The Political Economy of Labor Market Regulation with R&D

Tapio Palokangas

April 2014
ABSTRACT

The Political Economy of Labor Market Regulation with R&D*

In this paper, I study the political rationale for labor market regulation. Oligopolists employ raw labor and human capital (i.e. key workers) for production and R&D. There are many jurisdictions, in each of which a self-interested policy maker can regulate/deregulate the local labor market. I show that the observed tendency to labor market deregulation results from labor market policies being set up at the local level. In small jurisdictions, the fall of income due to wage increases is so large that the labor markets are deregulated. With labor market integration, jurisdictions get larger and face less competition from outside. Then, the fall of income due to wage increases is reduced and labor market regulation becomes more attractive to workers’ lobbies.

JEL Classification: F15, J50, O40

Keywords: political economy, labour market regulation, R&D, union power

Corresponding author:

Tapio Palokangas
Economicum Building
P.O. Box 17 (Arkadiankatu 7)
FIN-00014 University of Helsinki
Finland
E-mail: Tapio.Palokangas@helsinki.fi

* The author thanks an anonymous referee for constructive comments.
1 Introduction

In this document, I examine the political economy of labor market regulation in an economy where firms have incentives to escape labor costs by performing research and development (R&D). The policy makers are assumed to be self-interested, subject to lobbying from employers and labor unions.

Unionization has declined in most OECD countries since the 1980s (Nickell et al. 2005, pp. 6-7). In particular, in the years 1975-2000, labor markets have been rapidly deregulated in the US and UK (Acemoglu et al. 2001). International trade (in particular outsourcing) has undermined union bargaining power (cf. Abraham et al. 2009, Dumont et al. 2005, 2012, Boulhol et al. 2011). Protection of regular employment contracts was diminished when globalization was proceeding rapidly (Potrafke 2010). On the other hand, there is little evidence of international trade having an impact on the workers’ bargaining power (Brock and Dobbelaere 2006). In this document, I explain declining union power as follows. Assume that economic integration increases the size of the economy, except that labor market policies are still set up at the local level. Because a local policy maker controls only a small proportion of the integrated labor markers, it has less changes to exercise independent policy. This makes lobbying for labor market regulation less attractive to workers’ lobbies.

The remainder of this document is organized as follows. Section 2 considers the related literature on the topic. Section 3 characterizes the institutional structure of the economy. Sections 4 and 5 construct the specific models of the households, final-good firms and inter-mediate-good industries, respectively. Section 6 establishes a common agency game where employers and workers lobby decision makers. Sections 7 and 8 construct the political equilibrium, on which the results are based, and section 9 focuses on welfare effects of labor market integration.

2 Related literature

Acemoglu et al. (2001) explain declining unionization by skill-biased technological change which increases the outside option of skilled workers, undermining the coalition among skilled and unskilled workers in support of
unions. In this document, I explain the same development by a political process in which workers and employers lobby policy makers on labor market regulation. Palokangas (2003) argues that distorting taxation causes labor market regulation. In a model where employers and workers bargain over wages and lobby the government for taxation and labor market regulation, he shows that if it is much easier to tax wages than profits, then the government protects union power by labor market regulation. In this document, I introduce in-house R&D as an alternative cause of labor market regulation: firms invest in R&D to escape labor costs due to high wages.

The growth effects of union power depend decisively on the structure of the economy. Labor unions impose minimum wages that cause unemployment. If the same technology were used both in production and in R&D, then union power would decrease profits, undermining incentives to invest in productivity-enhancing R&D (cf. Peretto 2011). In that case, an increase in union wages decreases both employment and the productivity growth rate. There is, however, contrasting empirical evidence. Caballero (1993) and Hoon and Phelps (1997) find a positive dependence between unemployment and productivity growth. Vergeer and Kleinknecht (2010) show that the annual percentage growth of real wages has a positive effect on growth in value added per labor hour. In that case, flexible (i.e. deregulated) labor markets can lead the combination of high employment and slow productivity growth.

Palokangas (1996, 2000, 2004) establishes a positive dependence between unemployment and productivity growth by the assumption that production and R&D are subject to different technology. He assumes two categories of labor: key workers (call human capital, for convenience) are used both in production and R&D, while ordinary workers (call “raw” labor) are used only in production. When the minimum wages for ordinary workers increase, firms lay out labor, but transfer human capital from production to productivity-enhancing R&D to escape labor costs. In this document, I assume that R&D plays a decisive role in labor marker policy. In the presence of cost-escaping R&D, workers can accept unemployment in exchange for higher prospective labor income. Labor market regulation increases wages, decreasing output and transferring human capital from production to R&D. This promotes R&D, raising productivity and prospective income.
Labor market policy can be endogenized either by majority voting (cf. Saint-Paul 2002a, 2002b), all-pay auctioning, in which the lobbyist making the greater effort wins with certainty (cf. Johal and Ulph 2002), or menu actioning, in which the lobbyists announce their bids contingent on the policy maker’s actions (cf. Grossman and Helpman 1994, Dixit et al 1997). Because I am interested in relative union bargaining power, which is not a discrete variable, majority voting is not applied in this document. In the all-pay auctioning, lobbying expenditures are incurred by all the lobbyists before the policy maker takes an action. This is the case e.g. when interest groups spend money to increase the probability of getting their favorite type of government elected. In menu auctioning, it is not possible for a lobbyist to spend money and effort on lobbying without getting what he lobbied for. Because the menu-auction model is better associated with the case in which the policy maker’s decision variable (e.g. relative union bargaining power) is continuous and interest groups can obtain marginal improvements for themselves by lobbying, I apply it in this document.

3 The economy

The households supply two primary inputs inelastically: human capital $H$, which consists of skilled key workers, and (raw) labor $L$. The institutional difference between these inputs is that human capital can be used both in production and in research (R&D), while labor only in production. There is a “continuum” of industries $i \in [0, 1]$. In each industry $i \in [0, 1]$, one oligopolist (labeled $i$) produces a different high-tech good (labeled $i$). Competitive firms produce the consumption good from the intermediate goods $i \in [0, 1]$.

The market for human capital is competitive. Oligopolist $i$ bargains over the wage with a labor union (labeled $i$) that represents its labor. The industries $i \in [0, 1]$ are organized in a number $n$ of equal but disjoint jurisdictions, each of which determines relative union bargaining power independently:

$$[0, 1] = \bigcup_{k=1}^{n} B_k, \quad B_k \cap B_\ell = \emptyset \text{ for } k \neq \ell, \quad \frac{1}{n} = \int_{i \in B_k} di, \quad (1)$$

where $B_k$ is the set of industries belonging to jurisdiction $k \in [0, n]$. 
In jurisdiction $k \in [0, n]$, there is a policy maker (labeled $k$) which determines relative union bargaining power, an employer lobby (labeled $k$) that represents oligopolists $i \in B_k$, and a union lobby (labeled $k$) that represents the workers of those oligopolists. Because human capital is fully employed, it has no lobby of its own. The lobbies influence the policy maker by their political contributions. Labor market integration decreases the mass $n$, but increases the size $\frac{1}{n}$ of jurisdictions.

In industry $i \in [0, 1]$, the supply of labor is given by $L_i$, the supply of human capital by $H_i$, the demand for labor and human capital in production by $l_i$ and $h_i$, respectively, and the demand for human capital in R&D by $z_i$. Labor and human capital can freely move between the industries $i \in [0, 1]$:

$$L = \int_{i\in[0,1]} L_i \, di, \quad H = \int_{i\in[0,1]} H_i \, di. \tag{2}$$

The market clearing conditions for human capital and the full-employment constraints for labor are given by:

$$h_i + z_i = H_i, \quad l_i \leq L_i, \quad i \in [0, 1]. \tag{3}$$

In this document, the common agency model (c.f. Bernheim and Whinston 1986, Grossman and Helpman 1994, and Dixit et al. 1997) is used to establish the political equilibrium (cf. Fig. 1). The players are households that consume, competitive firms that produce the consumption good, oligopolists that make intermediate goods, labor unions, labor and employer lobbies, and policy makers. Their decisions form the following sequence:

1. Labor and human capital choose their location among industries.
2. Employer and union lobbies influence policy makers, relating their prospective political contributions to the latter’s decisions.

