397	0.03252
		2.0536	1	0.82546

Table 13: Standard deviation. Monthly CPS data, Employment and Unemployment, 1976Q1 - 2013Q1, Men and Women, HP filter with smoothing parameter 10^5. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.086295	0.069806	0.11134	0.16103
		0.62694	1	1.4463
JFR	0.15671	0.14693	0.16161	0.20154
		0.90912	1	1.247
u	0.21221	0.16513	0.2388	0.28343
		0.6915	1	1.1869

A.5 Robustness check on Elsby et al. (2010)'s data

We use the data for Figure 8 of their paper. Notice that there are several differences with our computations. First, they include men and women. Their numbers shall then be compared with our results in Appendix A.4. Secondly, Elsby et al. (2010) use Shimer (2012)'s formula based on stocks of unemployed and employed workers (in which separations are proxied by short-term unemployment) rather than disaggregated data in our case. Their approach yields higher levels of transition rates. Finally, old workers are 55 years old and more while we restrict our sample to workers prior to retirement (55-61). We compute business cycle statistics on their database. Since

transition rates are set at their historical mean. We log and HP-filter the time series using a smoothing parameter of 10^5 and compute the variance decomposition of the cyclical component of steady state unemployment based on βs. We find that $\beta^{EU} + \beta^{UE} = 0.7502$

the time series are quarterly, the smoothing parameter on the HP filter is 1600. The results are reported in tables 14 and 15. We find that the mean of transition rates fall with age while the standard deviation increases with age.

Table 14: Mean. Elsby et al. (2010) data, 1977Q2 - 2009Q4, Quarterly data, Men and Women.

	All: 16+	Young: 16-24	Prime-age: 25-54	Old: 55+
JSR	0.03527	0.10047	0.023898	0.015542
		4.2043	1	0.65034
JFR	0.54514	0.7111	0.46652	0.43876
		1.5243	1	0.94049

Table 15: Standard deviation. HP-filtered, smoothing parameter 10^5, 1977Q2 - 2009Q4, Quarterly data, , Men and Women. Elsby et al. (2010) data.

	All: 16+	Young: 16-24	Prime-age: 25-54	Old: 55+
JSR	0.044485	0.046366	0.060651	0.092973
		0.76447	1	1.5329
JFR	0.10627	0.095022	0.113	0.15079
		0.8409	1	1.3344

A.6 Rolling standard deviation

In this appendix, we present rolling standard deviations computed over a 10-year window, using the same dataset as in the main text (CPS data, Men, 2 states (E, U), Log HP-filtered quarterly time series). Results are displayed in figures 7 and 8. The window is 10 years meaning that each point in the graph provides a measure of volatility over the 10 previous years.

Panel (a) of each figure displays the volatility of each age group while panel (b) provides this measure relative to the volatility of prime-aged workers. Figures 7 and 8 confirm that the responsiveness of old workers to business cycles is larger than the ones found for their younger counterparts.

A.7 Taking into account the change in age composition

In this section, we present a simple decomposition exercise to get a sense of how much of our results on aggregate data is attributable to the change in the age distribution of the workforce. In order to do so, as in Jaimovich & Siu (2009), we first assess the age structure of the workforce using OECD Labour Force Statistics annual database on unemployed and employed populations.

Figure 7: Rolling standard deviation on separation rates by age group, 10 year window, Log HP-filtered Monthly CPS data, Men. Recession in shaded area.

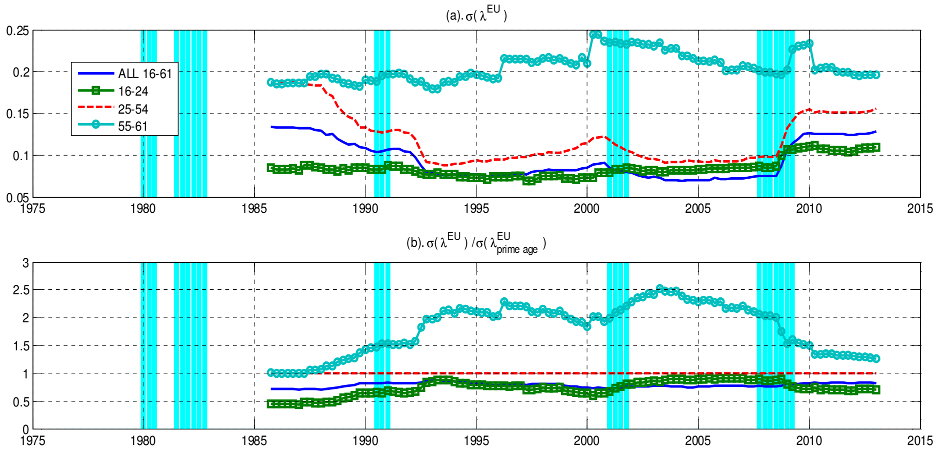
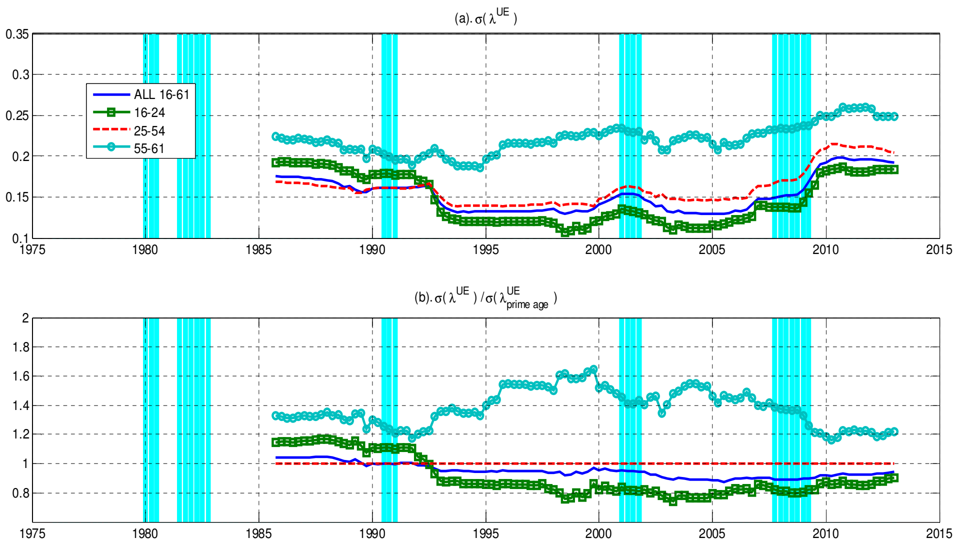


Figure 8: Rolling standard deviation on finding rates by age group, 10 year window, Log HP-filtered Monthly CPS data, Men. Recession in shaded area.