3. Policy makers set relative union bargaining power.

4. Oligopolists and unions bargain over wages for labor.

5. Oligopolists produce and perform R&D.

6. Salaries adjust to balance the markets for human capital.

7. Oligopolists employ human capital in production.

8. Competitive firms make the final good from the oligopolists’ outputs.

9. The households plan their consumption over time.

This game is solved in reverse order: stages 9 and 8 in section 4, stages 7, 6, 5 and 4 in section 5, and stages 3, 2 and 1 in section 6.

4 Households and final-goods producers

Provided that all households in the economy share the same preferences, they can be represented by a single household that chooses its flow of consumption \( c \) to maximize its utility starting at time \( T \),

\[
\int_T^\infty (\log c)e^{-\rho(\theta-T)}d\theta, \tag{4}
\]

where \( \theta \) is time, \( c \) consumption and \( \rho > 0 \) the constant rate of time preference. This utility maximization leads to the Euler equation

\[ \dot{X}/X = r - \rho \] \text{ with } \ X = P^e c, \tag{5} \]

where \( P^e \) the consumption price index, \( X \) consumption expenditure, \( r \) the interest rate and \( \dot{X} \equiv dX/d\theta \). Because in the model there is no money that would pin down the nominal price level at any time, one can normalize the households’ total consumption expenditure \( X \) at unity. This and (5) yield

\[ P^e c = X = 1, \quad P^e = 1/c, \quad r = \rho = \text{constant} > 0. \tag{6} \]

6
The outputs $y_i$ of oligopolists $i \in [0, 1]$ are substitutes. Competitive firms make the consumption good from these through CES technology:

$$c = \left( \int_0^1 A_i y_i^{1-1/\epsilon} \, di \right)^{\epsilon/(\epsilon-1)} \text{ with constant } \epsilon > 1,$$

where $\epsilon$ is the *elasticity of product substitution* and $A_i$ the *productivity of good* $i$ in providing services to the households. Oligopolist $i$ can increase its productivity $A_i$ by investing in R&D.

Because all consumption-good producers are competitive, they can be represented by a single firm that maximizes its profit $Pc - \int_0^1 p_i y_i \, di$ by its inputs $y_i$, $i \in [0, 1]$, subject to technology (7), given the output price $P$ and the input prices $p_i$, $i \in [0, 1]$. Given this and (6), the profit maximization yields the *inverse demand curve* of oligopolist $i$:

$$p_i = P \frac{\partial c}{\partial y_i} = PA_i \left( \frac{c}{y_i} \right)^{1/\epsilon} = A_i c^{1/\epsilon - 1} y_i^{-1/\epsilon}$$

## 5 Industry $i$

Oligopolist $i$ and union $i$ take the macroeconomic variables, the interest rate $r$ and aggregate consumption $c$, as given. They pay political contributions $R_{io}^i$ and $R_{iu}^i$, respectively, to the policy maker of their jurisdiction. Because $R_{io}^i$ and $R_{iu}^i$ are determined by lobbying at the level of the jurisdiction, oligopolist $i$ and union $i$ take them given as well.

### 5.1 Technological change

Following Grossman and Helpman (1991) and Aghion and Howitt (1998), the arrival rate of innovations $\Lambda_i$ is assumed to follow a Poisson process being in fixed proportion $\lambda$ to innovation intensity $z_i$ for each oligopolist $i \in [0, 1]$:

$$\Lambda_i = \lambda z_i, \quad \lambda > 0, \quad z_i \geq 0.$$  

The serial number of technology for oligopolist $i$ is denoted by $t_i$ and productivity corresponding to that technology by $A_i(t_i)$. It is assumed that an invention of a new technology raises $t_i$ by one and $A_i(t_i)$ by constant $a > 1$:

$$A_i(t_i + 1) = aA_i(t_i), \quad a > 1.$$  

7
In Appendix A, I show that the expected average growth rate \( g_i \) of productivity \( A_i(t) \) is in fixed proportion to labor devoted to R&D, \( z_i \),

\[
g_i = (\log a) \lambda z_i, \tag{11}
\]

and the expected value of the flow of productivity \( A_i \) at time \( T \) is given by

\[
E \int_T^\infty A_i e^{-r(\theta-T)} d\theta = \frac{A_i T}{r + (1-a)\lambda z_i}, \tag{12}
\]

where \( E \) is the expectation operator and \( A_i T \) is productivity \( A_i \) at time \( T \).

### 5.2 Production and R&D

Oligopolist \( i \) produces its output \( y_i \) from labor \( l_i \) and human capital \( h_i \) according to the CES function

\[
y_i = F(l_i, h_i), \quad F_l = \frac{\partial F}{\partial l_i} > 0, \quad F_h = \frac{\partial F}{\partial h_i} > 0, \quad F_{ll} = \frac{\partial^2 F}{\partial l_i^2} < 0, \quad F_{hh} = \frac{\partial^2 F}{\partial h_i^2} > 0, \quad F_{lh} = \frac{\partial^2 F}{\partial l_i \partial h_i} > 0, \quad F_l F_h F_{lh} = \varphi \in (0, 1) \cup (1, \infty), \tag{13}
\]

where \( \varphi \) is the constant elasticity of factor substitution. It employs human capital \( z_i \) for its in-house R&D and pays the wage \( W_i \) for labor \( l_i \) and the salary \( S_i \) for human capital \( l_i + z_i \). Its profit \( \Pi_i \) is equal to sales revenue \( p_i y_i \) minus wages \( W_i l_i \), salaries \( S_i (l_i + z_i) \) and political contributions \( R_o \). Noting the inverse demand curve (8) and the production function (13), this yields

\[
\Pi_i = p_i y_i - W_i l_i - R_o = \epsilon^{1/\epsilon-1} A_i y_i^{1-1/\epsilon} - W_i l_i - S_i (h_i + z_i) - R_o
\]

\[
= \epsilon^{1/\epsilon-1} A_i F(l_i, h_i)^{1-1/\epsilon} - W_i l_i - S_i (h_i + z_i) - R_o. \tag{14}
\]

Oligopolist \( i \) employs human capital in production, \( h_i \), up to the level where the marginal product of human capital is equal to the salary \( S_i \):

\[
S_i = \frac{\partial}{\partial h_i} \left[ \epsilon^{1/\epsilon-1} A_i F(l_i, h_i)^{1-1/\epsilon} \right] = \left( \frac{1}{\epsilon} \right)^{1/\epsilon-1} A_i \frac{F_h(h_i, l_i)}{F(l_i, h_i)^{1/\epsilon}}. \tag{15}
\]

Because oligopolist \( i \) is the only employer of human capital in industry \( i \), it takes the market-clearing conditions (15) and \( h_i + z_i = H_i \) [cf. (3)] into account in the next stage. To obtain a stationary-state equilibrium where
the inputs of labor and human capital are constant over time, I assume that oligopolist $i$ and union $i$ bargain over the productivity-adjusted wage

$$w_i = W_i/(c^{1/\epsilon-1}A_i),$$

(16)

where $A_i$ and $c^{1/\epsilon-1}$ are the levels of productivity due to past investment in R&D and aggregate consumption $c$, correspondingly. Noting (3), (15) and (16), the profit of oligopolist $i$, (14), then becomes

$$\Pi_i = c^{1/\epsilon-1}A_i\pi_i - R_o$$

with

$$\pi_i = F(l_i, H_i - z_i)^{1-1/\epsilon} - w_i l_i - \left(1 - \frac{1}{\epsilon}\right) \frac{F_h(l_i, H_i - z_i)H_i}{F(l_i, H_i - z_i)^{1/\epsilon}}.$$ 