At any point in time t, the aggregate JSR_t is

$$\begin{aligned}
JSR_t &= \frac{EU_{Yt} + EU_{At} + EU_{Ot}}{E_t} \\
JSR_t &= \frac{EU_{Yt}}{E_{Yt}}\frac{E_{Yt}}{E_t} + \frac{EU_{At}}{E_{At}}\frac{E_{At}}{E_t} + \frac{EU_{Ot}}{E_{Ot}}\frac{E_{Ot}}{E_t} \\
JSR_t &= JFR_{Yt}\frac{E_{Yt}}{E_t} + JSR_{At}\frac{E_{At}}{E_t} + JSR_{Ot}\frac{E_{Ot}}{E_t} \qquad (22)
\end{aligned}$$

with E the total employment stock ($E = E_Y + E_A + E_O$), and EU_i the number of transitions from employment to unemployment for workers of age i at time t. Equation (22) states that the aggregate JSR is a weighted average of JSR by age, the weights being given by each age group share in total employment. To isolate the effect due purely to the change in composition, we construct counterfactual JSR series that holds the age structure fixed at the values observed either at the beginning

$$\widetilde{JSR}_{1976,t} = JFR_{Yt}\frac{E_{Y,1976}}{E_{1976}} + JSR_{At}\frac{E_{A,1976}}{E_{1976}} + JSR_{Ot}\frac{E_{O,1976}}{E_{1976}}$$

or at the end of the sample.

$$\widetilde{JSR}_{2012,t} = JFR_{Yt}\frac{E_{Y,2012}}{E_{2012}} + JSR_{At}\frac{E_{A,2012}}{E_{2012}} + JSR_{Ot}\frac{E_{O,2012}}{E_{2012}}$$

Doing this for every month generates counterfactual time series $\widetilde{JSR}_{1976,t}$ and $\widetilde{JSR}_{2012,t}$. The OCDE data are consistent with population aging: $\frac{E_{Y,1976}}{E_{1976}} = 22.27\%$, $\frac{E_{Y,2012}}{E_{2012}} = 13.65\%$ and $\frac{E_{O,1976}}{E_{1976}} = 8.3\%$, $\frac{E_{O,2012}}{E_{2012}} = 11\%$.

We compare the standard deviation of filtered counterfactual JSR with the earlier and later demographic structure. As expected, the counterfactual time series with the larger share of young people $\widetilde{JSR}_{1976}$ is characterized by a larger sample mean (13% higher) and a lower standard deviation after HP-filtering with smoothing parameter of 10^5 (10% lower).

A similar decomposition is performed for the JFR where the weights are given by each age group share in total unemployment. The OCDE data are consistent with population aging: $\frac{U_{Y,1976}}{U_{1976}} = 50\%$, $\frac{U_{Y,2012}}{U_{2012}} = 31.5\%$ and $\frac{U_{O,1976}}{U_{1976}} = 4.4\%$, $\frac{U_{O,2012}}{U_{2012}} = 8\%$. As expected from tables 1 and 2, the counterfactual time series with the larger share of young people $\widetilde{JFR}_{1976}$ is characterized by a larger sample mean (4% higher) and a lower standard deviation after HP-filtering with smoothing parameter of 10^5 (3% lower).

The effects might not seem larger. In our view, this is due to the age groups chosen in our analysis. The demographic changes have affected the group of prime age workers that also include workers in their early 50s.

A.8 Employment and Unemployment by education

The data per age can mix an age effect and a skill effect. Indeed, the older workers are on average less educated than the younger workers. Thus, our age effect can be a skill effect. In order to deal with this identification problem, we propose to separate in two skill groups our population: the labor market participants with an high school diploma and less, and those having a higher education (More than high school).

Table 16: Mean. Monthly CPS data, Employment and Unemployment, 1976Q1 - 2013Q1, Men. Authors' calculations.

	High school and less			More than high school		
	All: 16+	16-54	Old: 55+	All: 16+	16-54	Old: 55+
JSR	0.034297	0.036999	0.016455	0.014686	0.015344	0.0085209
		1	0.44473		1	0.55534
JFR	0.43205	0.43819	0.35042	0.41524	0.42485	0.30959
		1	0.7997		1	0.72871
u	0.077717	0.082009	0.050573	0.03659	0.037343	0.029889
		1	0.61667		1	0.80039

Table 17: Standard deviation. Employment and Unemployment, Monthly CPS data, 1976Q1 - 2013Q1, Men, HP-filtered, smoothing parameter 10^5, Authors' calculations.

	High school and less			More than high school		
	All: 16+	16-54	Old: 55+	All: 16+	16-54	Old: 55+
JSR	0.13485	0.14002	0.24422	0.14218	0.14512	0.27525
		1	1.7441		1	1.8967
JFR	0.16708	0.16673	0.27954	0.17743	0.17878	0.28459
		1	1.6766		1	1.5919
u	0.23679	0.23546	0.36584	0.2777	0.27982	0.38679
		1	1.5537		1	1.3823

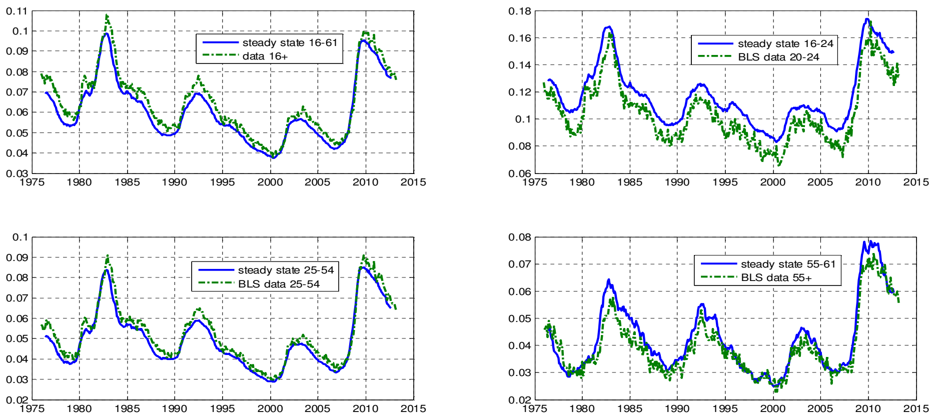
After controlling for education, levels are age-decreasing and volatilities are age-increasing. Thus, our stylized facts account for a phenomena liked to workers' age, and is not the result of a composition effect.

A.9 Steady state unemployment rate and actual unemployment rate

Figure 9 displays the steady state unemployment found when using monthly CPS data for men and women in a 3 state labor market à la Shimer (2012) and the corresponding BLS unemployment rates.

Notice that the age groups are not exactly the same. Fluctuations in steady state unemployment closely match those found in the data.

Figure 9: Steady state unemployment by age and BLS unemployment



B Computing male real hourly wage by age

In CPS data, questions on wages are asked only of about one-quarter of the entire sample on the Outgoing Rotation Group. Monthly data on weekly earnings of wage and salary workers are collected. All self-employed persons are excluded, regardless of whether their businesses are incorporated. Data represent earnings before taxes and other deductions and include any overtime pay, commissions, or tips usually received. Hourly wage is computed as the ratio of usual weekly earnings to usual weekly hours. We restrict our sample to male workers in 4 age groups : 16-24 (young), 25-54 (prime age) and 55-61 (old) and all age groups (16-61). We delete outliers: individuals whose hourly wage is greater than 250 dollars or lower than half the net minimum wage prevailing during the year the data is collected. We also delete young workers working more than 45 hours per week. These raw time series are detrended to take into account inflation and technological progress. In order to do so, we use the time series of male weekly earnings for all age groups. This increasing time series captures the growing trend in aggregate productivity and inflation. We compute the trend on this time-series and use it to deflate the raw hourly wage time series for each age group. After applying the x12 method to correct for seasonality, we take quarterly averages of these monthly time series, log them and extract the cyclical component using the HP filter (10^5). We check that levels of real hourly wages are consistent with BLS weekly earnings by age (Median usual weekly earnings, LEU0252881800, LEU0252882100, LEU0252888100, LEU0252891100) as well as Heathcote et al. (2010)'s data. As for business cycle features on our real hourly wage, their volatility is in line with those found by Jaimovich et al. (2013) on annual CPS data.

C Model

In order to save space, we directly present here the model including young workers ($i = Y$). The markov age process is

$$\Pi = \begin{bmatrix} \pi_Y & 1-\pi_Y & 0 & 0 & \cdots & 0 \\ 0 & \pi_A & 1-\pi_A & 0 & \cdots & 0 \\ 0 & 0 & \pi_{O_1} & 1-\pi_{O_1} & \cdots & 0 \\ 0 & 0 & 0 & \ddots & \ddots & 0 \\ 1-\pi_{O_7} & 0 & 0 & 0 & \cdots & \pi_{O_7} \end{bmatrix}$$

C.1 Stock-flow dynamics

C.1.1 Levels of unemployment and employment

The mass of age-i workers employed during the month t in a firm such that $\tau \in [R_{i,t}, x]$, is given by

$$n_i(z,x) = \int_{R_i(z)}^{x} \mu(\tau)d\tau$$

where $\mu(\tau)$ the mass of firms with a productivity z. This stock of jobs evolves as follows:

$$\begin{aligned} & \text{If } i = Y \\ n_Y(z',x) \;=\; & \pi_Y \begin{bmatrix} [(1-s_e)\lambda(m_Y - u_Y(z)) + e_Y(z)p(\theta_Y(z))u_Y(z)][G(x) - G(R_Y(z'))] \\ +(1-s_e)(1-\lambda)[n_Y(z,x) - n_Y(z,R_Y(z'))] \end{bmatrix} \end{aligned} \tag{23}$$

$$\begin{aligned} & \text{If } i \neq Y \\ n_i(z',x) \;=\; & \pi_i \begin{bmatrix} [(1-s_e)\lambda(m_i - u_i(z)) + e_i(z)p(\theta_i(z))u_i(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda)[n_i(z,x) - n_i(z,R_i(z'))] \end{bmatrix} \\ & +(1-\pi_{i-1}) \begin{bmatrix} [(1-s_e)\lambda(m_{i-1} - u_{i-1}(z)) + e_i(z)p(\theta_i(z))u_{i-1}(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda)[n_{i-1}(z,x) - n_{i-1}(z,R_i(z'))] \end{bmatrix} \end{aligned} \tag{24}$$

where, as in Hairault et al. (2010), we assume that when worker ages (from $i-1$ to i), his job contact probability ($e_i(z)p(\theta_i(z))$), and his reservation productivity $R_i(z)$ are those of a worker of the age i.

C.1.2 Unemployment and employment rates

The dynamics of the unemployment rates by age are given by:

$$u_i(z) = m_i - n_i(z,1) \Leftrightarrow \underbrace{\frac{u_i(z)}{m_i}}_{\equiv u_i^r(z)} = 1 - \underbrace{\frac{n_i(z,1)}{m_i}}_{\equiv n_i^r(z,1)} \quad \forall i, z$$

The dynamics of the employment rate is given by

$$
\begin{aligned}
& \text{If } i = Y \\
n_Y^r(z', x) &= \pi_Y \left[\begin{array}{l} [(1-s_e)\lambda(1-u_Y^r(z)) + e_Y(z)p(\theta_Y(z))u_Y^r(z)][G(x) - G(R_Y(z'))] \\ +(1-s_e)(1-\lambda)[n_Y^r(z,x) - n_Y^r(z, R_Y(z'))] \end{array} \right] \qquad (25) \\
& \text{If } i \neq y \\
n_i^r(z', x) &= \pi_i \left[\begin{array}{l} [(1-s_e)\lambda(1-u_i^r(z)) + e_i(z)p(\theta_i(z))u_i^r(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda)[n_i^r(z,x) - n_i^r(z, R_i(z'))] \end{array} \right] \\
& +(1-\pi_{i-1})\frac{m_{i-1}}{m_i} \left[\begin{array}{l} [(1-s_e)\lambda(1-u_{i-1}^r(z)) + e_i(z)p(\theta_i(z))u_{i-1}^r(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda)[n_{i-1}^r(z,x) - n_{i-1}^r(z, R_i(z'))] \end{array} \right] \qquad (26)
\end{aligned}
$$