(17)

Because the system has a stationary state equilibrium, the optimum can be solved by choosing inputs $(l_i, z_i)$ from the class of constant controls. The expected value of the profits (17) starting at time $\theta = T$ is then [cf. (12)]

$$E \int_T^\infty \Pi_i e^{-r(\theta - T)} d\theta = \pi_i c^{1/\epsilon-1} E \int_T^\infty A_i(t_i) e^{-r(\theta - T)} d\theta - R_o \int_T^\infty e^{-r(\theta - T)} d\theta$$

$$= \frac{\pi_i c^{1/\epsilon-1} A_i T}{r + (1 - a)\lambda z_i} - \frac{R_o}{r},$$

(18)

Oligopolist $i$ maximizes its value (18) by inputs $(l_i, z_i)$ subject to technological change in industry $i$ (cf. subsection 5.1), given aggregate consumption $c$, the productivity-adjusted wage $w_i$, the supplies labor of and human capital in industry $i$, $(L_i, H_i)$, and political contributions $R_o$. This maximization leads to the following value and demand functions (cf. Appendix B):

$$W_o(w_i, c, R_o) = \max_{l_i, z_i} E \int_T^\infty \Pi_i e^{-r(\theta - T)} d\theta$$

$$= c^{1/\epsilon-1} A_i T \max_{l_i, z_i} \frac{\pi_i}{r + (1 - a)\lambda z_i} - \frac{R_o}{r},$$

(19)

$$l_i = \tilde{l}(w_i, H_i), \quad y_i = \tilde{y}(w_i, H_i), \quad z_i = \tilde{z}(w_i, H_i),$$

$$\tilde{w}_w < 0, \quad \tilde{z}_w > 0 \iff \epsilon > \varphi, \quad \tilde{y}_w < 0 \text{ for } \epsilon > \varphi.$$ 

(20)

The results (20) can be explained as follows. The demand for labor falls with the productivity-adjusted wage, $\tilde{w}_w < 0$. The higher the price

\footnote{With some complication, the same result can be obtained by dynamic programming (cf. Dixit and Pindyck 1994). These calculations will be provided to the reader on request.}
elasticity of output demand for an oligopolist, \( \epsilon \), the stronger the output effect: output and the demand for human capital in production, \( \tilde{h} \), fall with a higher productivity-adjusted wage \( w_i \). The higher the elasticity of factor substitution, \( \gamma \), the stronger the substitution effect: the demand for human capital in production, \( e_h \), rises with a higher productivity-adjusted wage \( w_i \). If the elasticity of product substitution, \( \epsilon \), is greater than that of factor substitution, \( \gamma \), then the output effect dominates over the substitution effect and an increase in the productivity-adjusted wage \( w_i \) lowers the demand for human capital in production. Because human capital is fully employed, this generates a transfer of human capital from production to R&D, \( \tilde{z}_w > 0 \).

5.3 Collective bargaining

The workers belonging to union \( i \) earn wages \( W_i l_i \) minus their political contributions \( R_i^u \). Given (16) and (20), this implies

\[
V_i = W_i l_i - R_i^u = w_i l_i A_i c^{1/\epsilon - 1} - R_i^u = w_i \tilde{l}(w_i, H_i) A_i c^{1/\epsilon - 1} - R_i^u. \tag{21}
\]

Union \( i \) observes technological change (cf. subsection 5.1). Because inputs \( (\tilde{l}, \tilde{z}) \) and the productivity-adjusted wage \( w_i \) are constants in equilibrium, then, given (12), (20) and (21), the expected present value of the flow of the union members’ income (21) at time \( \theta = T \) is

\[
W_u(w_i, c, R_u^i) = E \int_T^{\infty} V_i e^{-r(\theta-T)} d\theta = \frac{c^{1/\epsilon - 1} A_i T w_i \tilde{l}(w_i, H_i)}{r + (1 - a) \tilde{z}(w_i, H_i)} - \frac{R_u^i}{r}. \tag{22}
\]

Oligopolist \( i \) maximizes its value function (19) and labor union \( i \) its value function (22) by the productivity-adjusted wage \( w_i \) in an alternating-offers game, given aggregate consumption \( c \), the interest rate \( r \), the supply of labor and human capital, \( (L_i, H_i) \), and contributions \( (R_i^u, R_o^i) \). Both parties can alone forestall production. Because oligopolist \( i \) (union \( i \)) earns nothing but pays contributions \( R_o^i \) (\( R_u^i \)) in the case of no production, its fall-back income is the discounted value of the flow of these contributions \(-R_o^i/r \) \((-R_u^i/r \)).

The outcome of the alternating-offers game is obtained by maximizing the Generalized Nash Product (GNP) of the parties’ utilities (19) and (22),

\[
\Theta(w_i, c, R_u^i) \equiv \alpha_i \log [W_u(w_i, c, R_u^i) - (-R_u^i/r)] + (1 - \alpha_i) \log [W_o(w_i, c, R_o^i) - (-R_o^i/r)]
\]
by the productivity-adjusted wage $w_i$, where $\alpha_i \in [0,1]$ is the relative bargaining power of union $i$. It is equivalent to maximize $\Theta/\alpha$ by $w_i$. Given (17), this leads to the first-order condition

$$\frac{1}{\alpha_i} \frac{\partial \Theta}{\partial w_i} = \frac{\partial W_u(c_i)}{\partial w_i} + \left(\frac{1}{\alpha_i} - 1\right) \frac{1}{\pi_i} \frac{\partial \pi_i}{\partial w_i} = \frac{\partial W_u(c_i)}{\partial w_i} + \left(1 - \frac{1}{\alpha_i}\right) \frac{l_i}{\pi_i} = 0.$$ 

On the assumption that the equilibrium is unique, the second-order condition $\frac{1}{\alpha_i} \frac{\partial^2 \Theta}{\partial w_i^2} < 0$ holds true. Because from (17) and (20) it follows that $\frac{\partial}{\partial \alpha_i} \left(\frac{1}{\alpha_i} \frac{\partial \Theta}{\partial w_i}\right) = \frac{1}{\alpha_i} \frac{l_i}{\pi_i} > 0$, the productivity-adjusted wage increases with relative union bargaining power:

$$w_i = \bar{w}(\alpha_i, H_i), \quad \frac{\partial w_i}{\partial \alpha_i} > 0. \quad (24)$$

6 Lobbies and policy makers

Employer lobby $k$ represents the oligopolists $i \in B_k$ and union lobby $k$ the workers in jurisdiction $k$. Relative union bargaining power $\alpha_i$ and political contributions $(R^l_{ku}, R^l_{ko})$ are uniform throughout the industries $i \in B_k$ of the same jurisdiction $k$:

$$\alpha_i = \beta_k, \quad R^l_u = R_{ku} \quad \text{and} \quad R^l_o = R_{ko} \quad \text{for} \quad i \in B_k. \quad (25)$$

This equalizes the productivity-adjusted wages (24) in jurisdiction $k$:

$$w_i = \bar{w}(\beta_k) \quad \text{for} \quad i \in B_k. \quad (26)$$

In this document, technology spillover is modeled in line with Aghion and Howitt (1998, pp. 87-88) as follows. Although the arrival rates in different sectors are independent of each other, the innovations themselves all draw on the same pool of shared technological knowledge. The state of this knowledge is represented by a leading-edge technology, whose productivity parameter is $A$. If oligopolist $i \in [0,1]$ innovates, then it can start producing with the
leading edge of technology. When this happens, the technology parameter $A_i$ in industry $i \in [0,1]$ jumps discontinuously to $A$:

$$A_i = A \text{ for } i \in [0,1].$$

(27)