Thus, given the definition of $n_i^r(z,x)$ (equations (25) and (26)), $G(1) = 1$ and $u_i^r(z) = 1 - n_i^r(z,1)$, we obtain

$$
\begin{aligned}
& \text{If } i = Y \\
u_Y^r(z') &= \pi_Y \left[\begin{array}{l} [1 - e_Y(z)p(\theta_Y(z))(1 - G(R_Y(z')))]u_Y^r(z) \\ +(1-s_e)(1-\lambda)n_Y^r(z, R_1(z')) \\ +[s_e + (1-s_e)\lambda G(R_Y(z'))](1 - u_Y(z)) \end{array} \right] \\
& +(1-\pi_{O_7 O_7})\frac{m_{O_7}}{m_Y} \qquad (27) \\
& \text{If } i \neq Y \\
u_i^r(z') &= \pi_i \left[\begin{array}{l} [1 - e_i(z)p(\theta_i(z))(1 - G(R_i(z')))]u_i^r(z) \\ +(1-s_e)(1-\lambda)n_i^r(z, R_i(z')) \\ +[s_e + (1-s_e)\lambda G(R_i(z'))](1 - u_i^r(z)) \end{array} \right] \\
& +(1-\pi_{i-1})\frac{m_{i-1}}{m_i} \left[\begin{array}{l} [1 - e_i(z)p(\theta_i(z))(1 - G(R_i(z')))]u_{i-1}^r(z) \\ +(1-s_e)(1-\lambda)n_{i-1}^r(z, R_i(z')) \\ +[s_e + (1-s_e)\lambda G(R_i(z))](1 - u_{i-1}^r(z)) \end{array} \right] \qquad (28)
\end{aligned}
$$

Unemployed workers of age i in month $t+1$ are those of age i in month t who do not age, and

- who do not find a job (first term of the first line of the right-hand side of equations (27) and (28)),
- employed workers of age i who loose their job in week $t+1$ due to a change in aggregate productivity leading to a change in the reservation productivity[42] (second term of the first line),
- the age-i employed workers who loose their jobs due to a separation, which can result from an exogenous reason with a probability s_e and from endogenous decisions with a probability $(1-s_e)\lambda G(R_i(z'))$ (first term of the second line),

[42]When $R_i(z) < R_i(z')$, the mass of obsolete jobs depends on the job creations over the past. Obviously, if $R_i(z) > R_i(z')$, these jobs do not exist.

- and the new participants (last term of the second line).

Due to the aging, in these age-i unemployed, there is the mass of the age-$i-1$ unemployed who age without finding a job (the two last lines of (28), which are composed by the same flows than the two first, except that the age of the agents is not the same). Finally, the second line of the equation (27) shows that newly born agents enter in the labor market as unemployed workers. Note that the unemployment dynamics is a function of $n_i(z, R_i(z'))$ and $n_{i-1}(z, R_i(z'))$, which are themselves function of the past values if the unemployment.[43] This underlines the interdependence of the age-i unemployment stock to the unemployment level at previous age. The average unemployment rate is $u^r_t = \sum_{i=1}^{T} u_{i,t}$.

C.1.3 Transition rates

The job finding rate (JFR) and the job separation rate (JSR) are respectively:

$$\begin{aligned} JFR_i(z) &= \frac{e_i(z)p(\theta_i(z))(1-G(R_i(z'))\left[\pi_i u^r_i(z) + (1-\pi_{i-1})\frac{m_{i-1}}{m_i}u^r_{i-1}(z)\right]}{u^r_i(z)} \\ JSR_i(z) &= \frac{(1-s_e)(1-\lambda)\left[\pi_i n^r_i(z,R_i(z')) + (1-\pi_{i-1})\frac{m_{i-1}}{m_i}n^r_{i-1}(z,R_i(z'))\right]}{n^r_i(z,1)} \\ &\quad + \frac{[s_e+(1-s_e)\lambda G(R_i(z'))]\left[\pi_i(1-u^r_i(z)) + (1-\pi_{i-1})\frac{m_{i-1}}{m_i}(1-u^r_{i-1}(z))\right]}{n^r_i(z,1)} \end{aligned}$$

In the basic infinite horizon model, we have $\pi_i = 1, \forall i$, $m_i = 1$, and $n_i(z, R_i(z')) = n_{i-1}(z, R_i(z')) = 0$ leading to $JFR(z) = e(z)p(\theta(z))(1-G(R(z'))$ and $JSR(z) = s_e + (1-s_e)\lambda G(R(z'))$. These definitions of worker flows have an empirical counterpart and are used by Fujita & Ramey (2012) to test the ability of the MP model to match labor market features. In the data, it is only possible to detect the worker age before a transition. Thus, we compute the transition rate conditionally on being of a given age prior to the transition. In this case, all workers have "the same" age in our measure of the transition rates by age. The counterparts in the model are:

$$\begin{aligned} JFR_i(z) &= e_i(z)p(\theta_i(z))[1-G(R_i(z'))] \\ JSR_i(z) &= \frac{(1-s_e)(1-\lambda)n^r_i(z,R_i(z')) + [s_e+(1-s_e)\lambda G(R_i(z'))]n^r_i(z,1)}{n^r_i(z,1)} \end{aligned}$$

[43] Indeed, given the equation (26), we deduce $n^r_i(z, R_i(z'))$, where z^- denotes the realization of the aggregate shock at the previous period:

$$\begin{aligned} n^r_i(z,R_i(z')) &= \pi_i \left[\begin{array}{l} [(1-s_e)\lambda(1-u^r_i(z^-)) + e_i(z^-)p(\theta_i(z^-))u^r_i(z^-)][G(R_i(z')) - G(R_i(z))] \\ +(1-s_e)(1-\lambda)[n^r_i(z^-,R_i(z')) - n^r_i(z^-,R_i(z))] \end{array} \right] \\ &\quad +(1-\pi_{i-1})\frac{m_{i-1}}{m_i} \left[\begin{array}{l} [(1-s_e)\lambda(1-u^r_{i-1}(z^-)) + e_i(z^-)p(\theta_i(z^-))u^r_{i-1}(z^-)][G(R_i(z')) - G(R_i(z))] \\ +(1-s_e)(1-\lambda)[n^r_{i-1}(z^-,R_i(z')) - n^r_{i-1}(z^-,R_i(z))] \end{array} \right] \end{aligned}$$

and we have the same expression for $n^r_{i-1}(z, R_i(z'))$.

where $n_i^r(z,1) = 1 - u_i^r(z)$. We use this usual approximation of the worker flows per age in order to measure the ability of the theory to explain the observed data, computed using the same formula.

C.2 Steady state surplus

The surplus function is defined by

$$S_i(z,\epsilon) = \max\left\{ \begin{array}{l} z(\epsilon - R_i(z)) + \beta\pi_i(1-\lambda_i)(1-s_e)E_z[S_i(z',\epsilon) - S_i(z',R_i(z))] \\ +\beta(1-\pi_i)(1-\lambda_{i+1})(1-s_e)E_z[S_{i+1}(z',\epsilon) - S_{i+1}(z',R_i(z))] \end{array} ;0 \right\}$$

Thus, at age $i+1$ and for $\epsilon = R_i(z)$, we have, at the conditional steady state

$$S_{i+1}(z,R_i(z)) = \max\left\{ \begin{array}{l} z(R_i(z) - R_{i+1}(z)) + \beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)S_{i+1}(z,R_i(z)) \\ +\beta(1-\pi_{i+1})(1-\lambda_{i+2})(1-s_e)[S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z))] \end{array} ;0 \right\}$$

Assuming that $S_{i+1}(z,R_i(z)) > 0$, we obtain:

$$\begin{aligned} S_{i+1}(z,R_i(z)) &= \frac{z(R_i(z) - R_{i+1}(z))}{1 - \beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)} \\ &\quad + \frac{\beta(1-\pi_{i+1})(1-\lambda_{i+2})(1-s_e)}{1 - \beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)}[S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z))] \end{aligned}$$

For age $i+2$, we then have

$$\begin{aligned} S_{i+2}(z,R_i(z)) &= \frac{z(R_i(z) - R_{i+2}(z))}{1 - \beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)} \\ &\quad + \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1 - \beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_i(z)) - S_{i+3}(z,R_{i+2}(z))] \\ S_{i+2}(z,R_{i+1}(z)) &= \frac{z(R_{i+1}(z) - R_{i+2}(z))}{1 - \beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)} \\ &\quad + \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1 - \beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_{i+1}(z)) - S_{i+3}(z,R_{i+2}(z))] \end{aligned}$$

We deduce the value for $S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z))$, which is

$$\begin{aligned} S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z)) &= \frac{z(R_i(z) - R_{i+1}(z))}{1 - \beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)} \\ &\quad + \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1 - \beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_i(z)) - S_{i+3}(z,R_{i+1}(z))] \end{aligned}$$

Introducing this result in the expression of $S_{i+1}(z,R_i(z))$, we obtain,

$$\begin{aligned} S_{i+1}(z,R_i(z)) &= \frac{z(R_i(z) - R_{i+1}(z))}{1 - \beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)} \\ &\quad + \frac{\beta(1-\pi_{i+1})(1-\lambda_{i+2})(1-s_e)}{1 - \beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)}\left[\frac{z(R_i(z) - R_{i+1}(z))}{1 - \beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}\right. \\ &\quad \left. + \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1 - \beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_i(z)) - S_{i+3}(z,R_{i+1}(z))]\right] \end{aligned}$$

This leads to an expression for $S_{i+1}(z, R_i(z))$ such that

$$S_{i+1}(z, R_i(z)) = \Omega_{i+1} z (R_i(z) - R_{i+1}(z))$$

More generally, the surplus is given by, $\forall \epsilon \geq R_i(z)$

$$S_i(z, \epsilon) = \Omega_i z (\epsilon - R_i(z))$$

where $\Omega_i = a_i \left\{1 + a_{i+1} b_{i+1} \left[1 + a_{i+2} b_{i+2} (\ldots)\right]\right\}$ with $a_i = \frac{1}{1-\beta\pi_i(1-\lambda_i)(1-s_e)}$ and $b_i = \beta(1-\pi_i)(1-\lambda_{i+1})(1-s_e)$ and until $i + n \leq O_7$. Thus, we have, e.g., $\Omega_{O_7} = \frac{1}{1-\beta\pi_{O_7}(1-\lambda_{O_7})(1-s_e)}$ and $\Omega_{O_6} = \frac{1}{1-\beta\pi_{O_6}(1-\lambda_{O_6})(1-s_e)} \left[1 + \frac{\beta(1-\pi_{O_6})(1-\lambda_{O_7})(1-s_e)}{1-\beta\pi_{O_7}(1-\lambda_{O_7})(1-s_e)}\right] \ldots$

C.3 Employment distribution

The employment distribution, assuming that $1 - \pi_i \approx 0$ and the z is the permanent level of productivity:

$$dn_i(z, \epsilon) \approx \frac{e_i(z)p(\theta_i(z)) + \left[(1-s_e)\lambda_i - e_i(z)p(\theta_i(z))\right] \int_{R_i(z)}^{1} dn_i(z, x)dx}{1 - (1-s_e)(1-\lambda_i)} dG(\epsilon)$$

where $dn_i(z, x)$ is the mass of age-i workers employed on a x-productivity job. The impact of the change in $R_i(z)$ is given by

$$\frac{d}{dR_i(z)} dn_i(z, \epsilon) = -\frac{\left[(1-s_e)\lambda_i - e_i(z)p(\theta_i(z))\right] dn_i(z, R_i(z))}{1 - (1-s_e)(1-\lambda_i)} dG(\epsilon) > 0$$

iff $(1 - s_e)\lambda_i < e_i(z)p(\theta_i(z))$, which is always satisfied to have $R_i < R_{i+1}$.