Given (7), (20), (26) and (27), aggregate consumption $c$ is determined by the productivity-adjusted wages as follows (cf. Appendix C):

$$c(\varpi_k, \varpi_k, A, n) = \frac{1}{c} \frac{\partial c}{\partial \varpi_k} \bigg|_{\varpi_k = \varpi} = A^{\gamma/(\gamma-1)} \tilde{y}(\varpi, H),$$

(28)

$$\frac{1}{c} \frac{\partial c}{\partial \varpi_k} \bigg|_{\varpi_k = \varpi} = \frac{\tilde{y}_w}{\tilde{y}_w} < 0, \quad \varpi_k \neq \{\varpi_\ell | \ell \neq k\}. \quad (28)$$

Plugging (25), (26) and (28) into the utilities of oligopolist $i$ and labor union $i$, (19) and (22), yields the utility functions of employer lobby $k$ and union lobby $k$:

$$F_k(\varpi_k, \varpi_k, n, R_{ko}) = W_o(\varpi_k, c, R_{i0}),$$

(29)

$$U_k(\varpi_k, \varpi_k, n, R_{ku}) = W_u(\varpi_k, c, R_{i0}).$$

(30)

The contribution schedules of the lobbies, $R_{ku}$ and $R_{ko}$, depend on the arguments $(\varpi_k, \varpi_k, n)$ of their utility functions (29) and (30):

$$R_{ku}(\varpi_k, \varpi_k, n), \quad R_{ko}(\varpi_k, \varpi_k, n).$$

(31)

Policy maker $k$ collects the flow of the political contributions $R_{ko} + R_{ku}$ from all oligopolists and labor unions in jurisdiction $k \in [0, n]$, $\int_{i \in B_k} (R_{ko}^i + R_{ku}^i) di$. It maximizes the present value of this flow of income (cf. (1), (26) and (31)):

$$G_k(\varpi_k, \varpi_k, n) = E \int_T^\infty \left[ \int_{i \in B_k} (R_{ko}^i + R_{ku}^i) di \right] e^{-r(\theta-T)} d\theta = \frac{R_{ko} + R_{ku}}{rn}$$

$$= \frac{1}{rn} \left[ R_{ko}(\varpi_k, \varpi_k, n) + R_{ku}(\varpi_k, \varpi_k, n) \right].$$

(32)

7 Political Equilibrium

Given (1), (20) and (26), the full-employment constraints (3) become

$$\bar{L}(\varpi_k, H) \leq \int_{i \in B_k} L_i di / \int_{j \in B_k} dj, \quad k \in [0, n].$$

(33)
Employer lobby \( k \) and union lobby \( k \) influence policy maker \( k \) over union bargaining power \( \beta_k \). These three agents take the productivity-adjusted wages elsewhere, \( \varpi_{-k} \) [cf. (28)], as given and observe the full-employment constraint (33). Because there is a one-to-one correspondence from \( \beta_k \) to \( \varpi_k \) through (26), it is equivalent to assume that the lobbies influence policy maker \( k \) over the productivity-adjusted wage \( \varpi_k \) subject to (33), given \( \varpi_{-k} \).

According to proposition 1 of Dixit et al. (1997), a subgame perfect Nash equilibrium for the game between the employer lobby, the union lobby and the policy maker in jurisdiction \( k \) is a set of contribution schedules (31) and a policy \( \varpi_k \) s.t. the following conditions (\( i \) – \( iv \)) hold:

(i) The contributions of the labor and employer lobbies, \( R_{ku} \) and \( R_{ko} \), are non-negative but no more than the contributor’s income.

(ii) The policy \( \varpi_k \) maximizes the policy maker’s welfare (32):

\[
\varpi_k = \arg \max_{\varpi_k \text{ s.t. (33)}} G_k(\varpi_k, \varpi_{-k}, n).
\]  

(iii) The employer (labor) lobby cannot have a feasible strategy

\[
R_{ko}(\varpi_k, \varpi_{-k}, n) \quad (R_{ku}(\varpi_k, \varpi_{-k}, n))
\]

that yields it higher utility (29) ((30)) than in equilibrium, given the policy maker’s expected policy:

\[
\varpi_k = \arg \max_{\varpi_k \text{ s.t. (33)}} F_k(\varpi_k, \varpi_{-k}, n, \epsilon, R_{ko}(\varpi_k, \varpi_{-k}, n)),
\]

\[
\varpi_k = \arg \max_{\varpi_k \text{ s.t. (33)}} U_k(\varpi_k, \varpi_{-k}, n, \epsilon, R_{ku}(\varpi_k, \varpi_{-k}, n)).
\]  

(iv) The employer (labor) lobby provides the policy maker at least with the level of utility than in the case where the lobby offers nothing \( R_{ko} = 0 \) \( (R_{ku} = 0) \), and where the policy maker responds optimally given the other lobby’s contribution function (30) ((29)).

Noting (29) and (30), the equilibrium conditions (35) become

\[
0 = \frac{dF_k}{d\varpi_k} - \frac{\partial F_k}{\partial \varpi_k} \frac{1}{r} \frac{\partial R_{ko}}{\partial \varpi_k},
\]

\[
0 = \frac{dU_k}{d\varpi_k} - \frac{\partial U_k}{\partial \varpi_k} \frac{1}{r} \frac{\partial R_{ku}}{\partial \varpi_k}.
\]
These equations are equivalent to

\[
\frac{1}{r} \frac{\partial R_{ku}}{\partial \varpi_k} = \frac{\partial U_k}{\partial \varpi_k}, \quad \frac{1}{r} \frac{\partial R_{ko}}{\partial \varpi_k} = \frac{\partial F_k}{\partial \varpi_k}.
\]  

(36)

Conditions (36) say that in equilibrium the change in the discounted contributions of labor (employer) lobby \( k \) due to a change in the productivity-adjusted wage \( \varpi_k \) equals the effect of that wage on the inter-temporal welfare of that lobby. Thus, the contribution schedules are locally truthful. As in Berhheim and Whinston (1997) or in Grossman and Helpman (1994), this concept can be extended to a globally truthful contribution schedule. This type of schedule represents the preferences of labor (employer) \( k \) at all policy points. Given (36), the truthful contribution functions take the form

\[
R_{ku} = \max[U_k - U_k, 0], \quad R_{ko} = \max[F_k - F_k, 0],
\]  

(37)

where \( U_k \) (\( F_k \)) is the welfare of labor (employer) lobby \( k \) when it does not pay contributions but policy maker \( k \) chooses its best response, given the contribution schedule of employer (labor) lobby \( k \).

The threat points \( U_k \) and \( F_k \) are determined by (24), (26), (29) and (30) and (33) as follows. If union lobby \( k \) does not pay contributions to policy maker \( k \), \( R_{ku} = 0 \), then the latter decreases the relative bargaining power \( \alpha_i \) of union \( i \), and consequently the productivity-adjusted wage \( \varpi_k \), to the level \( \varpi_k \) corresponding to full employment \( \bar{\varpi}(\varpi_k, H) = L \). Thus, \( U_k = U_k(\varpi_k, \varpi_{-k}, n, R_{ku}) \). If employer lobby \( k \) does not pay contributions to policy maker \( k \), \( R_{ko} = 0 \), then the latter increases the relative bargaining power of union \( i \) to the maximum \( \alpha_i = 1 \). In that case, \( \varpi_k = w(1) \) and \( F_k = F_k(w(1), \varpi_{-k}, n, R_{ku}) \). It follows that \( U_k \) and \( F_k \) are given for union lobby \( k \) and employer lobby \( k \).

In the equilibrium conditions (24), (25), (33), (34) and (35) with the productivity-adjusted wages \( \varpi_k \) as unknown variables, there is perfect symmetry throughout all jurisdictions \( k \in [0, n] \), provided that labor \( L_i \) and human capital \( H_i \) were uniformly distributed throughout industries \( i \in [0, 1] \). Because labor and human capital face equal career prospects in all industries \( i \in [0, 1] \), they settle themselves down uniformly throughout \( i \in [0, 1] \). Given (2) and (25), relative union bargaining power and the supplies of labor and
human capital then become uniform throughout the economy:

\[ w_i = \varpi_k = \varpi, \quad L_i = L \quad \text{and} \quad H_i = H \quad \text{for} \quad k \in [0, 1] \quad \text{and} \quad i \in [0, 1]. \tag{38} \]

Noting (8), (20), (26), (28), (29), (30), (32), (36) and (38), the policy maker’s equilibrium conditions (34) become (cf. Appendix D)

\[ \tilde{l} < L \iff \Delta = 0 \quad \text{with} \quad \frac{\partial \Delta}{\partial \varpi} < 0, \quad \tilde{l} = L \iff \Delta < 0, \tag{39} \]

where

\[ \Delta(\varpi, n) = \left( \frac{a - 1}{r + (1 - a)\lambda \bar{z}_w} \right) + \left( \frac{1}{\epsilon - 1} \right) \left( \frac{\pi}{\varpi \nu} + 1 \right) \frac{1}{n} \frac{\bar{y}_w}{y} \tag{40} \]

with

\[ \frac{\partial \Delta}{\partial \left( \frac{1}{n} \right)} = \left( \frac{1}{\epsilon - 1} \right) \left( \frac{\pi}{\varpi \nu} + 1 \right) \frac{\bar{y}_w}{y} > 0 \quad \text{for} \quad \epsilon > \varphi. \tag{41} \]

The result (39) and (40) can be explained as follows. The growth effect of the productivity-adjusted wage \( \varpi \) is given by

\[ \frac{(a - 1)\lambda \bar{z}_w}{r + (1 - a)\lambda \bar{z}_w}. \tag{42} \]

Its sign depends on the sign of \( \bar{z}_w \). The level effect is given by

\[ \frac{\tilde{l}_w}{l} + \left( \frac{1}{\epsilon - 1} \right) \left( \frac{\pi}{\varpi \nu} + 1 \right) \frac{1}{n} \frac{\bar{y}_w}{y}. \tag{43} \]

If the level effect (43) is negative and dominates over the growth effect (42), i.e. \( \Delta < 0 \), then the labor market is deregulated: the productivity-adjusted wage \( \varpi \) falls until the full-employment \( \tilde{l}(\varpi_k, H) = L \) is attained [cf. (33)]. Otherwise, an increase in \( \varpi \) raises the welfare of the union lobby, creating incentives for labor market regulation. The level effect (43) is an increasing function (41) of the relative size \( \frac{1}{n} \) of a jurisdiction. Thus, it is the weaker, the more industries \( i \in B_k \) in jurisdiction \( k \) face competition from elsewhere.

8 Labor market integration

Without R&D, \( \lambda \to 0 \) [cf. (9)], there is no growth effect (42) and \( \Delta < 0 \) [cf. (40)]. From (39) it then follows that \( \tilde{l} = L \). In other words:
Proposition 1 The existence of R&D (i.e. \( \lambda > 0 \)) enables an equilibrium with labor market regulation and unemployment \( \overline{l} < L \).

Without R&D, all human capital is devoted to production. In that case, both lobbies attain their highest level of welfare in the presence of full employment, having no incentives to lobby for labor market regulation.

If \( \epsilon > \varphi \), then, from (20), (39) and (40), it follows that \( \tilde{z}_w \leq 0 \), \( \Delta < 0 \) and \( \overline{l} = L \). Thus, unemployment \( \overline{l} < L \) is possible only if \( \epsilon > \varphi \). Thus:

Proposition 2 Labor market regulation \( \overline{l} < L \) is possible only if the elasticity of product substitution is higher than that of factor substitution, \( \epsilon > \varphi \).

If the output effect dominates over the substitution effect (i.e. \( \epsilon > \varphi \)), then the growth effect is positive and can outweigh the level effect. Otherwise, a decrease in the productivity-adjusted wage \( \varpi \) benefits the lobbies and the political process ends up with labor market deregulation.

The mass of a jurisdiction is \( \frac{1}{n} \) [cf. (1)]. Given (39) and (41), \( \Delta < 0 \) and \( \overline{l} = L \) holds for low values and \( \Delta = 0 \) and \( \overline{l} < L \) for high values of \( \frac{1}{n} \). Thus:

Proposition 3 Assume that there exists a positive growth effect [i.e. \( \epsilon > \varphi \)]. In that case, the labor markets are deregulated (\( \overline{l} = L \)) for small and regulated (\( \overline{l} < L \)) for big jurisdictions.

If competition from outside the jurisdiction is weak (i.e. \( \frac{1}{n} \) close to one), then the growth effect (42) outweighs the level effect (43) and lobbying leads to labor market regulation. Otherwise, the labor markets are deregulated.

Differentiating the first-order condition \( \Delta = 0 \) [cf. (39)] and noting (39) and (41), one obtains

\[
\frac{d\varpi}{d(\frac{1}{n})} = - \left[ \frac{\partial \Delta}{\partial(\frac{1}{n})} \right] \frac{\partial \Delta}{\partial \varpi} > 0.
\]  

(44)

Given (11), (20), (26) and (38), the productivity growth rate becomes

\[
g = (\log a)\lambda \tilde{z}(\varpi, H), \quad \frac{dg}{d\varpi} = (\log a)\lambda \tilde{z}_w > 0.
\]

(45)

According to Proposition 2, inequality \( \epsilon > \varphi \) holds true for \( \overline{l} < L \). From this, (44) and (45) it follows that when \( \overline{l} < L \), both \( \varpi \) and \( g \) increase with an increase in \( \frac{1}{n} \). In other words:
Proposition 4  If the labor markets are initially regulated, \( l < L \), then labor market integration (i.e. an increase in the size \( \frac{1}{n} \) of jurisdictions) raises both the productivity-adjusted wage \( \varpi \) and the productivity growth rate \( g \).

If the labor markets are initially regulated, then the growth effect is positive. The expansion of jurisdictions weakens the negative level effect, for there will be less competition from outside the jurisdiction. This strengthens the incentives to lobby for labor market regulation, promoting R&D and growth.