C.4 The derivation of the model elasticity to the business cycle

The decision rule on θ leads to

$$p(\theta_i(z)) \int_{R_i(z)}^{1} S_i(z, x)dG(x) = \frac{1}{(1-\gamma)\beta\pi_i} c\theta_i(z)$$

The decision rule on e leads to

$$\phi'(e_i(z)) = \frac{\gamma}{1-\gamma} c\theta_i(z)$$

Using the functional form, we obtain

$$\frac{e_i(z)^{1+\phi}}{1+\phi} = \frac{1}{1+\phi} \frac{\gamma}{1-\gamma} c e_i(z)\theta_i(z) \Rightarrow \widehat{e}_i(z) = \frac{1}{\phi} \widehat{\theta}_i(z)$$

The surplus function, defined by

$$S_i(z, \epsilon) = \max \left\{ \begin{array}{l} z(\epsilon - R_i(z)) + \beta\pi_i(1-\lambda_i)(1-s_e)E_z[S_i(z', \epsilon) - S_i(z', R_i(z))] \\ +\beta(1-\pi_i)(1-\lambda_{i+1})(1-s_e)E_z[S_{i+1}(z', \epsilon) - S_{i+1}(z', R_i(z))] \end{array} ; 0 \right\}$$

leads to

$$S_{i+1}(z,R_i(z)) = \Omega_{i+1}z(R_i(z)-R_{i+1}(z))$$

More generally, the surplus is given by, $\forall \epsilon \geq R_i(z)$

$$S_i(z,\epsilon) = \Omega_i z(\epsilon - R_i(z))$$

implying

$$\int_{R_i(z)}^1 S_i(z,x)dG(x) = \Omega_i z \int_{R_i(z)}^1 (x-R_i(z))dG(x) = \Omega_i z \int_{R_i(z)}^1 (1-G(x))dx$$

If we denote $I(R_i(z)) = \int_{R_i(z)}^1 (1-G(x))dx$, we have

$$\widehat{\widehat{S}}_i(z) = \widehat{z} - \varepsilon_{I|R}\widehat{R}_i(z)$$

where $\varepsilon_{I|R} = \left|\frac{I'R}{I}\right|$.

Given the free entry condition, the FOC w.r.t. e and the solution for the surplus, the impled solution for $\Sigma_i(z)$, $\Lambda_i(z)$ and $\Gamma_i^r(z)$ are

$$\begin{aligned}
\Sigma_i(z) &= \frac{\gamma}{1-\gamma}c\left[\frac{\phi}{1+\phi}e_i(z)\theta_i(z) + \frac{1-\pi_i}{\pi_{i+1}}e_{i+1}(z)\theta_{i+1}(z)\right] \\
\Lambda_i(z) &= (1-s_e)\frac{c}{1-\gamma}\left[\lambda_i\theta_i(z)^\eta + \lambda_{i+1}\frac{1-\pi_i}{\pi_{i+1}}\theta_{i+1}(z)^\eta\right] \\
\Gamma_i^r(z) &= \beta(1-s_e)(1-\pi_i)(1-\lambda_{i+1})\Omega_{i+1}z(R_i(z)-R_{i+1}(z))
\end{aligned}$$

The Log-linear approximations of the free entry condition, the FOC w.r.t e and the separation decision rule are:

$$\begin{aligned}
\widehat{\theta}_i &\approx \frac{1}{\eta}\left[\widehat{z} - \varepsilon_{I|R}\widehat{R}_i\right] \\
\widehat{e}_i &\approx \frac{1}{\phi}\widehat{\theta}_i \\
\widehat{R}_i &\approx -\frac{b+\Sigma_i-\Lambda_i-\Gamma_i}{b+\Sigma_i-\Lambda_i}\widehat{z} + \frac{\Sigma_i\frac{1+\phi}{\phi}-\Lambda_i\eta}{b+\Sigma_i-\Lambda_i}\widehat{\theta}_i
\end{aligned}$$

By combining these equation, we obtain (20), (21) and (19).

The elasticity of the average productivity. In order to compute the elasticities of the average wage $\mathcal{W}$, it is necessary to use the properties of the employment distribution. Indeed, we have

$$\widehat{\mathcal{W}}_i = \gamma\frac{z\mathcal{G}_i}{\mathcal{W}_i}(\widehat{z}+\widehat{\mathcal{G}}_i) + (1-\gamma)\frac{\Sigma_i}{\mathcal{W}_i}\widehat{\Sigma}_i$$

where $\mathcal{G}(R_i(z)) = \frac{1}{n_i(z)}\int_{R_i(z)}^1 xdn_i(z,x)$. The Log-linear approximation of this expression is

$$\widehat{\mathcal{G}}_i = -\widehat{n}_i + \frac{1}{\int_{R_i}^1 xdn_i(x)}\left(\kappa_i^r\widehat{R}_i + \kappa_i^e\widehat{e}_i + \kappa_i^\theta\widehat{\theta}_i\right)$$

where $\kappa_i^e = e_i \int_{R_i}^1 x \frac{\partial dn_i(x)}{\partial e_i} > 0$, $\kappa_i^\theta = \theta_i \int_{R_i}^1 x \frac{\partial dn_i(x)}{\partial \theta_i} > 0$ and $\kappa_i^r = R_i \left[-R_i dn_i(R_i) + \int_{R_i}^1 x \frac{\partial dn_i(x)}{\partial R_i} \right] \lesseqgtr 0$. Using the elasticity of the search efforts ($\widehat{e}_i$ and $\widehat{\theta}_i$), which are $\widehat{e}_i = \frac{1}{\phi}\widehat{\theta}_i$ and $\widehat{\theta}_i = \frac{1}{1-\eta}\widehat{J}_i$, and the log-linear approximation the age-i employment rate [44]

$$\widehat{n}_i = \vartheta_i^e \widehat{e}_i + \vartheta_i^\theta \widehat{\theta}_i + \vartheta_i^r \widehat{R}_i$$

where $\vartheta_i^e = \frac{1}{1-\eta}\vartheta_i^\theta > 0$ and $\vartheta_i^r < 0$, we obtain

$$\widehat{\mathcal{G}}_i = \left(-\vartheta_i^r + \frac{\kappa_i^r}{\int_{R_i}^1 x dn_i(x)} \right) \widehat{R}_i + \frac{-\left(\vartheta_i^e \frac{1}{\phi} + \vartheta_i^\theta\right) + \frac{\kappa_i^e \frac{1}{\phi} + \kappa_i^\theta}{\int_{R_i}^1 x dn_i(x)}}{1-\eta} \left\{ \begin{array}{c} \frac{ze(R_i)}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))} \widehat{z} \\ - \frac{\Sigma_i(1-G(R_i))}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))} \widehat{\Sigma}_i \end{array} \right\}$$