## 9 Welfare considerations

The arrival rate of innovations of the leading-edge technology \( A \) [cf. (27)] follows a Poisson process being in fixed proportion \( \lambda \) to aggregate innovation intensity in the economy, \( \int_0^1 z_i di \) [cf. (9), (20), (26) and (38)]:

\[
\Lambda = \lambda \int_0^1 z_i di = \lambda \tilde{z}(\varpi, H). \tag{46}
\]

An invention of a new technology raises the serial number of the leading-edge technology, \( t \), by one and \( A(t) \) by constant \( a > 1 \) [cf. (10)]:

\[
A(t + 1) = a A(t), \quad a > 1. \tag{47}
\]

From (7), (26), (28) and (38) it follows that consumption \( c \) is a function of the productivity-adjusted wage \( \varpi \):

\[
c(\varpi) \equiv A^{\epsilon/(\epsilon - 1)} \tilde{y}(\varpi, H). \tag{48}
\]

This shows that the growth rate \( g^c \) of consumption \( c \) is in fixed proportion \( \epsilon/(\epsilon - 1) \) to the growth rate \( g \) of productivity \( A \). Given (6), (4) and (48), the expected welfare of the household at time \( T \) becomes

\[
U(\varpi, T) \equiv E \int_T^\infty [\log c(\varpi)] e^{-\rho(\theta - T)} d\theta
\]

\[
= E \int_T^\infty \left[ \log \tilde{y}(\varpi, H) + \frac{\epsilon \log A}{\epsilon - 1} \right] e^{-r(\theta - T)} d\theta. \tag{49}
\]

Let \( \varpi \) be the productivity-adjusted wage corresponding to full employment \( \tilde{I}(\varpi, H) = L \). Ignoring the full employment constraint \( \tilde{I}(\varpi, H) \leq L \)
for a while, I maximize welfare (49) subject to technological change (46) and (47) by the productivity-adjusted wage $\varpi$. In Appendix E, I show that the optimal value for the productivity-adjusted wage with unemployment is

$$\varpi^* = \arg \max_\varpi \left[ \log \bar{y}(\varpi, H) + \frac{\lambda \epsilon \log a}{(\epsilon - 1) r} \bar{z}(\varpi, H) \right].$$

(50)

If $\varpi^* > \varpi$, then, noting (44) and (45), there is an optimal size $\left(\frac{1}{n}\right)^*$ of a jurisdiction that corresponds to $\varpi\left(\left(\frac{1}{n}\right)^*\right) = \varpi^*$, and an optimal growth rate

$$g^* = (\log a) \lambda \bar{z}(\varpi^*, H).$$

Given (44) and (45), one obtains the following result:

**Proposition 5** If jurisdictions are smaller than $\left(\frac{1}{n}\right)^*$, then the growth rate is too low ($g < g^*$) and employment too high, and welfare can be improved by increasing relative union bargaining power $\varpi$ to $\varpi^*$. If they are larger than $\left(\frac{1}{n}\right)^*$, then the growth rate is too high ($g > g^*$) and employment too low, and welfare can be improved decreasing relative union bargaining power $\varpi$ to $\varpi^*$.

10 Conclusions

This document studies the political rationale for labor market (de)regulation. Firms are oligopolists who employ human capital (i.e. key workers) in production and R&D and (raw) labor in production. Human capital is fully employed, but the labor market can be regulated which causes unemployment. The main result is that the observed tendency to labor market deregulation comes from labor market policies being set up at the local level.

The political equilibrium is modeled as a common agency game where workers and firms lobby policy makers to shift labor market regulations in their favor. There are many jurisdictions, in each of them a self-interested policy maker can (de)regulate the local labor market. Human capital always fully employed and are the only one that can perform R&D production. The tension between employers and employees comes from the assumption that oligopolists perform R&D: without R&D, both workers and employers would like to have deregulated markets.

When markets get more regulated (i.e. wages raise), the oligopolists increase their output price and decrease their output so that there are less human capital in production (output effect). At the same time, the oligopolists
replace labor by human capital (substitution effect). If the elasticity of product substitution is higher than that of factor substitution, then the output effect dominates over the substitution effect: labor market regulation decreases human capital devoted to production. Because human capital is fully employed, more human capital is devoted to productivity-enhancing R&D. If this positive growth effect outweighs the negative effect of wage increases on income, then there are incentives to lobby for labor market regulation. Otherwise, the labor markets are deregulated.

When labor markets become more integrated (i.e. the size of jurisdictions increase), they face less competition from outside the jurisdiction so that the fall of income due to wage increases is reduced and labor market regulation gets more attractive to union lobbies. When labor markets are very little integrated (i.e. jurisdictions are small), they are deregulated.

Appendix

A Equations (11) and (12)

During a short time interval $d\theta$, oligopolist $i$ has an innovation $dq_i = 1$ with probability $\Lambda_i d\theta$, and no innovation $dq_i = 0$ with probability $1 - \Lambda_i d\theta$. Because technology changes from $t_i$ to $t_i + 1$ with probability $\Lambda_i d\theta$, and does not change with probability $1 - \Lambda_i d\theta$ during interval $d\theta$, then, given (9) and (10), one obtains (11):

$$g_i = \Lambda_i E[\log A_i(t_i + 1) - \log A_i(t_i)] = (\log a)\Lambda_i = (\log a)\lambda z_i.$$  

Define the expected value

$$\Omega(t_i) = E \int_T^\infty A_i(t_i) e^{-r(\theta - T)} d\theta. \tag{51}$$

The Bellman equation is (cf. Dixit and Pindyck 1994)

$$r\Omega(t_i) = A_i(t_i) + \Lambda_i \left[\Omega(t_i + 1) - \Omega(t_i)\right], \tag{52}$$

where $r\Omega(t_i)$ is the revenue from assets $\Omega(t_i)$ at the market interest rate $r$, $A_i(t_i)$ current income from assets $\Omega(t_i)$ and $\Lambda_i \left[\Omega(t_i + 1) - \Omega(t_i)\right]$ the expected
increase of the value of assets \( \Omega(t_i) \). Let us try the solution

\[
\Omega(t_i) = A_i(t_i)/\omega,
\]

in which the discount factor \( \omega > 0 \) is independent of \( t_i \). Inserting (53) into the Bellman equation (52) yields

\[
r = \frac{A_i(t_i)}{\Omega(t_i)} + \Lambda_i \left[ \frac{\Omega(t_i + 1)}{\Omega(t_i)} - 1 \right] = \omega + (a - 1)\Lambda_i.
\]

Solving for \( \omega \) from (54), inserting this into (53), noting (51) and (9) and denoting productivity \( A_i \) at time \( T \) by \( A_{iT} \), one obtains (12):

\[
E \int_T^\infty A_i(t_i) e^{-r(\theta-T)} d\theta = \Omega(t_i) = A_{iT} \frac{r}{r + (1-a)\Lambda_i} = A_{iT} r + (1-a)\lambda z_i.
\]

**B Functions (19) and (20)**

Given (18), the planning problem of oligopolist \( i \) takes the form

\[
(l_i, z_i) = \arg \max_{l_i,z_i} E \int_T^\infty \Pi_i e^{-r(\theta-T)} d\theta = \arg \max_{l_i,z_i} \frac{\pi_i}{r + (1-a)\lambda z_i} = \arg \max_{l_i,z_i} \Xi,
\]

where \( \Xi = \{ \log \pi_i - \log[r + (1-a)\lambda z_i] \} \). If oligopolist \( i \) has a unique equilibrium, then, noting (17), these are equivalent to the first-order conditions

\[
\frac{\partial \Xi}{\partial l_i} = \frac{1}{\pi_i} \frac{\partial \pi_i}{\partial l_i} = 0, \quad \frac{\partial \Xi}{\partial z_i} = \frac{(a - 1)\lambda}{r + (1-a)\lambda z_i} + \frac{1}{\pi_i} \frac{\partial \pi_i}{\partial z_i} = 0,
\]

and the second-order conditions

\[
\frac{\partial^2 \Xi}{\partial l_i^2} = \frac{1}{\pi_i} \frac{\partial^2 \pi_i}{\partial l_i^2} < 0, \quad \frac{\partial^2 \Xi}{\partial z_i^2} < 0, \quad J = \frac{\partial^2 \Xi}{\partial l_i \partial z_i} \left( \frac{\partial^2 \Xi}{\partial l_i \partial z_i} \right)^2 > 0.
\]

Furthermore, from \( h_i = H_i - z_i \), (13), (17) and (55) it follows that

\[
\frac{\partial^2 \Xi}{\partial l_i \partial w_i} = \frac{1}{\pi_i} \frac{\partial^2 \pi_i}{\partial l_i \partial w_i} - \frac{1}{\pi_i} < 0, \quad \frac{\partial^2 \Xi}{\partial z_i \partial w_i} = \frac{1}{\pi_i} \frac{\partial^2 \pi_i}{\partial z_i \partial w_i} = 0,
\]

\[
\frac{\partial \Xi}{\partial l_i} = \frac{1}{\pi_i} \frac{\partial \pi_i}{\partial l_i} = \left(1 - \frac{1}{\epsilon}\right) \frac{1}{\pi_i} \left[ F^{-1/\epsilon} F_i - \frac{F_i F_{H_i}}{F^{1/\epsilon}} \left( \frac{L}{F_{H_i}} - \frac{1}{F} \right) - \frac{\epsilon w_i}{\epsilon - 1} \right] = \left(1 - \frac{1}{\epsilon}\right) \frac{1}{\pi_i} \left[ F^{-1/\epsilon} F_i \left(1 + \frac{1}{\epsilon - \varphi} \right) H_i \frac{F_{h_i}}{F} - \frac{w_i}{\epsilon - 1} \right] = 0,
\]
Differentiating the production function (13) and noting these partial derivatives as follows:

\[ F^{-1/\epsilon} F_i \left[ 1 + \left( \frac{1}{\epsilon} - \frac{1}{\varphi} \right) H_i \frac{F_i}{F} \right] = \frac{\epsilon w_i}{\epsilon - 1} > 0, \]

\[
\frac{\partial^2 \Xi}{\partial l_i \partial z_i} = \left( 1 - \frac{1}{\epsilon} \right) \frac{1}{\pi_i} \left\{ F^{-1/\epsilon} F_i \left[ 1 + \left( \frac{1}{\epsilon} - \frac{1}{\varphi} \right) H_i \frac{F_i}{F} \right] \left( \frac{F_{hh} F_i}{F_h} - \frac{F_h}{F_h} \right) \right\} dl_i \frac{d}{dz_i} \]

\[ = \left( \frac{1}{\epsilon} - 1 \right) \frac{1}{\pi_i} \left\{ \frac{\epsilon w_i}{\epsilon - 1} \left( \frac{F_{hh} F_i}{F_h} - \frac{F_h}{F_h} \right) \right\} + F^{-1/\epsilon} F_i \left( \frac{1}{\varphi} - \frac{1}{\epsilon} \right) H_i \frac{F_i}{F} \left( \frac{F_h F_i}{F_h} - \frac{F_h}{F_h} \right) \]

\[ = \left( \frac{1}{\varphi} - \frac{1}{\epsilon} \right) \left( \frac{1}{\epsilon - 1} \right) \frac{1}{\pi_i} H_i \frac{F_i}{F} \left\{ \frac{\epsilon w_i}{\epsilon - 1} + H_i F_i F_h \left( \frac{F_h}{F} - \frac{F_{hh}}{F_h} \right) \right\} < 0 \]  

\[ \iff \frac{1}{\varphi} > \frac{1}{\epsilon} \iff \epsilon > \varphi. \quad (57) \]

Differentiating the first-order conditions (55) yields the matrix equation

\[
\begin{bmatrix}
\frac{\partial^2 \Xi}{\partial l_i \partial z_i} & \frac{\partial^2 \Xi}{\partial l_i \partial z^2} \\
\frac{\partial^2 \Xi}{\partial z_i \partial z_i} & \frac{\partial^2 \Xi}{\partial z_i ^2}
\end{bmatrix}
\begin{bmatrix}
dl_i \\
dz_i
\end{bmatrix}
+ \begin{bmatrix}
-\frac{1}{\pi_i} \\
0
\end{bmatrix}
dw_i = 0.
\]

Noting (56) and (57), this can be written as partial derivatives as follows:

\[ \frac{dl_i}{dw_i} = -\frac{1}{J} \begin{bmatrix}
-\frac{1}{\pi_i} \\
0
\end{bmatrix} \frac{\partial^2 \Xi}{\partial l_i \partial z_i} = \frac{1}{J} \begin{bmatrix}
0 \\
1
\end{bmatrix} \frac{\partial^2 \Xi}{\partial z_i ^2} < 0, \]

\[ \frac{dz_i}{dw_i} = -\frac{1}{J} \begin{bmatrix}
\frac{\partial^2 \Xi}{\partial l_i \partial z_i} \\
0
\end{bmatrix} = -\frac{1}{J} \begin{bmatrix}
0 \\
1
\end{bmatrix} \frac{\partial^2 \Xi}{\partial l_i \partial z_i} > 0 \iff \epsilon > \varphi. \]

Differentiating the production function (13) and noting these partial derivatives and \( h_i = H_i - z_i \), one obtains

\[ \frac{dy_i}{dw_i} = F_i \frac{dl_i}{dw_i} + F_h \frac{dh_i}{dw_i} = F_i \frac{dl_i}{dw_i} - F_h \frac{dz_i}{dw_i} < 0 \text{ for } \epsilon > \varphi. \]

21
Noting (1), (7), (20), (27) and (38), consumption is determined as follows:

\[
c(\varpi, -k, A, n) = \left[ \int_0^n \left( \int_{i \in B_k} A_i y_i^{1-1/\epsilon} \right) dk \right]^{\epsilon/(\epsilon - 1)}
\]

\[
= A^{\epsilon/(\epsilon - 1)} \left[ \int_0^n \left( \tilde{y}(\varpi, H_i)^{1-1/\epsilon} \int_{i \in B_k} dk \right) di \right]^{\epsilon/(\epsilon - 1)}
\]

\[
= \left( \frac{A}{n} \right)^{\epsilon/(\epsilon - 1)} \left[ \int_0^n \tilde{y}(\varpi, H_i)^{1-1/\epsilon} dk \right]^{\epsilon/(\epsilon - 1)},
\]

where \( i \in B_k \). This implies

\[
c|_{\varpi = \varpi} = \varpi \quad \text{and} \quad H_i = H \quad \text{for} \quad \ell \in [0, n] = A^{\epsilon/(\epsilon - 1)} \tilde{y}(\varpi, H),
\]

\[
\frac{1}{c} \frac{\partial c}{\partial \varpi_k} = \frac{\partial \log c}{\partial \varpi_k} = \frac{\epsilon}{\epsilon - 1} \frac{\partial}{\partial \varpi_k} \log \int_0^n \tilde{y}(\varpi, H_i)^{1-1/\epsilon} d\ell
\]

\[
= \frac{\tilde{y}(\varpi, H_i)^{-1/\epsilon} \tilde{y}_w(\varpi, H_i)}{\int_0^n \tilde{y}(\varpi, H_i)^{1-1/\epsilon} d\ell},
\]

\[
\left[ \frac{1}{c} \frac{\partial c}{\partial \varpi_k} \right]_{\varpi = \varpi} = \varpi \quad \text{and} \quad H_i = H \quad \text{for} \quad \ell \in [0, n] = \frac{\tilde{y}_w(\varpi, H)}{ny(\varpi, H)} < 0.
\]

D Results (39) and (40)