We then deduce that

$$\widehat{\mathcal{W}}_i = \gamma \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i} \left((1+\Gamma_i^z)\widehat{z} + \Gamma_i^r \widehat{R}_i + \Gamma_i^s \widehat{\Sigma}_i \right) + (1-\gamma) \frac{\Sigma_i}{\mathcal{W}_i} \widehat{\Sigma}_i$$

where

$$\begin{aligned} \Gamma_i^z &= \frac{-\left(\vartheta_i^e \frac{1}{\phi} + \vartheta_i^\theta\right) + \frac{\kappa_i^e \frac{1}{\phi} + \kappa_i^\theta}{\int_{R_i}^1 x dn_i(x)}}{1-\eta} \frac{ze(R_i)}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))} \\ \Gamma_i^r &= -\vartheta_i^r + \frac{\kappa_i^r}{\int_{R_i}^1 x dn_i(x)} \\ \Gamma_i^s &= -\frac{-\left(\vartheta_i^e \frac{1}{\phi} + \vartheta_i^\theta\right) + \frac{\kappa_i^e \frac{1}{\phi} + \kappa_i^\theta}{\int_{R_i}^1 x dn_i(x)}}{1-\eta} \frac{\Sigma_i(1-G(R_i))}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))} \end{aligned}$$

C.5 Numerical algorithm

The model has three exogenous state variables: the worker's age i, the match-specific productivity ϵ and the aggregate productivity z. For the grid of the match-specific productivity ϵ, we don't follow Fujita & Ramey (2012): its highest value x^h is set to sufficient large value to generate mean match productivity of 1, given that $G(\epsilon)$ is approximated by a discrete distribution with support $X = \{x_1, ..., x_M\}$, satisfying $x_1 = 1/M$, $x_m - x_{m-1} = x_M/M$. The associated probabilities

[44] At this stage, we consider that the employment rate at the steady state can be approximated by

$$n_i = \frac{1 - \pi_i + \pi_i e_i p(\theta_i)(1 - G(R_i))}{1 - \pi_i + \pi_i \{e_i p(\theta_i)(1 - G(R_i)) + [s_e + (1 - s_e)\lambda G(R_i)]\}}$$

which means that we consider that the impact of the fluctuations of the people that change age negligible with respect to their mass. Thus, we have:

$$\frac{\partial n_i}{\partial e_i} = \frac{\pi_i p(\theta_i)(1 - G(R_i))\pi_i[s_e + (1 - s_e)\lambda G(R_i)]}{[1 - \pi_i + \pi_i\{e_i p(\theta_i)(1 - G(R_i)) + [s_e + (1 - s_e)\lambda G(R_i)]\}]^2} \Rightarrow \vartheta_i^e > 0$$

$$\frac{\partial n_i}{\partial \theta_i} = \frac{\pi_i e_i p'(\theta_i)(1 - G(R_i))\pi_i[s_e + (1 - s_e)\lambda G(R_i)]}{[1 - \pi_i + \pi_i\{e_i p(\theta_i)(1 - G(R_i)) + [s_e + (1 - s_e)\lambda G(R_i)]\}]^2} \Rightarrow \vartheta_i^\theta > 0$$

$$\frac{\partial n_i}{\partial R_i} = -\frac{\pi_i G'(R_i)\{\pi_i e_i p(\theta_i)[1 + s_e(1 - \lambda)] + (1 - \pi_i)(1 - s_e)\}}{[1 - \pi_i + \pi_i\{e_i p(\theta_i)(1 - G(R_i)) + [s_e + (1 - s_e)\lambda G(R_i)]\}]^2} \Rightarrow \vartheta_i^r < 0$$

$\{\gamma_1, ..., \gamma_M\}$ are $\gamma_m = g(x_m)/M$ for $m = 1, ..., M-1$, where $g(x)$ is the Log-normal density, and $\gamma_M = 1 - \sum_{i=1}^{M-1} \gamma_i$. For the aggregate shock, we also follow Fujita & Ramey (2012): we choose the method presented in Tauchen (1986) in order to represent the process z_t as a Markov chain with a state space $Z = \{z_1, ..., z_I\}$. The transition matrix of this process is $\Pi_z = [\pi_{ij}^z]$, where $\pi_{ij}^z = Pr(z_{t+1} = z_j | z_t = z_i)$. We then form two transition matrix: firstly, the matrix $\Pi_{z,\epsilon} = [\pi_{ij}^{z,\epsilon}]$ where $\pi_{ij}^{z,m} = Pr(z_{t+1} = z_j | z_t = z_i)\gamma_m$, which gives the joint probability when both aggregate and match-specific shocks can change simultaneously, and secondly, the matrix $\overline{\Pi}_z = [\overline{\pi}_{ij}^z]$, where $\overline{\pi}_{ij}^z = Pr(z_{t+1} = z_j | z_t = z_i)\mathbb{I}_m$, which gives the probability when only aggregate shock can change, for each level of match-specific productivity.

Let $\mathcal{S}_{O_7}$ the vector $[S(x_1, z_1), \cdots, S(x_M, z_1), \cdots, S(x_1, z_I), \cdots, S(x_M, z_I)]$, and $\mathcal{R}$ be the vector $Z \bigotimes X$. Then, for an initial guess for $e_{O_7}(z)$ and $\theta_{O_7}(z)$, we find the fix point of

$$\mathcal{S}_{O_7} = \max\left\{\mathcal{R} - z + \pi_{O_7}\beta\left[\lambda\Pi_{z,\epsilon}\mathcal{S}_{O_7} + (1-\lambda)\Pi_z\mathcal{S}_{O_7} - \Pi_{z,\epsilon}^{e,\theta,O_7}\mathcal{S}_{O_7}\right]; 0\right\}$$

where $\Pi_{z,\epsilon}^{e,\theta,O_7}\mathcal{S}_{O_7}$ is deduced from the definition of the opposite of the search value, which is $\phi(e_{O_7}) - \gamma e_{O_7} p(\theta_{O_7})\pi_{O_7}\beta\Pi_{z,\epsilon}\mathcal{S}_{O_7}$. At each iteration, we use the FOC w.r.t. e[45] to substitute $\phi(e_{O_7})$ by $\frac{1}{1+\phi}\gamma e_{O_7} p(\theta_{O_7})\pi_{O_7}\beta\Pi_{z,\epsilon}\mathcal{S}_{O_7}$ We then have $\Pi_{z,\epsilon}^{e,\theta,O_7}\mathcal{S}_{O_7} = \frac{\phi}{1+\phi}\gamma e_{O_7} p(\theta_{O_7})\Pi_{z,\epsilon}\mathcal{S}_{O_7}$. When convergence criteria are satisfied, we obtain the decision rules $\theta_{O_7}^\star(z)$, $e_{O_7}^\star(z)$ and $R_{O_7}^\star(z)$, and the optimal value for the surplus $S_{O_7}^\star(x, z)$, $\forall z$ and $\forall x$.

For $i = O_6$, we solve the same problem, except that we integrate the solution for the age $i = O_7$ in the agents' expectations. Then, we find the fix point of

$$\mathcal{S}_{O_6} = \max\left\{\begin{array}{l} \mathcal{R} - z + \pi_{O_6}\beta\left[\lambda\Pi_{z,\epsilon}\mathcal{S}_{O_6} + (1-\lambda)\Pi_z\mathcal{S}_{O_6} - \Pi_{z,\epsilon}^{e,\theta,O_6}\mathcal{S}_{O_6}\right] \\ (1-\pi_{O_6})\beta\left[\lambda\Pi_{z,\epsilon}\mathcal{S}_{O_7}^\star + (1-\lambda)\Pi_z\mathcal{S}_{O_7}^\star - \Pi_{z,\epsilon}^{e^\star,\theta^\star,O_7}\mathcal{S}_{O_7}^\star\right] \end{array}; 0\right\}$$

which gives, when the convergence criteria are satisfied, $\{\theta_{O_6}^\star(z), e_{O_6}^\star(z), R_{O_6}^\star(z)\}$, and $S_{O_6}^\star(x, z)$, $\forall z$ and $\forall x$. we repeat the procedure until $i = Y$. Given this complete set of decision rules, we can simulate the Markov chain for JFR and JSR and construct the theoretical distribution of the employment per age, using equations (3), (5) and (4).

C.6 Matching young workers' labor market

Vectors Φ_1 and Ψ do not change. We minimize the distance $\Psi^{theo}(\Phi_2) - \Psi$, where the 11 free parameters are

$$\Phi_2 = \{H, \chi, \phi, h_A, h_O, b_Y, b, \lambda_Y, \lambda_A, \lambda_O, \sigma_\epsilon\} \quad dim(\Phi_2) = 11$$

The difference between this calibration and the one presented in the text is that there is no composition effect here ($\mu_i = 0$, $\forall i$), but we introduce a home production which is specific to young workers (they have no children and no house). Calibration is reported in table 18.

[45]This FOC is $\phi'(e_{O_7}) = \gamma p(\theta_{O_7})\pi_{O_7}\beta\Pi_{z,\epsilon}\mathcal{S}_{O_7}$

Table 18: Calibration: Matching young workers' labor market

Calibration Φ_2						
	H	b	σ_ϵ	χ	ϕ	
HLS	0.0975	0.912	0.317	0.31	0.7	
FR	0.061	0.934	0.124	-	-	
	b_Y	λ_Y	λ_A	λ_O	h_A	h_O
HLS	0.7752	0.035	0.174	0.195	1.82	1.976
FR	-	-	0.085	-	-	-

HLS = our weekly calibration

FR = Fujita and Ramey (2011)'s weekly calibration

Simulation results are in the table 19. If we focuss on the youth labor market, this identification strategy leads to a larger gap between the data and the model predictions. Notice that the model is also able to predict the age-increasing volatilities in accordance with the data.

Table 19: Second order moments: data versus theory. Matching young workers' labor market

		All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR:	US Data	0.11	0.089	0.14	0.20
	Model 1	0.10	0.07	0.17	0.19
	Model 2	0.12	0.01	0.16	0.20
JFR:	US Data	0.16	0.16	0.17	0.22
	Model 1	0.11	0.04	0.12	0.15
	Model 2	0.10	0.036	0.11	0.13
u:	US Data	0.23	0.18	0.27	0.32
	Model 1	0.16	0.05	0.25	0.27
	Model 2	0.17	0.033	0.23	0.27
w:	US Data	0.015	0.026	0.019	0.018
	Model 1	0.0123	0.0127	0.0123	0.0122
	Model 2	0.0114	0.0127	0.0112	0.0111

US data: Standard deviation, CPS Monthly data, Men, 1976 June - 2012 Sept

HP filter with smoothing parameter 10^5. Authors' calculations.

Simulations reported in the table 20 show that the horizon effect is the main explanation of these results. These results underline the robustness of the ones discussed in the last section of the paper.

Table 20: Second order moments: Sensitivity analysis

		All: 16-61	Y: 16-24	A: 25-54	O: 55-61
JSR:	Model 2	0.12	0.01	0.16	0.20
	h & b	0.11	0.015	0.17	0.22
	h & b & λ	0.11	0.17	0.17	0.19
	Low b	0.024	0.016	0.045	0.05
JFR:	Model 2	0.10	0.036	0.11	0.13
	h & b	0.09	0.05	0.12	0.14
	h & b & λ	0.12	0.11	0.11	0.14
	Low b	0.041	0.041	0.05	0.06
u:	Model 2	0.17	0.033	0.23	0.27
	h & b	0.16	0.05	0.24	0.29
	h & b & λ	0.23	0.19	0.24	0.26
	Low b	0.052	0.038	0.094	0.095

h & b: Model with homogenous h & b: $h_i = h_A$ and $b_i = b_A$

h & b & λ: Model with homogenous h & b & λ: $h_i = h_A$, $b_i = b_A$ and $\lambda_i = \lambda_A$

Model with low b: $b_i = rh_i$ such that $r - \sum_i m_i \phi(e_i^{ss}) \approx 0.4$ as in Shimer (2005)