From (17), (19), (20) and (22) it follows that

\[
\mathcal{W}_o + \mathcal{W}_u = \left[ \max_{\pi_i} \frac{\pi_i}{r + (1 - a)\lambda z_i} + \frac{w_i \tilde{l}(w_i, H_i)}{r + (1 - a)\lambda z_i} \right] c^{1/\epsilon - 1} A_{IT}
\]

\[
- (R_o^i + R_u^i)/r,
\]

\[
\frac{\partial (\mathcal{W}_o + \mathcal{W}_u)}{\partial w_i} = \begin{cases} \frac{1}{r + (1 - a)\lambda z_i} \frac{\partial \pi_i}{\partial w_i} + \frac{\tilde{I} + w_i \tilde{l}_w}{r + (1 - a)\lambda z_i} \left( \frac{(a - 1)\lambda \tilde{z}_w w_i \tilde{l}}{r + (1 - a)\lambda z_i} \right) \right) \\
\times c^{1/\epsilon - 1} A_{IT} \end{cases}
\]

\[
= \left[ \frac{w_i \tilde{l}_w + (a - 1)\lambda \tilde{z}_w w_i \tilde{l}}{r + (1 - a)\lambda z_i} \right] c^{1/\epsilon - 1} A_{IT}
\]

\[
= \left[ \frac{\tilde{l}_w + (a - 1)\lambda \tilde{z}_w}{r + (1 - a)\lambda z_i} \right] w_i \tilde{l} c^{1/\epsilon - 1} A_{IT}.
\]

\[\text{(58)}\]
\[
\frac{\partial(W_o + W_u)}{\partial c} = \left(\frac{1}{\varepsilon} - 1\right) \frac{\pi_i + w \tilde{l}}{r + (1 - a)\lambda \bar{z}} e^{1/\varepsilon - 2} A_{IT}.
\] (59)

Noting (26), (28), (29), (30), (38), (58) and (59), one obtains

\[
\frac{\partial(F_k + U_k)}{\partial \omega_k} = \frac{\partial(W_o + W_u)}{\partial w_i} \frac{\partial w_i}{\partial \omega_k} + \frac{\partial(W_o + W_u)}{\partial c} \frac{\partial c}{\partial \omega_k}
\]

\[
= \frac{\tilde{l}_w}{l} + \frac{(a - 1)\lambda \bar{z}_w}{r + (1 - a)\lambda \bar{z}} \frac{w_i \tilde{l} e^{1/\varepsilon - 1} A_{IT}}{r + (1 - a)\lambda \bar{z}} + \left(\frac{1}{\varepsilon} - 1\right) \frac{(\pi_i + w \tilde{l}) e^{1/\varepsilon - 1} A_{IT}}{r + (1 - a)\lambda \bar{z}} \frac{\tilde{y}_w}{n \bar{y}}
\]

\[
= \Delta(\omega, n) \frac{\tilde{l} e^{1/\varepsilon - 1} A_{IT}}{r + (1 - a)\lambda \bar{z}},
\]

where \(\Delta(\omega, n) \equiv \frac{(a - 1)\lambda \bar{z}_w}{r + (1 - a)\lambda \bar{z}} + \frac{\tilde{l}_w}{l} + \left(\frac{1}{\varepsilon} - 1\right) \frac{(\pi_i + w \tilde{l})}{\tilde{l} e^{1/\varepsilon - 1} A_{IT}} \frac{\tilde{y}_w}{n \bar{y}}.\) (60)

From (32), (36) and (60), it follows that

\[
\frac{\partial G_k}{\partial \omega_k} = \frac{1}{m} \frac{\partial (R_{ko} + R_{ku})}{\partial \omega_k} = \frac{1}{m} \left( \frac{\partial F_k}{\partial \omega_k} + \frac{\partial U_k}{\partial \omega_k} \right) = \frac{\partial(F_k + U_k)}{\partial \omega_k}
\]

\[
= \Delta(\omega, n) \frac{\tilde{l} e^{1/\varepsilon - 1} A_{IT}}{r + (1 - a)\lambda \bar{z}}.
\] (61)

Conditions (34) are equivalent to the maximization of the Lagrangean

\[
\mathcal{L}_k = G_k(\omega_k, \omega_{-k}, n) + \xi_k[L - \tilde{l}(\omega_k)]
\]

by the wage \(\omega_k,\) where the multiplier \(\xi_k\) is subject to the conditions

\[
\xi_k[L - \tilde{l}(\omega_k)] = 0, \quad \xi_k \geq 0.
\] (62)

Given (20), (26), (38) and (61), this yields first-order and second-order conditions for the maximization:

\[
\frac{\partial \mathcal{L}_k}{\partial \omega_k} = \frac{\partial G_k}{\partial \omega_k} \Bigg|_{(26),(38)} - \xi_k \tilde{l}_w = \Delta \frac{\tilde{l} e^{1/\varepsilon - 1} A_{IT}/n}{r + (1 - a)\lambda \bar{z}} - \xi_k \tilde{l}_w = 0,
\]

\[
\frac{\partial \Delta}{\partial \omega_k} < 0 \iff \frac{\partial^2 G_k}{\partial \omega_k^2} < 0 \iff \xi_k = 0.
\] (63)

The conditions (62) and (63) are equivalent to

\[
\tilde{l} < L \iff \Delta = 0 \text{ with } \frac{\partial \Delta}{\partial \omega_k} < 0,
\]

\[
\tilde{l} = L \iff \xi > 0 \iff \Delta = \xi_k \tilde{l}_w \frac{r + (1 - a)\lambda \bar{z}}{\tilde{l} e^{1/\varepsilon - 1} A_{IT}} < 0.
\]

23
Define the value function
\[ \Phi(t) = \max_{\varpi} E \int_{t}^{\infty} \left[ \log \tilde{y}(\varpi, H) + \frac{\epsilon \log A(t)}{\epsilon - 1} \right] e^{-r(\theta - T)} d\theta. \] (64)

If there is a stationary state solution in which the productivity-adjusted wage \( \varpi \) is constant, then from (46), (47) and (64) it follows that
\[ \Phi(t + 1) - \Phi(t) = \frac{\epsilon}{\epsilon - 1} \int_{T}^{\infty} [\log A(t + 1) - \log A(t)] e^{-r(\theta - T)} d\theta \]
\[ = \frac{\epsilon \log a}{\epsilon - 1} \int_{T}^{\infty} e^{-r(\theta - T)} d\theta = \frac{1}{r} \frac{\epsilon \log a}{\epsilon - 1}. \] (65)

Noting (64) and (65), one can define the Bellman equation for the maximization (cf. Dixit and Pindyck 1994):
\[ r \Phi(t) = \max_{\varpi} \left\{ \log \tilde{y}(\varpi, H) + \frac{\epsilon \log A(t)}{\epsilon - 1} + \Lambda [\Phi(t + 1) - \Phi(t)] \right\} \]
\[ = \max_{\varpi} \left[ \log \tilde{y}(\varpi, H) + \frac{\epsilon \log A(t)}{\epsilon - 1} + \frac{\lambda \epsilon \log a}{\epsilon - 1} \right] \]
\[ = \max_{\varpi} \left[ \log \tilde{y}(\varpi, H) + \frac{\epsilon \log A(t)}{\epsilon - 1} + \frac{\lambda \epsilon \log a}{(\epsilon - 1)r} \tilde{z}(\varpi, H) \right]. \]

The solution for the value function is then given by
\[ \Phi(t) = \frac{1}{r} \max_{\varpi} \left[ \log \tilde{y}(\varpi, H) + \frac{\epsilon \log A(t)}{\epsilon - 1} + \frac{\lambda \epsilon \log a}{(\epsilon - 1)r} \tilde{z}(\varpi, H) \right]. \]

The optimal value for the productivity-adjusted wage is
\[ \varpi^* = \arg \max_{\varpi} \left[ \log \tilde{y}(\varpi, H) + \frac{\epsilon \log A(t)}{\epsilon - 1} + \frac{\lambda \epsilon \log a}{(\epsilon - 1)r} \tilde{z}(\varpi, H) \right] \]
\[ = \arg \max_{\varpi} \left[ \log \tilde{y}(\varpi, H) + \frac{\lambda \epsilon \log a}{(\epsilon - 1)r} \tilde{z}(\varpi, H) \right]. \]
References:


